The Slide Rule

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asserted that the same kind of strangling of initiative was occurring daily. Professor Louis Miall, in an interesting and informing speech, which was heartily applauded, pointed out that the true method of examination was that in which the teacher himself examined, with an outside examiner present to see fair play, as it were. This proposition was perhaps the best answer to the rather malicious question put by one of the lady members, "Who is to teach the teachers and examine the examiners?"

The discussion was closed by a short speech from the sectional President, Sir John Gorst, who gave some reminiscences of his early experiences in teaching mathematics to the Maories, and expressed his conviction of the value of such a discussion in which the teachers and the specialists are brought into direct contact.

I think there can be no doubt of the benefit in ventilating what is really a burning educational question, though, of course, it is difficult for such an assembly to get at the "hard pan" of the crucial questions in one sitting. May I hazard the opinion that the line of reform indicated by Professor Miall, that of emancipating the teacher from all detailed syllabuses, is a more hopeful one than that of merely laying down a fresh syllabus, which in the nature of things will in course of time be as much of a clog upon educational progress as that which it replaces? R. F. MUIRHEAD.

THE SLIDE RULE.

Having accepted with some mental reservation the assurance of the Editor of this Gazette that the Slide Rule, although invented by Gunter as far back as 1624, is still unknown to many mathematicians, the writer has ventured to make this article explanatory and descriptive. To such then as are unenlightened he offers the good news that half-a-guinea will purchase an instrument capable of multiplying and dividing, of squaring and cubing, of extracting square and cube roots, of solving triangles and performing rapidly many similar combinations of arithmetic and trigonometric operations, all without any mental effort and to an accuracy of from $\frac{1}{2}$ to $\frac{1}{3}$ %. To examiners having to reduce marks to percentages the Slide Rule is invaluable, to physicists it is indispensable, to engineers it is both invaluable and indispensable.

The Gravet Slide Rule has been selected for description, mainly because Gravet has for over thirty years set the fashion in the manufacture of accurate yet inexpensive instruments. The detailed explanation has been restricted to the more elementary operations, for when the principles of these have been grasped no mathematician can find difficulty in extending them.

The operations of the Slide Rule may best be illustrated by studying the effect of sliding one ordinary uniformly graduated scale along another such scale. Thus in Fig. 1 the scale *B* has been slid along the scale *A* until its extreme left-hand graduation, marked *O*, comes under the graduation 17 on *A*: we note now that each graduation on *B* marks on *A* the result of adding 17 to it; thus above the graduation 5 on *B* we find 22 (*i.e.* $5+17$) on *A*, above 32 on *B* we find 49 (*i.e.* $32+17$) on *A*. Hence "the addition ($x+y$) may be performed by setting the left end of *B* to *x* on *A* and reading the sum on *A* at the point marked by *y* on *B*." Conversely, the subtraction ($x-y$) may be performed by moving the slide *B* till the graduation *y* on *B* comes under *x* on *A*, the left end of *B* then marks ($x-y$) on *A*. In Fig. 1 the operation $44-27=17$ is indicated.

If, instead of making the graduations on *A* and *B* identical, we replace *B* by a scale *C* marked off to twice as big a scale as *A*, we have a ready method of halving or doubling numbers. For when *A* and *C* are placed "zero to

Fig. 1

Fig. 2

Fig. 8

Fig. 9

Fig. 11

Fig. 12.

Fig. 13

Fig. 4

Fig. 5

Fig. 3

Fig. 6

Fig. 10

Fig. 7

zero" we find at any point on C half of the number marked on A immediately above it. Fig. 2 shows how such a rule performs the operation $(x+2y)$. The left end of C is set to x on A , the sum $(x+2y)$ is then read on A at the point marked by y on C ; the figure gives

$$13+2 \times 10=33, \quad 13+2 \times 12=37, \text{ and so on.}$$

Similarly by interchanging the functions of A and C we may perform the operation $(x+\frac{1}{2}y)$; thus, set the left end of A to mark 8 on C , then $8+\frac{1}{2} \times 12=14$ is read on C at the point marked by 12 on A .

By constructing a rule in which the scale of C bears any other relation to that of A , say k times as big, we can in the same manner perform the operations $(x+ky)$ and $(x+\frac{y}{k})$.

The essential feature of the above processes is that by sliding one scale along another we can *mechanically add two lengths*, and these lengths being proportional to numbers, it follows that we at the same time add two numbers. If now instead of making the lengths proportional to numbers we make them proportional to the *logarithms* of numbers, we shall by the process of Fig. 1 mechanically add $\log x$ to $\log y$ and thus obtain a length proportional to $\log(xy)$, and by the converse subtraction-process we shall take $\log y$ from $\log x$ and thus obtain a length proportional to $\log(\frac{x}{y})$.

The construction of logarithmic scales is quite simple; Fig. 3 shows the method. Above the line is a uniform scale, below it the logarithmic scale.

Since $\log 1=0$, the graduation 1 is placed at the left end of the scale; since $\log 2=.301$, the graduation 2 is placed slightly beyond .3 of the upper scale and so on.

It is interesting to note how the graduations close together towards the right end of the scale, the intervals between them representing $\delta \cdot \log x$ being approximately proportional

to $\frac{1}{x}$. In the figure only the main graduations 1, 1.1, 1.2, etc.,

3, 3.5, 4, 4.5, etc., are shown, but on the ordinary 10-inch slide rule, subdivisions are marked at 1.02, 1.04, and so on from 1 to 2; 2.05, 2.1, 2.15, etc., from 2 to 5; 5.1, 5.2, 5.3,

etc., from 5 to 10; further subdivision is made by the eye, and very little practice is needed to attain an accuracy of 1 in 500. It might at first sight seem that there is a loss of accuracy at the right end of the scale, but this is only apparent, the percentage error being the same throughout.

Since $\log 20=1.301$, $\log 30=1.477$, and so on, it follows that the extension of the scale beyond 10 up to 100 involves merely the repetition of the scale from 1 to 10. So a further extension from 100 to 1000 involves another repetition of the same scale. But just as a single four-figure table of logarithms of numbers from 1000 to 9999 serves to give us the logarithms of say 2538, 25.38, 2.538, provided due regard be paid to the characteristic, so a single scale from 1 to 10 serves for all numbers provided a watchful eye be kept on the decimal point in the result. In practice it is convenient to have the scale repeated once.

Fig. 4 shows the logarithmic scale B slid along a similar scale A till its left end marks 1.7 thereon; the arrangement is analogous to that of Fig. 1, but here we add $\log 1.7$ to a series of $\log s$ of numbers. Hence above 2 on B we find 3.4 (i.e. 1.7×2) on A , above 3 on B we find 5.1 (i.e. 1.7×3) on A and so on. In general the operation $x \times y$ may be performed by setting the left end of B to x upon A , and reading the product on A at the point marked by y on B .

Fig. 4 also shows the process of division which is analogous to the subtraction shown in Fig. 1. Thus the division $\frac{x}{y}$ (e.g. $\frac{4.25}{2.5}$) may be performed

by moving the slide B until the graduation y (2.5) upon it comes under x (4.25) on A , the left end of B then marks $\frac{x}{y}$ (1.7) on A .

Now let us construct on the lower edge of the slide a logarithmic scale C twice as long as A or B , and also on the lower edge of the rule construct D identical with C . These scales C and D may be used as a pair like A and B to multiply and divide; but their main use is in combination with A or B .

Note that the pair of scales B and C [and also the pair A and D] are fixed relatively to each other in the position "unity to unity," and that this position corresponds to the position "zero to zero" of the scales A and C in Fig. 2. They, therefore, give automatically the halves and the doubles of logarithms, *i.e.* they "square" or they extract the square root. As the scales A and D are not adjacent, a sliding index or "cursor" is needed (Fig. 5); it consists of a light metal frame carrying a small plate of glass with a fine perpendicular line engraved on its under surface.

Set the index to mark any number x on A (or B), then it also marks $\sqrt{x}$ on D (or C). In Fig. 6 the index stands at 12 on A , 3.462 ($=\sqrt{12}$) is marked on D ; it stands at 6 on B , 2.448 ($=\sqrt{6}$) is marked on C .

Further, Fig. 6 shows how the rule performs the following operations:

(a) $\sqrt{x} \times y$: the left end of B is set to x on A , the product $\sqrt{x} \times y$ is then read on D at the point marked by y on C ; thus

$$\sqrt{2} \times 2 = 2.83, \sqrt{2} \times 3.2 = 4.525, \text{ and so on.}$$

(b) $\frac{\sqrt{x}}{y}$: the index is set to x (12) on A , y (2.45) on C is then pushed under the index, and the quotient 1.414 is marked on D by the left end of C .

(c) $x \times y^2$: the slide is set so that the left end of C marks y ($=1.414$) on D , the product xy^2 ($3 \times 1.414^2 = 6$) is then read on A at the point marked by x (3) on B .

(d) $\frac{x}{y^2}$: the index is set to x (12) on A , y (2.45) on C is then pushed under the index, the quotient $\frac{x}{y^2}$ (2) is marked on A by the left end of B . Note that this operation only differs from (b) in that the result is read on A instead of C ; this is, of course, because $\frac{\sqrt{x}}{y} = \sqrt{\frac{x}{y^2}}$.

The sliding index or cursor was added by M. Gravet of Paris about 1882, and added immensely to the utility of the rule, for it enables us to obtain the final result of several multiplications and divisions without recording the intermediate steps.

For example, to evaluate $\frac{42.5 \times 362}{2.5 \times 11.8}$.

(a) Set the index to 425 on A .

(b) Push the slide till 2.5 on B comes under index as in Fig. 4.

[The left end of B now marks the quotient 170 on A ; but there is *no need to read off this quotient*, for the first step towards the next operation—multiplication by 362—is to set the left end of B to 170, *i.e.* to the point at which it is now standing.]

(c) Set the index to 362 on B .

[This marks the product 617 on A , but this again being an intermediate result need not be read.]

(d) Push the slide till 11.8 on B comes under the index, then the final result 522 is read on A .

Rules may be formulated to decide where the decimal place should stand in the result, but the intelligent computator will know *a priori*, say, whether a mass he is dealing with is 522, 52.2, or 0.522 grammes.

By using the scales A , B , C , D in judicious combination operations involving any number of multiplications or divisions by simple numbers, by their squares or by their roots may be performed.

The process of extracting a cube root is interesting, it is analogous to finding one-third of a number with the rule shown in Fig. 2. Here if we set an index at 39, and adjust the slide until the number on A marked by the left end of B is identical with the number marked on B by the index, this number is obviously $\frac{1}{3}$ of 39, to wit 13.

So with the rule of Fig. 6: to find $\sqrt[3]{8}$; set the index at 8 on A , adjust the slide until the number on A marked by the left (or right) end of B is identical with the number marked on C by the index: the log of this number is obviously $\frac{1}{3}$ of log 8, hence the number is $\sqrt[3]{8}$, to wit 2.

Having set the index to 8 on A the above adjustment can be made in three and only three possible ways, one gives $\sqrt[3]{8}$, the others $\sqrt[3]{80}$ and $\sqrt[3]{800}$; just as the logarithms .90309 with its different characteristics 0, 1, 2, 3, 4 gives three distinct mantissae after division by 3.

In practice it is necessary to decide on the first figure of the result by inspection before beginning to adjust the slide; e.g. $\sqrt[3]{.0857}$, the first figure is 4.

The under surface of the slide has three scales engraved upon it. The middle one is uniformly divided, it is in fact the uniform scale from which D has been constructed (as in Fig. 3). This scale gives by means of a fixed index mark on a slot at the back of the rule the logarithm of the number to which the slide has been set on D . The upper scale S is constructed by marking off lengths proportional to log sines of angles to the same scale as A and B ; it readily gives sines, cosines, secants, cosecants, and their logs. The lower scale T is constructed from log tangents; in the French-made rules it also is to the same scale as A and B ; on most German and so-called English rules, it is to the same scale as C and D ; it gives tangents, cotangents, and their logs by means of a slot at the left end of the rule. [Some makers omit this slot—a very serious drawback.]

These three scales may also be used by taking the slide out of the rule and replacing it bottom face uppermost. This is convenient when several sines or tangents have to be multiplied by a constant as in reducing galvanometer readings; for if one end of the slide be set to the constant on A (possibly on D for tangents) the products for each angle are read off on A where marked by the graduation on S or T .

There is a distinct type of Slide Rule in which a circular logarithmic scale is constructed on a disc capable of rotation (Fig. 8).

Here the mechanical addition of logarithms is effected by means of two pointers; one A is fixed, the other B can be turned about the centre like a watch hand. If the disc be set so that "1" comes under the fixed index A , and the index B be then set to any number (e.g. 1.7), the angular distance A to B then represents log 1.7; if now the disc be turned to a new position, say, so that 22 comes under A , B remains at the same distance as before ahead of A , hence it adds log 1.7 to log 22, and we therefore find the product 37.4 ($= 1.7 \times 22$) marked by B on the disc. Division is equally simple.

Probably the best rule of this type is "Boucher's Calculator," it is of the size of a large watch, it has on one face in concentric circles a scale for sines, a scale for numbers, a scale of numbers twice the size occupying two complete circles and corresponding to the D scale of the straight rule—this scale gives square roots; on the other face is a scale of numbers enlarged three times—it therefore occupies three complete circles and gives cube roots, on this face also is a scale of log sines and a uniform scale giving logarithms. Two milled heads M and N turn the disc and the index respectively.

The advantage of these rules is that they are very compact and handy, and are beautifully engraved; the disadvantage is that they cost 27s. 6d., and

that there is great liability to parallax error in reading—for the movable index in turning has to clear the fixed index and therefore must be comparatively far from the disc; this cause limits the accuracy of calculation to about $\frac{1}{2}\%$ or 1%.

Abundant provision has been made for those computers for whom an accuracy of $\frac{1}{2}\%$ does not suffice. Thus straight rules of the Gravet pattern are made 20 inches long in place of 10 inches; the enlarged scales of course admit of more detail in sub-division of the graduations and can be read more exactly, hence the accuracy is doubled. But to raise a three-figure to a five-figure accuracy the rule would have to be lengthened 100 fold—an eighty-foot rule is unwieldy.

A five-figure accuracy has been secured in two ways—first by Fuller, who arranges a logarithmic scale (41 ft. 8 in. long) helically on the outer surface of a hollow cylinder *I*. This cylinder corresponds to the movable disc in Boucher's Rule, it slides smoothly on a second hollow cylinder *II*, which is twice as long as *I*, so as to allow *I* to be moved up or down through its own length as well as rotated upon it. The instrument is held by a handle attached to *II*. *II* also carries a fixed brass index *A* long enough to allow any number on *I* to be brought to its point. A movable index *B*, corresponding to the moving hand *B* in Boucher's Rule, is attached to a third cylinder *III* which slides within the hollow of *II*. The instrument is manipulated in exactly the same way as a Boucher. Thus for multiplication the graduated cylinder is slid round and down till its graduation "1" comes to the fixed index *A*. (This movement can be made rapidly by bringing a pin on the cylinder up to a fixed stop.) The movable index *B* now being brought to any number, say, 3216 on the scale, the distance from *A* to *B* measured along the helix represents the logarithm of 3216. Hence if the cylinder *I* be moved so that any other number, say 6158, comes to *A* the number which is now marked by *B* is log 3216 ahead of 6158, i.e. *B* marks 3216×6158 .

The Fuller rules are beautifully made, the only drawback to them is that they have no *C* or *D* scale for square roots, and they do not give trigonometric ratios except by tables printed on the spare surface of the fixed cylinder *II*.

The other five-figure rule was designed by Thatcher. It is simply a 60-foot rule of the Gravet type; the scale *B* of the slide is cut up into 40 equal lengths and these are arranged along 40 equidistant generating lines of a 2-ft. cylinder, which corresponds to the slide of the straight rule. In Fig. 10, which is diagrammatic, *B* is shown cut into 12 lengths only; also for clearness the graduated strips are not shown in the right half of the figure. The scales *A* and *D* are also cut into 40 equal lengths, and these are arranged on the 40 faces of 20 V-shaped ribs* of a skeleton cylinder, which constitutes the "fixed" part of the rule and within which the first cylinder slides. Although this instrument is of American build it is gratifying to know that the plates from which the scales are printed were engraved by Messrs. Stanley of London. The instrument costs about £12, but the luxury of its use in five-figure work makes it well worth the outlay; it deals with x^2 , x^3 , $\sqrt{x}$, $\sqrt[3]{x}$, $x^{\frac{2}{3}}$, and $x^{\frac{3}{2}}$ as readily as does the Gravet, but it has no trigonometric scales.

The foregoing descriptions are of instruments applicable to calculations of every manner and kind; before the introduction of the "cursor," work involving several operations was tedious owing to the necessity of recording intermediate results, hence special rules were designed and adapted for the requirements of various trades or individuals; these are often very ingenious, they bear mysterious gauge points representing constant factors which enter

* Fig. 10 is drawn as though there were only 12 ribs, and they are cut away on the left so that only one of the supporting end-rings is seen.

into the formulae on which the results depend: the engineers' shibboleth $7854 \left(\frac{\pi}{4}\right)$, the number of cubic inches in a gallon (277), and the like, are samples of these mysteries. Sometimes these rules carry several slides so as to give the product of several factors simultaneously. There should be a tendency for them to become obsolete, but the interest of unearthing the mode of their construction must always to a mathematician be as intense as that afforded to an archaeologist by any cuneiform inscription.

An interesting artifice in these special rules is the use of a reversed scale. With two uniformly divided scales as in Fig. 11, if any two numbers (as 8 and 3) be brought together then as we pass along the scales the sum of the readings is obviously constant, and we can read the sum $8+3$ at the zero of either scale. So with two logarithmic scales as we pass along the scale we have a constant product, hence if 3 and 8 be brought together, as in Fig. 12, the product 8×3 can be read at the end of either scale.

In the very pretty H.P. rule designed by Hudson and engraved by Messrs. Stanley there are two slides with two pairs of reversed scales, one of them, on which cylinder diameters are read, is of double size so that the diameters are automatically squared. The scales effect the product of the four variable factors involved in "H.P. = $\frac{p.l.a.n}{33,000}$ " and a suitable shifting of the zero on

the final scale provides for the constant factors π , 33,000, etc.

A full description of this rule would be too lengthy, we therefore describe a rule designed on the same lines which will give at sight the volume and weight of a cylinder of known diameter, length, and density. In Fig. 13 the scale *A* is double the reversed scale *B*, hence along these scales instead of having a constant product of simple numbers, the number on *A* is squared in the product; thus we find $6^2 \times 1$, $3^2 \times 4$, $2^2 \times 9$, $1.5^2 \times 16$, $1^2 \times 36$ all equal to 36, the number marked on *B* by the left end of *A*. The scales *C* and *D* are the same size as *B*, but the unity end of *D* has been set back to the left so that 7.854 on *D* comes over 1 on *A*. If the slide be removed from the rule, and we begin to replace it, the unity end of *B* first comes to the unity end of *A*, our constant product is $1^2 \times 1 = 1$; the right end of *C* then marks 7.854 on *D*, which is the number of grammes in a cylinder diameter 1 cm., length 1 cm., and density 10. In the figure the slide has been pushed in from this position through a length representing $\log 36$, therefore the right end of *C* now marks $7.854 \times 36 [=283]$ on *D*, while the unity indicated by a blurred arrow near the middle of *C* marks 28.3 on *D*; this is volume in c.c. of the cylinders, diameter 3 cm., length 4 cm., or diameter 2 cm., length 9 cm., etc. Finally, if we travel along the density line *C* from the arrow to say 7 the result on *D* is multiplied by 7, hence we read 198 grammes as the weight of each of these cylinders when the density is 7.

The following points should be looked to in selecting a slide rule:

(1) At each end of the rule a space of about $\frac{3}{4}$ " should be left clear beyond the graduations, so that the cursor may not wobble when used near the end of the scale: the writer and a colleague have for years attempted in vain to secure this improvement. Messrs. Faber have just issued a rule with 1.5 cm. clear at each end.

(2) The glass of the cursor should be quite flat on the rule and the index line should be very fine: any failing in these respects soon reduces the accuracy to $\frac{1}{2}\%$. It would be a great improvement if a break 2 mm. long were made in the line just where it crosses the top and bottom edge of the slide: in this way all readings would be made at the end of a fine line and the disturbing effect of thickness of line would be eliminated. Specimen cursors improved in this way have been promised to the writer by two English firms in succession, one promise dates three, the other six years back. M. Gravet of Paris has produced a specimen in three months, and

although the required improvement was not quite understood the writer hopes that at the next attempt he may attain his object.

In conclusion the writer would ask for indulgent criticism of the crude figures which illustrate this article; they were produced at a time when pressure of work prevented the use of the beam compass and proper dividing instruments, but it is hoped that they will serve their purpose as well as more artistic productions would have done.

F. R. BARRELL.

REVIEWS.

The Theory of Equations, with an Introduction to the Theory of Binary Algebraic Forms. By W. SNOW BURNSIDE, M.A., D.Sc., and A. W. PANTON, M.A., D.Sc. Fourth Edition. Vol. II., pp. xi. 292. (Dublin University Press, 1901).

Fourth editions do not often call for much reviewing. It, as a rule, suffices to give them a cordial welcome, and to congratulate the successful authors on still holding the field. There is no need to dwell again on the old merits which have long since secured recognition, and, as to old slips, such as there were have disappeared editions ago.

But the present notice must not be purely formal. Drs. Burnside and Panton do not rest content with stereotyping what has made its mark; they can always see room for improvement and expansion. The new edition appears in two volumes, and not like its predecessors in one only. This is a good thing. That the part which every mathematical schoolboy ought to possess, and the part which appeals to senior students should be procurable separately, is convenient for both classes of purchasers, and may not prove unprofitable to the publishers. Besides, the third edition was already too bulky for easy handling, and 82 additional pages have had to be found room for. The changes made in the old matter do not call for much remark. Suffice it to say that there is occasional rearrangement, and a fairly considerable addition to the number of worked examples which are of higher mathematical interest. Some additional space is devoted to semivariants, more particularly in a note which reproduces an investigation bearing on the question as to how many there are of a given type. The truly noteworthy addition to the book is, however, that of an entirely new last chapter of 50 pages, which marks quite a new departure in an English work on Equations.

This deals with the Theory of Substitutions and Groups of Substitutions, and its application. We have heard the slighting remark that English books on the 'Theory' of Equations do not justify their title, since the Resolvent 'Theory,' of which modern foreign treatises are full, has so far been absent from them. The stricture, no doubt far too sweeping, has yet been not without plausibility, so long as even the scientifically conceived, and, within its scope, thorough text-book of our present authors has only introduced reference to Abel and Galois once, and in a historical note. It may now be relegated to the well-grown category of taunts which are obsolete; for a new start has been made, and the one old defect in the admirable work which we are considering exists no longer.

Of course in but fifty pages of the bold type and ample spacing, for which poor eyes bless the press of the University of Dublin, there has been only room for a very brief treatment of so wide a theory as that of Substitutions and Resolvents. In the fifth edition we shall have more, no doubt. We are thankful, however, for what appears to be a very judicious selection from a great mass of material, and for an exposition which well exhibits the beauty of the Substitution-Group idea, and indicates convincingly the manner in which it has been brought to bear on deciding vexed questions of algebraical solubility. The subject is fascinating, but, as anyone who has tried to lecture on it will know, most difficult to grasp and to expound. Our authors are to be congratulated on their success in showing briefly its power as well as its attractiveness.

A final note of two pages deals with Abelian Equations. Let us hope for an ampler treatment of them in the next edition.

E. B. E.