point, the diffused light of a dull December day being quite sufficient to act upon the selenium*.

But I do not at present attach any serious importance to such practical applications of these devices. I regard them simply as affording somewhat attractive illustrations of the effect of light upon the resistance of selenium.

XXVIII. *Visibility of Interference-Fringes in the Focus of a Telescope.* By ALBERT A. MICHELSON.†

WHEN the angle subtended by an object viewed through a telescope is less than that subtended by a light-wave at a distance equal to the diameter of the objective, the form of the object can no longer be inferred from that of the image. Thus, if the object be a disk, a triangle, a point, or a double star, the appearance in the telescope is nearly the same.

If, however, the objective is limited by a rectangular slit, or better by two such, equal and parallel, then, as has been shown in a former paper‡, the visibility of the interference-fringes is in general a periodic function of the ratio of α , the angular magnitude of the source in the direction perpendicular to the length of the slits, and α_0 , the "limit of resolution."

The period of this function, and thence $\frac{\alpha}{\alpha_0}$, may be found with great accuracy; so that by annulling the greater portion of the objective the accuracy of measurement of the angular magnitude of a small or distant source may be increased from ten to fifty times. As ordinarily understood, this increase of "accuracy" would be at the cost of "definition" (which in this sense is practically zero); but if by "definition" we mean, not the closeness of the resemblance of the image to the object, but the accuracy with which the form may be inferred, then definition and accuracy are increased in about the same proportion.

In almost every case likely to arise in practice, the form of the source is a circular disk; and if the illumination over its surface were uniform, the only problem to be solved would be the measurement of its diameter. But in many cases the distribution is anything but uniform. If the curve repre-

* Experiments with the bell and the lamp were shown at the meeting.

† Communicated by the Author.

‡ "On the Application of Interference Methods to Astronomical Measurements" (Phil. Mag. July 1890).

senting the distribution along the radius: be $i = \psi(r)$, then the element of intensity of a strip $y_1 dx$ will be

$$\int_{-y_1}^{y_1} \psi(r) dy = \phi(x),$$

and it has been shown that the visibility-curve in this case is

$$V = \frac{\int \phi(x) \cos kx}{\int \phi(x) dx}.$$

This may be proved as follows:—

The intensity of the diffraction-figure of a luminous point in a telescope with a symmetrical aperture is *

$$I^2 = \left[\iint \cos \kappa \mu_1 x_1 \cos \kappa \nu_1 y_1 dx_1 dy_1 \right]^2, \quad . \quad . \quad . \quad (1)$$

in which $\kappa = 2\pi/\lambda$, μ_1 and ν_1 are the angular distances from the centre of the image, and x_1 and y_1 are the coordinates of the element of surface of the aperture.

If μ and ν are counted from the axis of the telescope and x, y, r , are the coordinates of the luminous point, the expression becomes

$$I^2 = \left[\iint \cos \kappa \left(\mu - \frac{x}{r} \right) x_1 \cos \kappa \left(\nu - \frac{y}{r} \right) y_1 dx_1 dy_1 \right]^2. \quad . \quad (2)$$

If now the source is a luminous surface whose elements vibrate independently,

$$\bar{I} = \iint I^2 dx dy. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the case of two equal apertures † whose centres are at $x_1 = -\frac{1}{2}a_{11}$ and $x_1 = +\frac{1}{2}a_{11}$,

$$I_{11}^2 = I^2 \cos^2 \frac{1}{2} \kappa a_{11} \left(\mu - \frac{x}{r} \right). \quad . \quad . \quad . \quad (4)$$

This substituted in (3) gives

$$\bar{I} = \iint I^2 \cos^2 \frac{1}{2} \kappa a_{11} \left(\mu - \frac{x}{r} \right) dx dy.$$

* ‘Wave Theory of Light,’ Rayleigh.

† More generally, for m equal equidistant apertures whose total area is constant,

$$I_m = I \frac{\sin \frac{1}{2} m \kappa \mu a}{m \sin \frac{1}{2} \kappa \mu a}.$$

Putting $\kappa a_{11}\mu = \mathfrak{S}$, $\kappa a_{11}/r = k_{11}$, and expanding,

$$2\bar{I} = \iint I^2 dx dy + \cos \mathfrak{S} \iint I^2 \cos k_{11} x dx dy + \sin \mathfrak{S} \iint I^2 \sin k_{11} x dx dy. \quad (5)$$

Let $y = \phi(x)$ be the equation of the curve bounding the luminous surface; or, better, let $\phi(x)dx$ be the "total intensity" of a strip of width dx .

Denoting $\int_{-\phi(x)}^{+\phi(x)} I^2 dy$ by $F(x)$, and omitting the factor 2, equation (5) becomes

$$\bar{I} = \int F(x) dx + \cos \mathfrak{S} \int F(x) \cos kx dx + \sin \mathfrak{S} \int F(x) \sin kx dx,$$

or $\bar{I} = P + C \cos \mathfrak{S} + S \sin \mathfrak{S}. \quad . \quad . \quad . \quad . \quad . \quad (6)$

If the width of the apertures is small compared with their distance, the variations of $F(x)$ with μ (or $\mathfrak{S}$) may be neglected, and in this case the maxima or minima occur when

$$\tan \mathfrak{S} = \frac{S}{C}, \quad \text{or when } \bar{I} = P \pm \sqrt{C^2 + S^2}.$$

If now the visibility of the interference-fringes be defined as the ratio of the difference between a maximum and an adjacent minimum to their sum

$$V^2 = \frac{C^2 + S^2}{P^2},$$

or

$$V^2 = \frac{[\int F(x) \cos kx dx]^2 + [\int F(x) \sin kx dx]^2}{[\int F(x) dx]^2}. \quad . \quad . \quad . \quad (7)$$

For narrow rectangular apertures,

$$F(x) = \int_{w_1}^{w_2} \frac{\sin^2 w}{w^2} dw.$$

In this expression, if $v=0$ and b =length of aperture,

$$w_1 = \frac{\kappa b}{2r} \phi_1(x) \quad \text{and} \quad w_2 = \frac{\kappa b}{2r} \phi_2(x).$$

So long as

$$\frac{2\phi(x)}{r} < \frac{\lambda}{b},$$

$F(x)$ is nearly proportional to $\phi(x)$; that is, so long as the angle subtended by the source is less than the limit of resolution of a telescope with aperture b , the brightness is pro-

portional to the size of the object. For larger angles the proportionality may still be made to hold by a slight alteration in the focal adjustment ; and to this degree of approximation we have

$$V^2 = \frac{[\int \phi(x) \cos kx dx]^2 + [\int \phi(x) \sin kx dx]^2}{[\int \phi(x) dx]^2} \dots \quad (8)$$

If the source is symmetrical the second term vanishes, and the expression reduces to the original form.

It is possible that, in addition to the uses already mentioned, the "visibility-curve" may have an important application in the case of small spherical nebulæ. For from the form of this curve the distribution of luminous intensity in the globular mass may be inferred, which would furnish a valuable clue to the distribution of temperature and density in gaseous nebulæ.

When the source is so small as to be indistinguishable from a star, it would seem that this method is the only one capable of giving reliable information ; but even in the case of bodies of larger apparent size it is equally applicable, may be made to give results at least as accurate as could be obtained by photometric measurements, and is far more readily applied.

XXIX. *On the Heating of Conductors by Electric Currents, and on the Electric Distribution in Conductors so heated.* By J. McCOWAN, M.A., B.Sc., Assistant Lecturer on Natural Philosophy, University College, Dundee*.

SO far as I am aware, no problems in thermal distribution involving an internal generation of heat—such, for instance, as takes place in the passage of an electric current through a conductor—have hitherto been discussed. Fourier, the founder of the mathematical theory of thermal conduction, takes no account of possible internal sources of heat in forming his equations, and considers only problems in which the sources are superficial. The same may be said of Poisson, Lamé, Riemann, and others who have since applied and extended Fourier's methods. Thomson, in his synopsis of the mathematical theory †, has expounded Fourier's diffusion-analysis in a form directly applicable to cases where there are internal

* Communicated by the Author. Read before the Edinburgh Mathematical Society, January 9th, 1891.

† Appendix to article "Heat," *Encyc. Brit.* 9th edition.