to simple pelagic Cœlenterates in which cell boundaries and cell layers may have played only a secondary and belated part as the size of the organism increased."

The economic importance of these organisms is great, both as a food supply and also occasionally as a destructive agency. It is well known that they form a large percentage of the stomach contents of sardines and other small fish. At times they are the dominant forms of the plankton, and have been recorded by Kofoid as the cause of outbreaks of " red water " on the Californian coast and elsewhere which may be a menace to the health and life of slow-moving or bottom-living animals which, being unable to escape from the infested area, die in quantity and are cast up in masses on the shore. Such discoloration of the water, due to species of Gymnodinium and Gonyaulax, are recorded as extending sometimes (August, 1917) for a hundred miles or more along the coast.

To point out a few slips in such a splendid memoir may seem ungracious, but Prof. Kofoid would probably prefer to have friendly criticism : In the phylogenetic diagram on p. 84, have not Protodinifer and Oxyrrhis exchanged places, should not Protodinifer be Pelagorhynchus, and, near the top of the diagram, should not Nema- topsis be Nematodinium? The text-figure on p. 509 is evidently printed upside down, and in Fig. F (p. 30) the numbers 2, 3, and 4 are misplaced. Some of the references to figures in the text are not correct, but the careful reader will notice these for himself and will readily discover what is intended.

So many species are described, redescribed, or discussed, and the synonymy and history are given so fully, that the memoir is truly a monograph of the group, and will be found indispensable by all who work at these important lower organisms.

<div align="right">W. A. HERDMAN.</div>

The Theory of Probability.

A Treatise on Probability. By J. M. Keynes. Pp. xi + 466. (London : Macmillan and Co., Ltd., 1921.) 18*s.* net.

DR. KEYNES'S book is a searching analysis of the fundamental principles of the theory of probability and of the particular judgments involved in its application to concrete problems. He adopts the view that knowledge may be relevant to our rational belief of a proposition without amounting to complete proof or disproof of it, and treats the probability as a measure of this relevance.

Otherwise he does not attempt to define " probability," regarding it as a concept intelligible without further definition. In this respect, as in several others, he is in agreement with the views expressed by Dr. Wrinch and the present reviewer (*Philosophical Magazine*, vol. 38, 1919, pp. 715–31), and some comparison of the two presentations may not be out of place.

Previous writers have practically all assumed that probabilities can be expressed by numbers, and this assumption was put into precise form in the paper mentioned. Dr. Keynes departs completely from tradition on this point. Defining an " argument " as the process of passing to knowledge about one proposition by contemplation of it in relation to another of which we have knowledge, he denies not only that the probabilities of all arguments can be expressed by numbers, but also that they can be arranged in a one-dimensional series at all. Thus the probability of one argument may be neither greater than, equal to, nor less than that of another. The difference in actual application between this theory and ours appears likely to be slight, for the definitions and hypotheses are such that practically any two probabilities that one needs to compare are comparable. From these the formal theory is soundly developed.

The principle of non-sufficient reason, or indifference, asserts that we assign equal probabilities to propositions if we have no reason to do the contrary. The author criticises severely many previous applications of this principle (so severely that an unprepared reader is likely to be betrayed into expecting him to reject the principle altogether). He finally modifies it by saying that neither of the propositions deemed equally probable may be expressible as the disjunction of two mutually inconsistent propositions, of the same form as itself, and both consistent with the data. His precise statement of this important principle makes it possible to evaluate a large class of probabilities that could otherwise be only estimated, and is a most useful advance.

Dr. Keynes rejects definitely the view of Jevons and others that if any two alternatives are exhaustive and mutually exclusive, and we have no reason to prefer one to the other, the probability of each is $\frac{1}{2}$. His reasons for believing that this view leads to contradictions, however, appear incorrect. He says on p. 43 : " If, for instance, having no evidence relevant to the colour of this book, we could conclude that $\frac{1}{2}$ is the probability of ' This book is red,' we could conclude equally that the probability of each of the propositions, ' This book is black ' and ' This book is blue,' is also $\frac{1}{2}$. So that we

are faced with the impossible case of *three* exclusive alternatives all as likely as not." It appears to us that each of these estimates is based on different evidence, and, therefore, that it is quite possible that the sum of the probabilities should be greater than unity. A person who could recognise only one colour, say, blue, all others appearing alike to him, would estimate the probability that the book is blue at $\frac{1}{2}$. A person who could recognise only red would make a similar estimate for red. But one who could distinguish red, blue, and black, and no others, would estimate each as having a probability of $\frac{1}{3}$. In each case we follow the author in assuming no previous knowledge of the proportions of different colours among books.

The point is worth insisting upon, for we believe that the author has for such reasons refrained from estimating prior probabilities in many cases where such estimates would have been useful. In his discussion of sampling inference, for instance, he refuses to admit that any plausible estimate of the probable composition of a sample can be made, however large a sample has already been examined, unless we have further evidence that no disturbing cause exists. Admittedly the inference depends on the prior probabilities of different compositions, but we have shown that in ordinary cases a wide range of variation of the prior probability produces little variation in the inference made with regard to the composition of a large sample, and we think this is the only justification required. The acquirement of knowledge about a disturbing cause provides additional data and is valuable for that reason; its absence is no reason for denying a probability inference not based on it.

The author's insistence on the desirability of careful testing of the sample to see whether different subclasses from it have compositions similar to the whole is, however, very important on other grounds, for his careful discussion indicates the precise usefulness of a kind of additional information that is often obtainable and valuable. His conclusion (p. 426) that " sensible investigators only employ the correlation coefficient to test or confirm conclusions at which they have arrived on other grounds " is an exaggerated statement, but perhaps a salutary one.

A form of the frequency definition is discussed and rejected on the ground that it does not give any basis for induction. According to this the probability of a proposition p on evidence q is to be obtained by selecting a large number n of instances of q. If m of these are also instances of p, the probability of p given q is defined to be m/n. This theory is taken too seriously; it would be sufficient objection to point out that, unless m/n is o or 1, the probability would necessarily be

changed by having $n+1$ instances instead of n, and would therefore be conventional. In the form of the frequency theory discussed (and also rejected) in our paper the probability is defined as the *limit* of this ratio when n tends to infinity. This view, though it has been seriously advocated, is not mentioned by the author.

The faults attributed to the book above are all on the side of excessive caution, and the positive contributions are extremely valuable. It is clearly written, with a good index and a copious bibliography. The misprints are few. Whitehead and Russell's " Principia Mathematica " is, however, mentioned a few times as if it were by a single writer. The work should be read by every student of science who aims at a real understanding of his subject.　　　　　　　　　HAROLD JEFFREYS.

The Royal Society Catalogue.

Catalogue of Scientific Papers, Fourth Series (1884–1900). Compiled by the Royal Society of London. Vol. 17, *Marc–P.* Pp. v + 1053. (Cambridge : At the University Press, 1921.) 9*l.* net.

THE high standard set by the volumes already published in this series is fully maintained in the seventeenth volume of the Royal Society's "Catalogue of Scientific Papers." The work of preparing the material for the press and of proof-reading was carried out by Miss Vagner and Miss Barnard, and until December 1920 Miss Chapman was also engaged upon the work. The Cambridge University Press is to be congratulated on the typographical excellence of the volume, the small type which had to be used being quite easy to read.

The papers indexed are those published during the seventeen years 1884 to 1900 by authors whose names begin with the four letters M (from Marc onwards), N, O, and P. No less than 10,662 names are indexed, the number of separate papers being 57,474. Thus, on an average, each author has published one paper every three years. The volume brings up the total number of authors' names already printed for the period 1884–1900 to 49,750, and the total number of entries of papers published by authors whose names begin with letters from A to P inclusive to 279,902. The catalogue of papers by authors whose names begin with letters from Q to Z is still to be published.

The Committee say that the difficulties in the printing and publishing trade, which for a time delayed the regular delivery of proofs, have now been overcome, so that they look forward with