Civil and Mechanical Engineering.

GRAPHICAL AND ANALYTICAL DETERMINATION OF STRAINS IN A ROOF TRUSS

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The present article is written in compliance with a request made to the author, to present in as popular a manner as possible the different methods of finding the strains in the various pieces of a roof truss of frequent occurrence in practice. The example here worked out accompanied the above request, and although perhaps not exactly calculated to show to the best advantage the peculiar merits of a graphical solution, will, we trust, be sufficient to enable the reader unacquainted with the method, to extend it to other and more irregular forms of trusses.

We shall first find the strains in the various pieces of the frame, due to dead load, by diagram; then indicate the methods of calculation, thus checking the results of diagram; and finally find the strains due to *wind force*, for which roof trusses are too rarely investigated, and for the determination of which the graphic method is peculiarly suited.

I. OUTER FORCES.—The truss proposed for solution is represented in Fig. 1. The span is 50 ft.; height of truss 12·5 ft.; diagonal bracing as shown in the Fig.; bottom tie horizontal without camber. The skeleton of the truss, drawn carefully to scale according to the given dimensions, we call the *frame diagram*. We have given it in Fig. 1, to a scale of 12 ft. to an inch for convenience of publication merely. In the actual determination of the strains which we shall give, it was drawn to a scale of 5 ft. to an inch. Now the *frame diagram* being thus carefully drawn to a suitable scale (in general the larger the better), we have first to determine the *outer forces* which

act upon the truss. In the example proposed each truss similar to Fig. 1, supported 5 ft. in length of roofing, and the total weight of this section of 5 ft., including slate, truss, roofing, etc., was 15 lbs. per sq. ft. Snow was taken at 5 lbs. per sq. ft. We have then for the length of each rafter, $\sqrt{25^2 + 12 \cdot 5^2} = \sqrt{625 + 156 \cdot 25} = \sqrt{781 \cdot 25} = 27 \cdot 95$ ft. or 28 ft. The total area of roof, therefore, for each section of 5 ft. will be $28 \times 5 \times 2 = 280$ sq. ft. At 15 lbs per sq. ft. this will give 4200 lbs. For snow, we have in addition $280 \times 5 = 1400$. Hence, total maximum load including snow, is $4200 + 1400 = 5600$ lbs. Now this load we may consider as divided into a series of loads at each apex; *omitting the two ends*, at which strictly we have 400 lbs. apiece. The strain in the rafter is in reality cumulative, increasing towards the ends. By the above method we find this maximum strain at the end correctly, and then consider the cross-section uniform to next apex, thus getting somewhat more than the area necessary. At each of the seven apices, then, omitting the ends, we have a vertical weight of 800 lbs., as represented in Fig. 1 by the vertical arrows. At the ends we have a vertical upward reaction of half the total weight, or 2800 lbs. Thus all the *outer forces* which act upon the truss are known, and it remains to find the strains caused by these forces in the various pieces. This we shall do in two ways—1st, by diagram; 2d, by calculation.

I. GRAPHIC SOLUTION.

II. NOTATION.—Let us first explain the notation which we have adopted for Fig. 1. Let all the space above the truss be denoted by the letter A, and all below it by the letter B, and then number the spaces within the truss by 1, 2, 3, 4, 5, 6, etc. Then the four pieces composing the rafter are A 1, A 2, A 4 and A 6; the three divisions of the tie are B 1, B 3, B 5, while the braces are 1 2, 2 3, 3 4, 4 5, 5 6, and 6 6', respectively; corresponding spaces in the right half being denoted by primes. The great convenience of this system of notation will soon appear. [For the above elegant system of notation we are indebted to *R. H. Bow's "Economics of Construction,"* where the reader will find the following method of graphic solution very clearly set forth, and illustrated by numerous examples.] The preceding system of notation being understood, we can now proceed to the principles of which we shall make use, in the solution of the problem before us.

III. GENERAL PRINCIPLES.—The principles of which we make use in this and all similar problems, are very simple and familiar to all. They are indeed among the most elementary in mechanics. The problem proposed is essentially a *statical* problem. That is the end to be obtained is rest or *equilibrium*. Thus at each apex, as we have seen, we have an outer force acting. The tendency of this force is, of course, to cause motion of the material point upon which it acts. But the force at each apex causes strains in the pieces which meet at that apex, and these strains must be so great and must act in such direction, that they exactly oppose and neutralize the action of the outer force. That is, *at each apex the strains in all the pieces which meet at that apex, are in equilibrium with the outer force which acts upon that apex.*

Now the principles of which we make use, and which here directly apply, are as follows :

1st. *If any number of forces, all in the same plane, are in equilibrium, the polygon formed by drawing lines parallel to the directions of these forces, and equal by scale to their intensities, must close.*

It will be seen at once, that, according to this principle, if we have only three forces acting upon a point and holding that point in equilibrium, and if *one* of these forces is known; then, if we draw a line parallel to the known force, and make it equal by scale to the known intensity of that force, the other two forces can be easily and directly determined *to the same scale*, by lines respectively parallel, which are prolonged till they intersect and thus close the polygon, which in this case is a triangle. Generally, no matter how many forces act upon the point in question, if they are in equilibrium, and if we know all but two of them, we have only to lay off to scale the known forces in the proper directions, and then “close” the polygon thus commenced by lines parallel to the known directions of the two unknown forces. We thus find to scale the intensities of these forces necessary and sufficient for equilibrium. If more than two forces are unknown, the problem is in general indeterminate.

Corresponding to this method of diagram, we have a method of *calculation*, which consists simply in solving trigonometrically the triangles and polygons thus obtained.

2d. *If any number of forces acting upon a point are in equilibrium, and if we take any point in the common plane of the forces as a centre of moments, then the tendency to rotation about this point is*

zero, or the sum of the moments of all the forces tending to cause rotation in one direction is exactly equal and opposite to that tending to cause rotation in the opposite direction.

The principle is self evident if there is to be equilibrium or no motion, and upon it we may base a perfectly general method both of diagram and calculation. It will thus be seen, that by these two principles we have *four* different and distinct methods of solution; two by diagram and two by calculation. The first method of diagram which we shall here make use of, is known very generally as *Prof. Clerk Maxwell's* method, the corresponding method of calculation may be called the method by "composition and resolution of forces."

The second principle gives rise to a method of calculation which is simple, easily acquired and applied, and perfectly general, provided all the *outer forces* are known. This method we may call the *method of moments* or "*Ritter's method.*" Prof. August Ritter in his work entitled *Dach und Brücken Constructionen*, has shown so exhaustively the generality of the method, and applied it in such detail to so many forms of construction, that although the principle is itself as "old as the hills" it well deserves to be thus known by his name.

Now, corresponding to this general method of moments, we have a *method of diagram* equally general, which lies properly at the basis of the new science of "Graphical Statics," and which, for similar reasons, we may call "*Culmann's method*"—in compliment to the man who, more than any other, has done most for its perfection and elucidation. This second method of diagram we shall not touch upon here, but refer the reader for an exposition of *all* these methods, together with their dependent principles, illustrated by numerous examples, to the author's work upon the "*Elements of Graphical Statics,*" etc.; John Wiley & Son. We shall, however, in what follows, make use of the first method of diagram above, as also of its correlative method of calculation, and shall also notice briefly the method of calculation by moments, which rests upon the second principle above enunciated. Although we shall here illustrate these principles by a single concrete example, the reader will, we trust, find no difficulty in applying them to *any* example which can occur in practice. Even in bridges, if he will thus find the strains in every piece due to each *separate* apex load considered by itself, and then form a table showing the strains thus found for each load, he can easily and unerringly pick out the **maximum strain** which, under any possible combination of

loads, can occur in any given piece. [For an exposition of this method, see also the work of the author above cited.] Thus for every framed structure, of whatever kind, we have a simple and general method, and the real solution of the problem in any case is in reality confined to finding the *outer forces* which act upon the structure. These in roof trusses, simple bridge girders, etc., are easily found. The weights are always known, the reactions at the ends easily found by the "law of the lever." For continuous girders or girders continuous over more than two supports, for braced arches, etc., these outer forces, more especially the "reactions," are not so easily found. Once known, the solution is easy. The whole problem of solution of framed structures is thus reduced to finding the *outer forces*. In all ordinary engineering structures these are easily found, and hence, no difficulty is experienced. For those structures of less frequent occurrence, already noticed, the determination of the outer forces for *each separate apex load considered by itself*, is by no means so difficult as has generally been supposed. This, as we have seen, is, however, all that we need; and for such methods of determination, we refer again to the above work of the author.

IV. APPLICATION OF THE ABOVE GENERAL PRINCIPLES.—Let us now proceed to apply the general principles of Art. III, to the case in hand. We take up, therefore,

1st. *Maxwell's Method of Diagram*—In Fig. 1, we have acting at the toe of the rafter, an upward reaction of 2800 lbs. This upward reaction causes strains in the pieces A 1 and B 1 (see Art. II for notation). According to our first principle then (Art. III), the strains in A 1 and B 1 must, with the reaction of 2800 lbs., form a system in equilibrium; and hence, the polygon formed by parallel lines must *close*. This polygon we call the *strain polygon*. Thus in Fig. 2, we lay off the distance A B, equal to 2800 lbs., by scale. In the figure we have taken 3200 lbs. to an inch. In the actual example we took 800 lbs. per inch, but have reduced the original diagram, for convenience of size. A B, then, is by scale 2800 lbs. We draw, then, A 1 in Fig. 2 parallel to A 1 in Fig. 1, and B 1 parallel to B 1, and produce these lines to intersection 1. The polygon is then closed, and A 1 and B 1 measured off to scale, will give at once the strains in the corresponding pieces.

We have now to call attention to a *very* important point. We have thus far the intensity of the strains in A 1 and B 1, but have yet to

find whether these strains are compressive or tensile. For this we have the following rule:

Follow round the strain polygon, in the direction of the known forces. We thus obtain certain directions for the unknown forces. Refer these directions to the apex at which the forces considered are supposed to act; a direction away from this apex is tension—towards, compression.

Thus, in the above case, we *know* that A B acts up. We follow round then, with this to guide us, from B to A; and then continuing round from A to 1, and 1 to B again. Now, looking back to the frame, we find A 1 acting towards the foot of rafter, which is the apex now considered, therefore compression; while 1 B acts away, therefore tension. The amount of compression or tension is indicated by the length to scale of the corresponding lines.

Now, having found A 1 a certain number of pounds compression, as shown in Fig. 2, we can find 1 2 and A 2; because at the apex, where these forces meet, we know two forces acting there, viz.: A 1 already found, and A A = 800 pounds given, and there are only two others unknown, viz.: 1 2 and A 2. A 1 and A A are shown in Fig. 2, already drawn to scale. A 2 and 1 2 closing the polygon commenced by these two forces, give at once to the same scale the strains in the corresponding pieces. Again as to the *quality* of the strains, observe that A A the weight, acts *down*. We pass round then, with this to help us, from A to 2 and 2 to 1, and so around. Then referring to the apex *at which all these forces act*, we have A 2 towards, therefore compression, and 1 2 ditto, and if we should still go round, A 1 also towards the new apex, or compression, as before found, thus checking our previous result.

Now we can pass to the first apex on the lower tie, because we know already B 1 and 1 2, and have only to find two unknown strains, 2 3 and B 3. We draw then in Fig. 2, 2 3 to intersection with B 1. We have then B 1, 1 2, 2 3 and B 3, closing the polygon. B 1 has already been found to be tension. With reference to the apex we are *now* considering, therefore, it acts *away*; we follow round then from B to 1, then 1 to 2, and then 2 to 3 and 3 to B again, and thus find by reference to the frame, 1 2 compression as already found, 2 3 tension, and 3 B tension. Now knowing A 2, 2 3, and the weight 800 pounds = A A, we can find A 4 and 3 4 as before, both compression. So we pass through the frame. Next time with B 3 and 3 4 known, we

find 4 5 and B 5 ; then with A 4, 4 5 and the weight A A all known, find A 5 and 5 6. Now knowing A 6 and the weight A A = 800 pounds at summit, we find A 6' and 6 6', the one compression, the other tension. Observe that as we proceed, now finding the strains in the right hand half of the truss, we *check* each time our previous results. The diagram thus checks its own accuracy. The points 6 and 6', 4 and 4', 2 and 2', are vertically over each other. If at any point we have committed an error, we can thus detect and remedy it. Observe, also, that while all the seven weights A A = 800 pounds, act downwards, the two reactions A B and B A act upwards, and just come back to the point of beginning, thus closing the "polygon," which in this case is a straight line, in accordance with our first principle, Art. III. In the case of wind force, or inclined reactions, this polygon is a true polygon instead of a straight line, as we shall see hereafter. This polygon we call the *force polygon*. We have thus the *frame diagram*, the *strain diagram*, and the *force polygon*. In every case, since the reactions are called forth by the outer forces, this latter must close. Unless it does close, we have not found the outer forces correctly, and cannot proceed to form the strain diagram. The strain diagram being completed as above, and checking its own accuracy, we can scale off with the dividers the strains in every piece. Representing compression by + and tension by — we thus find, for the lower flanges: B 1 = — 5624, B 3 = — 4832, B 5 = — 4024; for the upper flanges: A 1 = + 6280, A 2 = + 5816, A 4 = + 4712, A 6 = + 3584, and for the diagonals: 1 2 = + 720, 2 3 = — 720, 3 4 = + 1048, 4 5 = — 928, 5 6 = + 1472, 6 6' = — 2410. As the liability to error of the author working rapidly does not exceed $\frac{3}{100}$ ths of an inch, these strains may be depended upon within 2 4 or 2 5 pounds. In the Fig. 2, compression is represented by heavy lines. The checking of both halves of Fig. 2 gives us assurance of the correctness of our results.

2d. *Corresponding Method of Calculation*.—We can still check our results by calculation. We have only to solve trigonometrically the triangles of Fig. 2. This from the known weights, reactions and proportions of the frame, we can easily do. The *quality* of the strains, whether tension or compression, is easily determined as before. Thus, Fig. 2 comprises a set of *rules*, by which we may calculate the strains directly, without tedious and elaborate reasoning as to the way in which strains are "resolved," etc.

For instance, from Fig. 2, we see at once that $AB \times \sec B A 1 =$ strain in A 1, and $AB \times \tan B A 1 =$ strain in B 1. Now $\sec. B A 1 = \frac{27.95}{12.5}$ and $AB = 2800$, therefore str. in A 1 $= 2800 \times \frac{27.95}{12.5} = 6260$, against 6280 as found by diagram. Again, $\tan B A 1 = \frac{25}{12.5} = 2$, hence strain in B 1 $= 2800 \times 2 = 5600$ lbs., against 5624 as found by diagram. Again, directly from Fig. 2, we have, strain in 12 $= (AA \times \cos A 1 B) \times \sec.$ of the angle which 1 2 makes with the perpendicular to A 1. For the secant of this angle we easily find 1.008. Now, $AA = 800$ and $\cos A 1 B = \frac{25}{27.95}$ hence strain in 1 2 $= \left(\frac{25}{27.95} \times 800 \right) 1.008 = 721$ lbs., against 720 found by diagram.

Again from the Fig. $A 1 - AA \sin A 1 B - [AA \cos A 1 B \times \tan \text{ of angle of } 1\ 2 \text{ with perp. to } A 1] = A 2$. But A 1 we have already found 6260, $AA = 800$, $\sin A 1 B = \frac{12.5}{27.95}$ $\cos A 1 B = \frac{25}{27.95}$ and for the tan. above, 0.126, hence $A 2 = 6260 - 357.8 - 90.1 = 5812$ lbs., against 5816 as found by diagram. So we can go through the whole system of triangulation shown in Fig. 2 and check our results. It is advisable to use logarithms, instead of natural sines, etc., as above.

The above methods are of course perfectly general and apply to *any* form of roof truss. In the case, however, of a curved roof truss, the method of diagram is always equally simple and ready of application, while the corresponding method of calculation is often tedious and involves much intricate trigonometrical computation. In fact, just here is the great advantage of the graphic method, that while perfectly general, it is unaffected by the irregularity of the frame. One case is no more difficult than another.

3d. *Method of Calculation by Moments.*—We have still to consider the method of calculation based upon our 2d principle, Art. III. For the method of diagram corresponding to it, we must refer to our work upon "*Graphical Statics*," already quoted, as its development would far exceed the limits of the present article. We may only remark here that upon it has been founded an entire system of statics, not less elegant and far more general than the above method of dia-

gram. Thus it will be observed that the method of diagram already noticed, applies only to *framed* structures or structures in which the strains must follow certain *definite directions*; the latter method, on the contrary, applies as well to solid structures also. For the application of the first method all the outer forces must first be known; by the second, these forces may themselves be found. Though some few of the principles of this method have long been known and applied, notably by *Mery* (*Ann. d. ponts et chauss.*, 1840, *sem.* 1 p. 50) to the stone arch, yet its thorough development and the deduction of a regular system applicable to all statical problems, of which the arch is but one of many diverse problems, is due entirely to *Culmann*, ("Die Graphische Statik," 1866), and by his name it may well be called. The corresponding method of *calculation* owes, as already stated, much of its development to *Ritter*,* and to that we shall now proceed.

Thus, according to our 2d principle, Art. III, if we suppose a section made entirely through the truss, cutting all the pieces which it meets, then for equilibrium the algebraic sum of the moments of the strains in these pieces with reference to any point, must be balanced by the algebraic sum of the moments of all the outer forces acting upon the section of the truss right or left of the points of division, with reference to this same point. Now we know all the outer forces. Since our centre of moments may be *anywhere* in the plane, we have only to assume it at the *common point of intersection of all the cut pieces except one*. The moments of the strains in these pieces will then be zero, and we have only the moment of one unknown strain balanced by known moments. If now we know the lever arm of this unknown strain, it is itself easily found. Thus suppose in Fig. 1, we were to take a section cutting the pieces A 2, 2 3 and B 3. If we take the centre of moments at the common intersection of 2 3 and B 3, the moments of the strains in these pieces with reference to this point will be zero. Considering then the left hand section, we have the moment of the reaction acting up and causing, therefore, compression in A 2, *minus* the moment of the first apex weight, balanced by the moment of the strain in A 2. The lever arm for A 2 we easily find to be 3.72 ft. Hence strain in A 2 $\times 3.72 = 2800 \times 8.333 - 800 \times 2.08 =$

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* In the author's work upon Graphical Statics the method is further applied to the continuous girder, draw-span, braced iron and stone arch, etc., and shown to be of universal application.

21669·333 hence strain in A 2 = $\frac{21669 \cdot 333}{3 \cdot 72} = + 5855$, against 5816 as found by diagram.

Again suppose A 1 cut, we have the same lever arm as before, *i. e.* 3·72, but no apex weight left of the section, hence simply $A 1 \times 3 \cdot 72 = 2800 \times 8 \cdot 33$ or $A 1 = \frac{23333 \cdot 333}{3 \cdot 72} = 6272$ against 6280 as found by diagram.

Again, cutting B 1 and taking the point of moments at intersection of 1 2 and A 1, we have $B 1 \times 3 \cdot 12 = - 2800 \times 6 \cdot 25$, or $B 1 = \frac{17500}{3 \cdot 12} = - 5607$, against 5624 as found by diagram.

So we proceed, cutting each flange and taking the apex opposite to that flange as a point of moments, and taking the moments of all the forces left of the cut flange, with proper signs, and equating to the moment of the strain in the cut flange. The method for the diagonals is precisely similar. Here the intersection of the other pieces is at the end of the truss, which may then be taken as a common point of moments for all the diagonals. In any case, the main and only difficulty is the determination of the various lever arms; for a curved roof truss this involves much calculation. Such calculations, though sometimes long, are, however, always simple, and the lever arms may even be measured off to scale from the frame diagram with sufficient accuracy.

We have thus, as we see, two independent methods of calculation, and one (properly two) method of diagram, either of which are simple and perfectly general. Of the methods of calculation, the last is to be preferred as the simplest, and easiest of application. The method of diagram will, in most cases, be found preferable to all. Especially is this the case when we consider *wind force*, to the discussion of which we shall now proceed.

II. WIND FORCE.

V. It is of considerable importance to investigate the influence of a partial load, such as that caused by the wind blowing upon one side of the roof, and this, by the aid of our graphic method, we can easily do.

If P is the intensity of the wind pressure in pounds per square foot upon a surface perpendicular to its direction; i , the inclination

of any plane surface to this direction; P_n the normal pressure, P_h the horizontal component of this normal pressure, and P_v its vertical component; then we have

$$\begin{aligned} P_n &= P \sin i \frac{1.84 \cos i}{1} \\ P_h &= P \sin i \frac{1.84 \cos i}{1} \\ P_v &= P \cot i \sin i \frac{1.84 \cos i}{1} \end{aligned}$$

(See "*Iron Bridges and Roofs*," Unwin, p. 120, also, "*Elements of Graphical Statics*," DuBois, Art. 11.) If the wind blows horizontally, and if $P = 40$ pounds, then

For $i = 20^\circ$ we have $P_n = 18.1$, $P_v = 17$, $P_h = 6.2$.

For $i = 30^\circ$ we have $P_n = 26.4$, $P_v = 22.8$, $P_h = 13.2$.

Now in the case under consideration, $i = 26^\circ 34'$ nearly, therefore we have $P_n = \frac{18.1 + 26.4}{2} \times 6.98 \times 5 = 800$ pounds nearly, or the normal pressure at each apex, for $P = 40$ pounds, is about the same as the load upon the roof itself. We have also,

$P_v = \frac{17 + 22.8}{2} \times 5 \times 6.98 = 700$ nearly, and $P_h = 514$ at each apex.

If now at each apex we have 700 pounds acting vertically downwards, if the wind blows on, say from the left, and if the left end is fixed, the right being placed upon rollers to allow expansion; then the reaction at the left is for the first apex load,

$R_1 \times 50 = 700 \times 43.75 - 514 \times 3.12$, or $R_1 = 580$.

For the second, $R_2 \times 50 = 700 \times 37.5 - 514 \times 6.24$, or $R_2 = 525$.

For the third, $R_3 \times 50 = 700 \times 31.25 - 514 \times 9.36$, or $R_3 = 437.5$.

For the centre, $R_4 = \frac{350}{2} - \frac{257}{4}$ or $R_4 = 111$ pounds.

The entire reaction at the left end due to wind, therefore is,

$$R = 580 + 461 + 341 + 111 = 1493 \text{ pounds.}$$

We can now, therefore, form the force polygon of Fig. 3, by laying off in succession, to scale, the apex normal forces of 800 pounds each, viz.: A A, etc., equal to P_1 , P_2 , P_3 , and the last, $P_4 = 400$ pounds. Then drawing the verticals A a and B b, making A a equal to 1493 pounds, and drawing the horizontal a B. The force polygon thus closes, as it should. We have at the left end the vertical reaction A a, and the horizontal force a B. These two forces are in equilibrium with the strains in A 1 and B 1, hence lines parallel to these pieces must close the polygon. Thus we obtain B 1 and A 1, Fig. 3. So we go through the frame, precisely as before, and then

scale off the strains. We thus find $B\ 1 = -4464$ pounds, $B\ 3 = -3568$ pounds, $B\ 5 = -2680$ pounds, $A\ 1 = +3584$, $A\ 2 = +3480$, $A\ 4 = +2640$, $A\ 6 = +1784$, $A\ 6' = +1980$, also, $B\ 1' = -3608$, $B\ 3' = -2584$, $B\ 5' = -2232$, and $A\ 1' = +1688$, $A\ 2' = +3472$, $A\ 4' = +2584$. And for the diagonals $12 = +800$, $2\ 3 = -800$, $3\ 4 = +1208$, $4\ 5 = -1016$, $5\ 6 = +1608$, $6\ 6' = -1608$, $5'\ 6' = +320$, $4'\ 5' = -408$, $3'\ 4' = +480$, $2'\ 3' = -952$ $1'\ 2' = +800$. We can now pick out the greatest of these strains for any piece, and taking it together with the strains already found for dead load, we have the maximum strain in every piece. It will be seen that the strains for wind are very considerable, and should by no means be disregarded. In fact in some roof trusses, the effect of the wind is to *reverse* the strains due to dead load in some of the pieces, which should, therefore, be counterbraced.

We may, if we wish, check the above results, by either of our two methods of calculation, but the operations will be found still more tedious, and in curved roof trusses, very complex and intricate. By the graphic method however, no difficulty is experienced.

VI. ECONOMICAL ANGLE.—Let α be the angle between the rafter and tie. Then if u is the load per unit of horizontal length, we have for the compression at foot of the rafter, if s is the span,

$$\frac{us}{2} \sin \alpha + \frac{us}{2} \cot \alpha \cos \alpha.$$

But the length of the rafter is $\frac{s}{2 \cos \alpha}$, hence,

$$\frac{s}{2 \cos \alpha} \left[\frac{us}{2} \sin \alpha + \frac{us}{2} \cot \alpha \cos \alpha \right] = \frac{us^2}{4} (\tan \alpha + \cot \alpha),$$

represents the weight of the rafter. Differentiating and putting the first differential coefficient equal to zero, we have $\frac{1}{\cos^2 \alpha} - \frac{1}{\sin^2 \alpha} = 0$, or $\tan \alpha = 1$, or $\alpha = 45^\circ$. Therefore, 45° is the most economical angle for a girder of this kind with straight bottom flange.

If the rafter is considered as a beam continuous over five supports, as strictly it should be, this result is somewhat altered, and we have $43\frac{1}{2}^\circ$ as the angle of economy. In both cases the bracing is disregarded, as it is theoretically nearly constant. In reality, long compression members need extra material for stiffening, and hence our result is not strictly correct.

The above completes our discussion of the problem proposed. We trust that the attentive reader, interested in such questions, will be

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Fig. 1.

Fig. 3.

Fig. 2.

Scale of length, 12 ft. to an inch.
Scale of force, 3200 lbs. to an inch.

able to apply now the above principles for himself to any case that may arise. There are certainly several points of more or less importance which we have been unable to touch upon, but none which he will not, we think, be able to settle for himself. We have scarcely been able to more than hint at the graphical method based upon our second principle, Art. III, and yet it is upon this that the real method of graphical statics properly rests. Should the above serve to call attention to this branch of engineering science, which is at present attracting so much attention, our purpose will be abundantly accomplished.

The literature of the subject in English is already sufficiently extended. In the work of *R. H. Bow*, already quoted, a large number of diagrams for various forms of roof trusses will be found. Once familiar with the fundamental principle, the reader will find no difficulty in following through such examples at sight. In the *Engineering News* will be found a series of very excellent articles, by *Prof. Chas. E. Greene*, in which "Maxwell's Method," as applied to roof trusses, will be found very clearly and thoroughly elucidated. The same author has also in a work entitled "Graphical Method for the Analysis of Bridge Trusses," giving an original, and in many respects, very practical solution for continuous girders and draw spans. In German we have the works of *Culmann*, *Bauschinger* and *Winkler*. In French, of *Levy*. In the work of the author, entitled the "Elements of Graphical Statics," will be found a thorough presentation of *all* the various graphical methods, together with a complete list of the literature upon the subject, and applications to the continuous girder, draw span, braced arch, etc., illustrated by numerous practical examples.

Technical Terms.—The *Camberwell and Peckham Times*, in reporting the proceedings of the "Camberwell Vestry," (Anglice local governing board for Camberwell and Peckham), represents the following colloquy to have taken place during a discussion of a proposal to plant certain roads in the parish with trees, viz: "Mr. Grummant estimated the expense of the trees as being about 100*l*. He said that every tree would require a cradle, and then the ground round about would have to be paved with half sovereigns," (laughter). The surveyor explained that the term "half sovereigns" meant small "pitchers." Mr. Weller, amidst much confusion, protested against the term "half sovereigns" being applied to the stones when there was a proper name (*i. e.*, small pitchers) that might be used. Mr. Wesson (chairman): "Sit down; you shall have the order."—The *Builder*, London, Feb'y 5th, 1876.