have a room of its own for the purpose) ; and to the retiring members
of the Council for their services.

Mr. Tucker then read the following communication from Mr. G. O.
Hanlon, on

The Vena Contracta. By G. O. HANLON, Esq.

THE Vena Contracta has a very important bearing on the calculation
of the momentum of water flowing through an orifice, and the omission
of its consideration has given rise to mistakes in calculation which it is
the object of this paper to point out. It was stated in a controversy on
the " Nozzle Ship," about two years ago, in the " Engineer," by a
gentleman well known for his scientific attainments, that when water
flowed through an orifice in a tank, the pressure on the side of the tank
opposite to the orifice became doubled. Nothing could be more erro-
neous, and I pointed out, in a letter to that journal, the error of such a
supposition. The controversy, however, at the time had ended, so that
my views on the matter, although published, were not commented on.
I will begin by investigating the subject analytically, and will subse-
quently see how far my result agrees with observation.

I will proceed in my calculation as if the pressure remained the same
as before the orifice was opened ; which is clearly the case, since the
only pressure on the opposite side, after the orifice has been opened, is
the height of water above any area; and it is easy to see that the *motion*
of the water through the orifice could not add to the *pressure* of the water,
since it is that very pressure which produces the motion. Indeed, it may
be taken as an axiom, that where a force produces any motion, it is
impossible for that motion to add to the original force. The very idea
contains the solution of the problem of perpetual motion ; yet, simple
as the axiom appears, it is frequently overlooked by writers on Me-
chanics ; indeed, the error pointed out in the present paper is an instance
of such oversight.

Let $x = fy$ be the equation to the curve formed by the surface of the
rushing water in the plane of its least sectional area parallel to the
plane of the orifice, where the axis of X is vertical, and that of Y is
horizontal, the axis of X being wholly outside the orifice. Let a and b
be the two values of x when y has equal roots, and let y' and y'' be the
two values of y corresponding to any value of x. Then, the mass of
rushing water varying as the velocity, the momentum will be repre-
sented by the square of the velocity multiplied into the area. The
whole momentum will therefore be

$$\int_a^b (y' - y'') \, dx \, . \, 2gx.$$

This must equal the pressure in the opposite direction, which equals

gAC, where A is the area of the orifice, and C the distance from the surface to the centre of gravity of the orifice.   But

$$\int_{a}^{b} (y' - y'')\, x\, dx$$

equals the area of the curve $x = fy$ multiplied into the distance of its centre of gravity from the surface, which we will put equal to A'C';
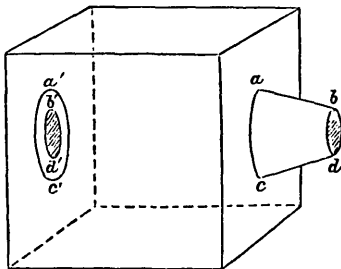
therefore gAC $= 2g$A'C'.

If we suppose C=C', we have   A = 2A'.

Whether C always equals C', or is always greater or less, is a result which probably can never be determined; but from actual measurement it appears that A=2A', and if this is true it follows that C=C'. But it is certain that the greater the ratio between the height of water and the vertical height of the orifice, the more nearly do the centres of gravity coincide (*i.e.*, C=C'); and on this assumption we may conclude that, generally, the least sectional area of the flowing stream is half that of the orifice.   This gives the coefficient of contraction equal to ·5, which agrees with the result obtained by actual measurement, and yet *was obtained on the supposition that no alteration of pressure took place when the water flowed through the orifice.*   Now I will endeavour to show the reason of the above practically.

Let us suppose an orifice *ac* cut in the tank in the figure, and that the shape assumed by the flowing water is *ab*, *cd*, where the area of the section *bd* equals half that of the section *ac*.   Now let us suppose a nozzle *abcd* to be fixed to the orifice, exactly the shape the water takes in flowing, and let us suppose the end of the nozzle *bd* closed, and the water at rest.   Let *a'c'* and *b'd'* be the projections of the sections *ac* and *bd*. There is a horizontal pressure on *ab* and *cd* all round the side of the nozzle equal to the pressure on the section *bd*, since it is equal to the pressure all round *a'b'*, *c'd'*, which is by hypothesis equal in area to *b'd'*. Now let us remove the stopper *bd* at the end of the nozzle.   The result is that the pressure ceases all round the nozzle, while the flowing water through *bd* gains that pressure which was round it, and consequently the pressure there is doubled; but that double pressure, since there is now no pressure from *ab*, *cd*, to give a corresponding pressure to *a'b'*, *c'd'*, has to be spread over *a'c'*, which is twice the area of *bd* or *b'd'*, and consequently this pressure on *a'c'* remains the same as before the stopper was taken from the nozzle.