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ART. XXXIII.—*On the Deflection of the Plumb-line and Variations of Gravity in the Hawaiian Islands;* by E. D. PRESTON, Sub-Assistant, U. S. Coast and Geodetic Survey.

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ON the return of the Solar Eclipse Expedition from the South Seas, in 1883, two of the party stopped in the Sandwich Islands for the purpose of determining the force of gravity. An old pendulum station on the island of Maui, occupied by DeFreycinet in 1819, was the point chosen. While there the latitude of the place was determined with precision, by the method of equal zenith distances. This latitude was carried back by triangulation to Honolulu by Professor Alexander, the Surveyor-General of the islands. It was there compared with the local astronomical latitude as given by Captain Tupman, of the British Transit of Venus Expedition of 1874. Their comparison revealed a discrepancy,—quite incompatible with the accuracy of the work,—as shown either by the star observations at the terminal points, or by the triangulation connecting them. This seemed to prove, as indeed had long been suspected by Professor Alexander, that there were unusual plumb-line deflections. Both the excess of matter in the high mountain masses above, and the defect of matter in the deep sea below, would indicate this.

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In view of the importance of the determination of these deflections in the survey of the islands, as well as the general scientific interest in the subject, the Hawaiian government was led to ask the coöperation of the United States in the work of determining with precision a number of astronomical latitudes. Accordingly, in December, 1886, with the sanction of the Honorable Secretary of the Treasury, the Superintendent of the Coast and Geodetic Survey loaned the necessary instruments, and granted one of the members of the service a leave of absence long enough to make the observations. On the part of the Hawaiian Survey, Messrs. F. S. Dodge and W. A. Wall assisted in the gravity observations. Mr. Wall was permanently attached to the party and recorded all the latitude work.

The credit of the work, therefore, belongs in part to both governments. The Island Survey bore all the expenses, and selected the stations; the Coast and Geodetic Survey loaned the instruments, furnished the observer, and is making the computations.

Later it was thought desirable to supplement the latitude work, by the determination of the force of gravity on the top of one of the mountains, and at the sea-level. This would give a check on the deflections determined at the foot of the mountain, by means of the zenith telescope. For, whether there exist great caverns under the visible surface or not, the pendulum will give us a value for the mean density of the whole mass. Then, knowing the volume and density, it is a simple arithmetical process to find its influence on the plumb-line suspended at a given distance from it. The liberality of the Hawaiian government can only be commended when we find it devoting time and money to the solution of scientific questions in which the rest of the world is quite as much interested.

The scope of the work was as follows: Fourteen latitude stations were to be established on the four principal islands of the group, viz: Kauai, Oahu, Maui and Hawaii. These were distributed in such a manner as to bring out the deflections of the plumb-line; being in general north and south of the high mountains. At each one of these stations it was proposed to observe thirty-three pairs of stars on three successive nights; which would bring the probable error of the resulting latitude below one-tenth of a second of arc.

Practically, however, this plan could not be rigidly adhered to. At all the windward stations the weather was very unfavorable for astronomical observations. At Kohala the wind was always too strong to allow us to pitch a tent in which to live. An old sugar storehouse, long since abandoned, served instead. At Hilo, where the rain-fall is sixteen feet in one

year—and this all falls during the time the trades blow—only one good night's observation could be obtained in a month's stay. At Ka Lae the station was eight miles from the water supply; the road leading part of the way over the rough lava known as the "aa." All these circumstances made it necessary to make the best possible use of good weather, when so favored, and not unfrequently over one hundred pairs of stars were selected for observation on a single night. The greatest number obtained, however, was seventy-five, these being taken at Kaupo in less than eight hours' observing. Satisfactory work was done, allowing only one minute and a half between pairs, and thirty seconds between stars of the same pair. In this time the micrometer was read and the instrument revolved 180° in azimuth. The whole number of observations was about 1500, being on the average more than one hundred for each station.

The mean places of these stars are being deduced by Mr. Henry Farquhar, at the Coast Survey office, and will be very reliable—about twenty of the best modern catalogues being consulted. Many of the stations, however, being nearly in the same latitude, pairs of stars occur which are common to several places; so that the difference of latitude of these stations is independent of any error in the declinations.

For the gravity work two pendulums were supplied by the Coast and Geodetic Survey. They were of the reversible pattern one measuring a meter between the knife edges and the other a yard. They were to be swung on top of one of the mountains, at the sea level, and at Honolulu, to connect the work with that done in 1883 by the U. S. Eclipse Expedition.

The mountain chosen was Haleakala on the island of Maui. It rises slightly over ten thousand feet above the sea and has on its summit one of the largest extinct craters of the world:—the crater itself being half a mile deep and twenty miles in circumference. At the southwestern corner of this gigantic pit and within fifty feet of its perpendicular walls the instruments were set up and the force of gravity determined.

The Yard Pendulum is common to both the determinations of 1883 and those of 1887.

The scheme for conducting the observations was identical for each station and was strictly adhered to as far as the circumstances would permit. The pendulum was swung day and night without interruption from the beginning to the end of the observations. Stars were first obtained for time, and the pendulum was started at the time of reversing the telescope. This being the mean epoch of the star observations, the oscillations were thus referred directly to the stars without depending on the rate of the chronometer. The same stars when

possible being observed on successive nights, the errors in right ascension were also in a great measure eliminated.

Six swings were made with the heavy end down. The pendulum was then revolved 180° around its longitudinal axis, and six more swings were made. It was then reversed end for end, and the same number of swings made with the heavy end up. This completed the work with one pendulum. The other was observed similarly, and stars were obtained every night—the mean epoch, as usual, being at the time of beginning a new series of oscillations with the pendulum.

The time was determined with a meridian telescope—the transits of stars as well as those for the pendulum being registered on a Fauth chronograph. Two small cells supplied the electric circuit in which were placed the chronograph, a sidereal chronometer, and two observing keys. The chronometer broke the circuit every two seconds.

The knife-edge plane of the pendulum head was made level. The precaution was taken to hang a weight equal to the pendulum on the projecting arm during the operation, to see that no appreciable deflection was caused. A similar test was made for the whole support. The barometers were carefully compared with Signal Service standards. The thermometers which were made by Baudin, and can be read to hundredths of a degree centigrade, had their corrections determined in this country both before and after the expedition. The zero point of one of them was determined in the islands, and they were compared on the mountain immediately after the experiments were concluded.

Although Haleakala is in the tropics and was occupied in midsummer, we found great ranges of temperature on the summit. Ice was formed at night to the thickness of quarter of an inch. The clouds do not usually come within several thousand feet of the top, which gave us a clear atmosphere for the astronomical part of the work. Indeed many stars were observed before sundown with a telescope of $2\frac{1}{2}$ inches aperture and magnifying power of 70. One low star for azimuth and four high ones for time were taken before reversing the telescope. The same program after reversal, completed the evening's observations—the usual level readings being made as often as possible during the work.

It was found impracticable to carry the pendulum stand to the summit; but a better natural support was found, and the pendulums were swung in a cavity between two rocks. This opening was closed behind by masonry laid in cement, and before by tarpaulins, blankets, and dry masonry. The room thus formed was about nine feet high by $3\frac{1}{2}$ feet in diameter and gave a quite uniform temperature.

These conditions were much more favorable than those that would have resulted from carrying out the first conceived plan of taking a stand to the top and swinging in a double tent. No more perfect support could be had than a two inch plank imbedded in solid masonry, and a cavern protected on three sides by twenty feet of rock gives exceptionally good temperature conditions. The support in its essential features was the same for all three stations.

Three thermometers placed near the top, middle, and bottom of the room were used throughout the work. The one below was attached to a brass rod, and the rod and thermometer were enclosed in tin foil—thus insuring as far as possible the same temperature for both. The brass rod and the pendulum being of the same metal, the lower thermometer was supposed to indicate the true temperature of the pendulum at its lower end. This thermometer was read continuously throughout the swing. The middle and upper ones were only read when it was necessary to enter the room to start the pendulum. By this means we get a very accurate idea of what the real temperature is, throughout the whole length of the bar. At one station a fourth thermometer was employed which gave the temperature of the air in the immediate neighborhood of the attached thermometer and rod. When these means are employed it is hard to believe that an error in the mean temperature of the whole pendulum can be made as great as one-tenth of one degree. The influence of such an error is quite within the general range of errors of observation, or accidental errors beyond our control.

The oscillations were observed from the transit tent twelve feet distant by means of a small theodolite. A window of plate glass was built in the wall of the pendulum house, and through it the observations of transits, amplitude of arc, and lower thermometer were made.

Each swing with heavy end down was made to consist of 15000 oscillations, beginning with an amplitude of $\frac{1}{10}$ of the radius. The instant marking the beginning of the swing was determined by forty transits across the vertical wire of the telescope. The probable error of the mean of this number of transits is considerably less than one hundredth of a second. After the pendulum had swung for four or five thousand oscillations, a few additional transits were taken in order that there might be no mistake in the whole number of oscillations made. At the end of every hour and a half is quite often enough to take the intermediate transits, because the uncertainty in the determination of one oscillation, does not, in this time, accumulate to be more than half a second at most; and inasmuch as transits always begin with the pendulum moving in the same direc-

tion, we know that the whole number of oscillations is even. Hence the uncertainty of half a second throws no doubt on the number made. However, if the variations in temperature are excessive, it may be necessary to take this into account when the intervals are long; for example, one degree centigrade changes the time of oscillation by one hundred thousandth part of a second; therefore an interval of 15000 oscillations, in which there was a change of say 5° , would be different from the normal period by about $\frac{1}{2}$ of a second; and this with the uncertainties arising from the other varying conditions, would make it extremely doubtful, how many oscillations were actually observed. But if intermediate transits are taken every two hours, none of the conditions can change enough to endanger the count.

The observations are reduced to similar conditions at the different stations, as regards temperature of the pendulum, pressure of the atmosphere, amplitude of oscillation, and rate of time piece. The temperature and pressure coefficients are those employed by Professor C. S. Peirce, and which he determined experimentally for these particular pendulums. The corrections for arc were calculated by several different formulæ, all of which however gave practically the same result. That of Borda was given the preference on account of the rapidity with which the numerical computations could be performed. It supposes that the arcs decrease in a geometrical ratio, while the time increases in an arithmetical one. Peirce's formula assumes that the differential coefficient of the arc, with reference to the time as the independent variable, may be expressed in terms of the ascending powers of the arc and constants. These constants are to be determined from the curves of decrement themselves, for each set of swings; or in case no abnormal decrement occurs, mean values for the constants may be employed to correct all the swings. Weddle's rule, which finds a value for the mean square from six equidistant values, gives approximately the same result.

The varying conditions of rate, temperature, pressure, amplitude, elevation, and latitude, all influence the period by nearly the same amount, when the conditions change by certain simple units. One second per day, one degree centigrade, one inch pressure, one hundredth of the radius of amplitude, one hundred meters of elevation and ten minutes of latitude, all changing the period of a seconds pendulum by about one hundred thousandth part of itself.

In the determination of differential gravity much labor can be saved by beginning each swing at the same amplitude, and making it consist of the same number of oscillations. This makes the corrections for arc the same for all, and in the

determination of periods, it is only necessary to deal with differences instead of the whole interval.

The determination of the relative forces of gravity at the base and summit of a mountain gives sufficient data for the computation of the ratio existing between the mean density of the mountain and that of the earth.

Whether we consider the matter lying between the summit and the base, as a cone, a cylinder, or the segment of a sphere, the mathematical expression for its attraction on the upper station is approximately the same, when the horizontal dimensions of the figure are great compared with the vertical ones.

In passing from the sea level to the top of Haleakala, the time of oscillation was found to have increased by its $\frac{1}{3120}$ th part, or since gravity varies inversely as the square of the time of oscillation, the decrease of gravity in passing to the summit is $\frac{1}{1860}$ th part of itself.

The two stations are not in the same latitude, nor is the base station exactly at the sea level; but corrections were first applied to make them comparable as to latitude. The time of oscillation at the base station was also reduced to what it would have been at the level, of the sea.

On account of distance alone we should expect the pendulum to lose 41 seconds per day on being transported from the sea to the summit. As a matter of fact it was observed to lose only 28 seconds. Hence the mass of the mountain accelerated the pendulum by 13 seconds daily.

Employing Young's rule we arrive at a value of 43 hundredths for the ratio of the mean density of the mountain to that of the earth. Assuming the earth's density to be 5.67 the resulting density of the mountain becomes 2.4.

This is not very far from the estimated density of the rocks composing the mass, so that this determination does not indicate any large cavern under the mountain, or any very great attenuation of the matter composing it. Mountains as a general rule show a defect of gravity on their summits. But this rule has been deduced from experiments made on continental mountains, notably in Peru and India. If we admit that the surface of the sea is elevated in the vicinity of continents by the attraction of the land, mountains need not necessarily be supposed light. The great plateau of India would raise the apparent sea level immediately under it by nearly a thousand feet. This would have a very perceptible effect on the determination of the mean density of the plateau. But in the case of a mountain rising in the midst of a deep sea the surface cannot be supposed to be influenced enough to materially affect the determination of its density. In fact a plateau having an extent equal to Haleakala and a height equal to its mean height

would only elevate the apparent sea level by about ten feet. This quantity may well be neglected when we consider that the mountain is 10,000 feet high and the mean density in any case is not certain beyond two or three significant figures. Therefore we should not expect to find in mountains in the middle of the Pacific Ocean, a mean density differing very much from that indicated by the rocks found on the surface.

The value 2.4 which the pendulums both agree in showing, certainly does not depart sufficiently from that furnished by the rocks themselves, to allow us to assert that the mountain is lighter than it should be, or that there is a defect of gravity on its summit. On the contrary, the indication inclines slightly to the other side, if we accept 2.3, which has been estimated by geologists for the density of the rocks on Maui. Between 9500 and 10,000 feet there exist rocks of a comparatively great density. This being the case at such an altitude, it is most reasonable to suppose a density, great enough, to counter-balance the beds of light lava and cinder, found on many parts of the summit.

In order to have a check on the whole work, as well as to compare pendulum results with those from star observations with the zenith telescope, the following plan was adopted: Latitude stations were made on the north and south side of the island, besides an intermediate station on the summit. All these stations have been connected by triangulation, as a regular part of the Government Survey by Professor Alexander. Comparing the astronomical deflections observed on either side of the mountain, with those calculated from its volume and density as furnished by the pendulum, we shall have a test of methods as well as results and if it is found that these agree, we should feel considerable confidence in our opinions concerning the constitution of the crust.

The attraction of a mountain may be calculated in several ways. The best and most accurate, when sufficient data is at hand is that due to Dr. Hutton and is briefly this: The country around the station is divided into compartments by concentric rings and radial lines. The attraction of an element of matter depends on its density and the square of its distance from the attracted point; and we have as a result a formula which must be integrated with reference to three variables. The first integration gives us a curve in the plane of the base of the mountain: after the second we have a vertical curved surface, and with the third results the volume and attraction of one of our elementary compartments. Performing the integration then for azimuth, distance, and elevation, we have all that is necessary to compute the attraction of the mountain in the direction of the meridian of the station. Here the work

of the integral calculus ceases. But the computation of the resultant attraction of the whole mountain is further facilitated, by so choosing the distances between the circles and lines, that each compartment has a component in the direction of the meridian depending only on its height. Then the attraction of one compartment has only to be multiplied by the sum of the heights of the compartments. These conditions are realized by taking the azimuths of the radial lines, such that their sines are in arithmetical progression, and the lines themselves, such that their lengths are in geometrical progression.

The formula is,

$$\rho \int_{a_1}^{a_2} \int_{r_1}^{r_2} \int_0^h \frac{r^2 \cos \alpha \, d\alpha \, dr \, dz}{(r^2 + z^2)^{\frac{3}{2}}}$$

and the final deflection is,

$$12'' \cdot 44 \frac{\rho}{\Delta} K l [\Sigma(h) - \Sigma(H)]$$

In calculating the deflection at Kaupo the whole island is considered in three parts, all of which lie to the north. The first is comprised between the prime vertical through the station and a semi-circumference of 10 miles radius. The second is also limited by the prime vertical and is a semi-circular ring included between radii of 10 and 26 miles. The third takes in that part lying beyond the valley, known as West Maui.

In the first part the ten radii are in arithmetical progression the common difference being one mile: in the second they have a ratio of $\frac{11}{10}$: the radius of the n th circle being $10 \times \left(\frac{11}{10}\right)^n$

or $\frac{(11)^n}{(10)^{n-1}}$ miles. In the third they are chosen arbitrarily depending on the configuration of the ground.

Radial lines are now drawn from Kaupo as a center and making angles with the meridian line whose sines have a constant difference of one-tenth. This divides the whole island into 318 compartments, the mean heights of which are determined from contour lines.* The first series of circles extends to the summit and includes the great crater. Within this limit, which makes up about $\frac{1}{4}$ the whole surface of the island, we have more than $\frac{2}{3}$ the whole attractive force. The second series goes to the valley, and practically includes all the remainder of the attraction. In computing the effect of West Maui the mountain is divided into two segments and a mean height is assumed for each: giving those parts nearest the attracted point greater weight. These heights are about

* Kindly furnished by Prof. C. H. Hitchcock.

2500 and 1500 feet and the resulting deflection is only $\frac{1}{8}$ of a second. This value compared with the total deflection caused by East Maui shows that the approximate data used for this distant part of the island are quite accurate enough. For a change of 10 miles in the distance or 10° in the azimuth only changes the deflection by a few hundredths of a second. A difference of 500 feet in elevation has about an equal influence. The distance of the center of West Maui from Kaupo is 35 miles and its mean azimuth 55° .

That part of the island lying to the south of the prime vertical is neglected. A computation of one of the compartments giving only a deflection of $''\cdot002$; besides whatever influence this land may have, is practically destroyed by the light sea lying north of the prime vertical on the east, as the two portions are nearly the same shape and size.

The sea lying to the southward will have the same influence as a volume of density equal to the difference of densities of rock and sea water. Supposing the mountain to continue to the bed of the ocean, with the same slope that it has from the summit to Kaupo, gives a cone of density of rock minus water, and volume equal to Haleakala, making up nearly all the disturbing effect. Hence we multiply the deflection found for the mountain by a factor depending on the relative densities of rock and water. Increasing the total deflection found or $17''\cdot8$ by $\frac{7}{17}$ of itself we get $29''\cdot1$ as the entire deviation of the plumbline at Kaupo. The deflection found by triangulation was $29''$. This mode of computation is the same as that employed by the Ordnance Survey in the treatment of deflections in the south of England. A shorter way and one that can be employed for an approximate value in some cases, is to consider the mountain as a paraboloid of revolution. This was done for Haleakala using the mean density furnished by the pendulum, and a value for the mass of the earth, previously employed in investigations of this kind. These data give for the deflection at Kaupo, (Ka Lae o Ka Ilio) a station on the south side of the mountain, 28 seconds, which is within one second of what was found by Prof. Alexander who carried the astronomical latitude of Haiku around the island, by triangulation to Kaupo, and there compared it with the locally determined latitude. Of course this very close agreement is only accidental because the data used are but approximate and the mountain is not a perfect paraboloid of revolution. In fact by this method the deflections cannot be known closer than a few seconds: for an uncertainty of a quarter of a mile either in fixing the radius of the mountain or in estimating the distance of the attracted point from the center of what would be an equivalent paraboloid, will produce an uncertainty of at

least several seconds, for a mountain equal to the one under consideration.

However, when the difference between the astronomical and geodetic latitudes is nearly a minute of arc, this method will give a value for the ratio of the density of the mountain, to that of the earth, in which the error is not greater than one-tenth of the whole amount. The difference between Playfair's reduction of the observations on Schiehallien and the generally accepted value is about one-fifth of the whole.

The last eruptions from Haleakala took place at least many hundred years ago. The natives have no traditions bearing on the fact; and geologists believe that the immense crater has come to its present condition by the sinking in of the top of the mountain and not by erosion. Both these would indicate a density of the mountain at least equal to its surface density: probably more. The pendulum adds testimony in this direction, and the zenith telescope observations at the foot of the mountain, indicate the same thing.

Therefore we may assert that the defect of gravity, usually noted on continental mountains, does not apply to those on small islands in the middle of a deep sea. Whether this supposed defect is caused by assuming an incorrect mean sea level, or whether the mountains are indeed only vertically and not horizontally displaced matter, we cannot now state. Both causes may exist: one certainly does.

On the Island of Hawaii there appears to be a striking example of plumb-line deflections. At Kohala we have a deflection of half a minute towards the south, and at Hilo, one, of quarter of a minute towards the north. These points are both on the windward coast and are distant from each other about sixty miles.

It is easy to see why there should be one to the southward, at Kohala, for the great mountain masses of Mauna Kea, Mauna Loa and Hualalai would all have this influence. But at Hilo there is no explanation unless we assume that the south side of the island, where the volcanoes are active, is much less dense, than the north side, where the fires have been slumbering for centuries.

In general the deflections everywhere in the islands are greater than we expect, judging from analogy with continental masses: but here we have a surrounding sea which is much lighter than the land on which the continental mountains rest; and it is a question whether the sea bottom is really very much more dense than the average land. If the earth in solidifying has simply contracted unequally, and if each prism, having its base on the surface and its apex at the center, contains the same amount of matter, this would not make the bed

of the ocean very much more dense than the land. The depth of the ocean is about $\frac{1}{1000}$ of the earth's radius, and the volume of the prism would therefore be decreased by its $\frac{1}{333}$ rd part. Hence, if we have the same amount of matter, the density should change by such amount. This is entirely inadequate to counteract the effect of the light matter above, or materially change the deflections brought to light by the zenith telescope or by the pendulum.

It has been found that variations that arise from hidden causes under the Himalayas are two or three times as great as those that arise from the mountains themselves. The computations on which this result rests assume Young's rule, and take the ratio of the surface density to the mean density to be one-half. Besides, it is well known that the rule supposes the matter lying under the station to be a plain of infinite extent. The same rule has been applied to mountains, supposing them to be either cones, cylinders or the segments of a sphere. Evidently, in these latter cases, the error is in the direction of making gravity on top of a mountain too great, because the rule corrects additively for too great an amount of matter. In fact, to suit Haleakala, the constant factor in Young's formula should be changed from 1.25 to 1.36, and we should have

$$\frac{g_z}{g_0} = \left(1 - \frac{4}{3} \frac{z}{r}\right)$$

where the factor $\frac{4}{3}$ of the ordinary formula is here replaced by $\frac{4}{3}$.

This *increase* of the factor seems at first a paradox—but we must remember that the essential tendency of gravity is to diminish as we rise, and that any correction for matter is a positive quantity which must be added to this negative correction. Hence, for a cone, which contains less matter than the plain, the decrease of gravity should be greater, or the factor which corrects for the matter should be greater.

In fact, the factor that corrects for an infinite plain has only to be multiplied by a quantity depending on the cosine of the vertical angle of the cone to make it applicable to this figure. Supposing the cone to have a vertical angle of 180° , the two formulæ coincide, as they should do, since in this case our cone becomes an infinite plain.

Using the new formula

$$\frac{dg}{g} = \frac{2h}{r} \left[1 - \frac{3}{4} \frac{\delta}{\Delta} (1 - \cos \beta)\right]$$

to get an improved mean density we arrive at 2.8—the decrease of gravity shown by the pendulums indicating a ratio of exactly $\frac{1}{2}$ for the two densities: gravity is first corrected for

elevation, the downward attraction of the cone is then added, and this compared with the actually observed decrease gives its density.

The conclusions drawn from the observations are therefore :

I. Deflections of the plumb-line are greater on island than on continental mountains, presumably on account of the lighter surrounding sea water; and gravity is not in defect because it is here estimated from the true sea level and not from a sea elevated by continental attraction.

II. Deflections appear to be greater in the vicinity of extinct volcanoes than near active ones.

III. The so-called "hidden causes," which in the case of the Himalayas give a variation of gravity several times as great as those arising from the attraction of the mountains themselves, do not exist in the Hawaiian Islands.