Monday, 18th January 1858.

THE RIGHT REV. BISHOP TERROT, V.P., in the Chair.

The following Communications were read :—

1. On the Mechanism of the Knee Joint. By Professor Goodsir.

After alluding to the comparatively superficial manner in which physiologists, with the exception of the brothers Weber, have hitherto investigated the structure and movements of the joints, the author gave an abstract of the general results which he had formerly obtained in an examination of the knee-joint, made with reference to Meyer's valuable observations. He had found that, as stated by Meyer, the thigh and leg rotate on one another in opposite directions,—at the close of extension, and at the commencement of flexion; and that the co-ordinated movements in the patella, the ligaments, and muscles correspond generally with the account given by that observer; but in addition he had ascertained what had previously escaped notice,

1. That the articular surfaces of the femur, tibia, and patella are not continuous but faceted surfaces.

2. That in consequence of this faceted configuration, and the peculiar manner in which the opposite articular surfaces move on one another, they are in no position of the joint congruent throughout, but gape more or less in different parts of their extent.

3. That in addition to their lubricating function, the so-called Haversian glands, or fatty folds of the synovial membrane, are arranged with reference to the resulting gaps or chinks between the opposite articular surfaces, each gap, as it opens out, being simultaneously occupied by the fatty synovial pad provided for it, and which is forced or dragged into the chink, and pulled or forced out again by special arrangements.

The author next proceeded to state, as introductory to the mechanism of the knee-joint, the results which he had latterly obtained in his examination of other diarthrodial articulations.

1. All diarthrodial surfaces are faceted, and consist of areas of distinct configuration and movement.

2. These facets and areas are marginal or terminal, and central or acting—the former giving steadiness to the action of the joint, and supplying surface on which it rests securely at the opposite ex-

tremities of its movements—the latter more especially regulating the movements themselves, and presenting the greatest extent of surface, which again consists of a moiety for each half of the movement, the one portion breaking contact, while the other is acting, and *vice versa*.

3. Even the acting facets of opposite articular surfaces are only congruent at one particular stage of their movement.

4. The movements of opposite diarthrodial surfaces upon one another appear to be in every instance a combination of gliding and rolling—the amount of the former being directly, and that of the latter inversely, as the congruence of the opposite articular surfaces.

Referring to the important simultaneous discovery recently made by Langer and Henke, and verified by Meissner, of the screwed structure of certain joints, the author proceeded to state, that he would in a future communication on the ankle and tarsal joints give the grounds on which he had come to the conclusion,

1. That in all the joints hitherto examined the screw is developed on a conical surface, and not on a cylindrical one, as is held by Langer to be generally the case.

2. That not only is it impossible accurately to prolong the screwed surface by uniting longitudinally a number of casts made from it, but that neither the original surface nor its cast admits of being screwed along the mould, with continued congruity of surface.

3. That this incongruity depends, in the first place, on the screwed surface being conical, and on the rapid increase in the obliquity of the thread ; and, in the second, on its consisting of at least two areas, each being a portion of a conical screw.

After exhibiting prolonged screws, made according to Langer's method, from the upper articular surface of the astragalus in the horse, panther, lion, and human subject, the author proceeded to state, that, induced to re-examine the knee-joint from this fresh point of view, he had ascertained, in the first place, that the path described by any point in the thigh, when the leg is fixed, and the knee put through its movements, does not lie in the presumed plane of flexion and extension, as it would do if the profile curvatures of the femoral condyles were circular arcs, or logarithmic spirals, according to the ordinary view, or that of the brothers Weber ; neither does the point in the upper part of its course describe the arc of a circle in a plane oblique to that in which it must afterwards move, if Meyer's

observations be absolutely correct, but on the contrary describes a helix, consisting of at least two parts, an upper and a lower. This observation led the author to the detection of two screw combinations in the knee-joint; and by a careful study of the anatomical relations of the elements of the articulation he came to the following conclusions,

1. The knee-joint consists essentially of two conical screw combinations.

2. One of these screw combinations forms the anterior, the other the posterior part of the joint.

3. The axes of these screw combinations, instead of being at right angles to the so-called plane of flexion and extension, as in the ankle and elbow joints, are parallel, or nearly so, to the axis of the limb, the vertices of the fundamental cones being directed upwards.

4. The femoral condyles form the concave; the tibial condyles, intercondyloid spine, and crucial ligaments, the convex elements of each screw combination.

5. Each of these screw combinations is double-threaded; the breadth and obliquity of the threads rapidly increasing from vertex to base.

6. A comparatively limited extent of the convex element of each combination is retained, so that the larger extent of the concave element employed moves on the former by a combination of gliding in the direction of the screw, and of rolling.

7. The gliding in the direction of the screw is due partly to the screwed configuration of the opposite cartilaginous surfaces; partly to the peculiar mode of attachment of the crucial ligaments.

8. In consequence of the peculiar attachments of the successive fasciculi of the crucial ligaments, these fasciculi, after having in succession co-operated in producing the gliding movement in the direction of the screw, bend over, and thus permit the rolling movement.

9. The path described by any point in the thigh or leg during flexion or extension of the knee-joint is a helix, produced by the movements of the two screw combinations in succession: but modified by the rolling.

10. The anterior screw combination is left-handed in the right knee, and right-handed in the left.

11. The posterior screw combination is right-handed in the left knee, and left-handed in the right.

12. The two screw combinations in each knee are united, so that the anterior half of the anterior combination, and the posterior half of the posterior, are alone retained ; while the external femoral and tibial condyles respectively consist of the united basal portions of one of the threads in each combination ; and the inner condyloid surfaces respectively of portions of the other thread in each, but consequently towards the vertices of the fundamental cones.

13. When the knee-joint is fully extended, its anterior screw combination is screwed home, and its posterior is unscrewed ; when it is completely flexed the anterior combination is unscrewed, and the posterior screwed home.

2. On the Exhibition of both Roots of a Quadratic Equation by one Series of Converging Fractions. By Edward Sang, Esq.

It had been long known that every periodic continued fraction expresses the root of a quadratic equation, and in 1808 M. Lagrange demonstrated the converse proposition, that the roots of every numerical equation of the second degree may be expressed by such fractions. The subject has since been examined by M. Legendre in his “*Theorie des Nombres*,” and also by Barlow, and the laws discovered have been applied to the resolution of certain classes of diophantine problems of the second order.

The writers on this subject have considered the two roots of such equations separately, and have regarded the two series of fractions which converge to them as distinct. In the development of these fractions the quotients become periodic after one or more terms have been found ; resembling in this way the digits of a recurring decimal fraction of which some of the earlier terms are not recurrent. The object of this notice is to show that these two series form in reality parts of one general series, the multipliers of which are periodic throughout. To this particular form of series it is proposed to give the name *duserr*, from the Persian *دو سر* *two heads*, or two leaders.

The character of the *duserr* progression was exemplified by considering the roots of the equation—

$$5x^2 - 32xy + 31y^2 = 0.$$

On approximating to these roots by the known process, the quotients

$$1, 5; 3, 1, 4; 3, 1, 4; \&c.$$

$$\text{and } 5; 4; 1, 3; 4, 1, 3; \&c.,$$

are found, whence the two progressions—

$$\begin{array}{r} 1 \quad 5; 3 \quad 1 \quad 4; 3 \\ \hline 1 \quad 1 \quad 6 \quad 19 \quad 25 \quad 119 \quad 382 \\ \hline 0 \quad 1 \quad 5 \quad 16 \quad 21 \quad 100 \quad 321 \quad \&c. \end{array}$$

$$\begin{array}{r} 5; 4 \quad 1 \quad 3; 4 \quad 1 \\ \hline 1 \quad 5 \quad 21 \quad 26 \quad 99 \quad 422 \quad 521 \\ \hline 0 \quad 1 \quad 4 \quad 5 \quad 19 \quad 81 \quad 100 \quad \&c. \end{array}$$

If, instead of proceeding forwards in either of these progressions, we compute backwards, using only the recurring quotients, we produce the other progression, thus :—

$$\begin{array}{r} 3 \quad 1 \quad 4 \quad 3 \quad 1 \quad 4 \quad 3 \quad 1 \quad 4 \\ \hline -99 \quad 26 \quad -21 \quad 5 \quad -1 \quad 2 \quad 1 \quad 6 \quad 19 \quad 25 \quad 119 \\ \hline \&c. \quad -19 \quad 5 \quad -4 \quad 1 \quad 0 \quad 1 \quad 1 \quad 5 \quad 16 \quad 21 \quad 100 \quad \&c. \end{array}$$

The case of the roots being on different sides of the zero, was exemplified by means of the equation

$$7x^2 - 8xy - 102y^2 = 0$$

which are contained in the two-headed progression—

$$\begin{array}{r} 7 \quad 2 \quad 3 \quad 7 \quad 2 \quad 3 \quad 7 \\ \hline 171 \quad -23 \quad 10 \quad -3 \quad 1 \quad 4 \quad 9 \quad 31 \quad 226 \\ \hline \&c. \quad -52 \quad 7 \quad -3 \quad 1 \quad 0 \quad 1 \quad 2 \quad 7 \quad 51 \quad \&c., \end{array}$$

and that case in which the roots lie equally on either side of the zero by the equation

$$10x^2 - 53y^2 = 0$$

the roots of which are contained in

$$\begin{array}{r} 4 \quad 3 \quad 3 \quad 4 \quad 3 \quad 3 \quad 4 \\ \hline 99 \quad -23 \quad 7 \quad -2 \quad 1 \quad 2 \quad 7 \quad 23 \quad 99 \\ \hline \&c. \quad -43 \quad 10 \quad -3 \quad 1 \quad 0 \quad 1 \quad 3 \quad 10 \quad 43 \quad \&c. \end{array}$$

It was remarked that the progressions indicate certain peculiarities in these continued fractions. First, that the order of recur-

rence in the quotients obtained for one root is inverse of that for the other root of the quadratic; second, that in continued fractions which express the square root of any rational fraction, the order of recurrence is symmetric within each period; and third, that the last quotient of the period must be double of the first or integer part of the root.

It was also observed, that the progressions may be separated into as many progressions as there are members in the period, each progression proceeding as with a single repetend. The consideration of this branch of the subject was said to lead to the formation of series representing the impossibility of the roots when the square of the middle coefficient is less than four times the product of the extreme ones, and when these have the same sign.

The following Gentleman was duly elected a Fellow of the Society:—

FREDERICK FIELD, Esq., Chili.

The following Donations to the Library were announced:—

Scheikundige Verhandelingen en Onderzoekingen uitgegeven door G. J. Mulder. Eerste deel, derde stuk. Het Bier scheikundig beschouwd door G. J. Mulder. Rotterdam, 1857. 8vo.
—*From the Author.*

Flora Batava, 182 Aflevering. Amsterdam. 4to.—*From the King of Holland.*

A Catalogue of 3735 Circumpolar Stars, observed at Redhill in the years 1854, 1855, and 1856, and reduced to mean positions for 1855.0. By Richard Christopher Carrington. London, 1857. Folio.—*From the Lords of Admiralty.*

Denkschriften der Kaiserlichen Akademie der Wissenschaften. Mathematisch-Naturwissenschaftliche Classe, Zehnter und elfter bände.—*From the Vienna Academy.*

Annales de l'Observatoire Physique Central de Russie. Par A. T. Kupffer. Année 1854. St Petersburg, 1856. 4to.—*From the Observatory.*

Bulletin de la Société Vaudoise des Sciences Naturelles. Tome v., No. 41. Lausanne, 1857. 8vo.—*From the Society.*

Proceedings of the Natural History Society of Dublin, for the Session 1856–57. Dublin, 1857. 8vo.—*From the Society.*

The Assurance Magazine and Journal of the Institute of Actuaries. Vol. VIII., Part 4. January 1858. 8vo.—*From the Institute of Actuaries.*

Isothermal and Rain Charts, illustrating the Climatology of the United States, and of the Temperate Latitudes of the North American Continent. By Lorin Blodget. Philadelphia, 1857. Folio.—*From Professor Henry D. Rogers.*

Monday, 1st February 1858.

SIR DAVID BREWSTER, V.P., in the Chair.

The following Communications were read :—

1. On the Form and Origin of the Symbols on the Ancient Sculptured Stones of Scotland. By Dr Wise.

In continuation of his former essay on the sculptured stones peculiar to the north-eastern portion of Scotland, the author produced, for the inspection of the Society, two specimens of the Buddhist *dorje*, which he considered to be the prototype of the well-known *spectacle-ornament*, found on no fewer than thirty-four of those stones ; and, in twenty-nine instances, in combination with zig-zag lines, the symbol of power. The *dorje* is a portable instrument, which represents the two original members of the Buddhist Triad, spirit and matter, and is employed by the Buddhist priests of Tartary, &c., in the performance of religious worship.

The *dorjes* which were exhibited consisted each of two hollow-ribbed spheres, or ovals of brass, united by a handle, two inches long, of the same metal. When represented on the black board, with its axes or poles towards the spectator, one of the *dorjes* resembled exactly the spectacle-ornament on the larger Aberlemno stone in Forfarshire.

The *dorje* is considered indispensable to the Buddhist priest, during certain religious ceremonies. It is held over the head of the kneeling penitent in the act of absolution ; and a sacred bell (likewise exhibited), having at the extremity of its handle a ball similar to those of the *dorje*, is rung. In some temples direct worship is paid to a *drje* kept for that purpose.