and there was no reason on paper why it should not be equally well done by two cylinders. It did not prove that three cylinders were better than two, but simply that it was better to use 150 lb. pressure than 50 lb. There were important practical questions which might make it inconvenient to use the high pressure in two cylinders, and there were also reasons for not doing so connected with the range of the temperature, with initial condensation, etc. He believed that inefficiency in the low pressure cylinder was caused mainly by defects of passages, by defects of receiver, etc., and could be remedied by alterations outside the low pressure cylinder itself. Large economies should be looked for rather in improvement in high pressure cylinders than in low pressure cylinders at present.

Mr. Mudd (Hartlepool) thought that the thanks of the Institution were specially due to Mr. Wyllie at this time, when every one was engaged in the design and manufacture of triple expansion engines, working out his own ideas, and feeling his way through the practical difficulties which higher pressure and higher temperature had brought into existence, and when the principle of triple expansion was only just emerging from the experimental stage. Indeed, he thought that the paper ought to be regarded as a kind of legacy left to the Institution and to the profession at large. The author stated that the most important conditions to be considered in order to obtain an efficient engine were that there should be approximate equality, first, in the range of temperature in each cylinder; secondly, in the initial strength of each crank; and thirdly, in the indicated horse power of each engine. That was much too general a statement; at all events, he was not willing to admit it without considerable additions and limitations. It was either too wide or too narrow a statement to correspond with the facts. It was possible for an engine to comply precisely with those conditions, and at the same time to be working most inefficiently; and therefore it was clear that those were not the whole of the conditions that determined the efficiency of the engine. If they were the test of efficiency in a triple expansion engine, they must also be so in an ordinary compound engine and in a single engine, but the test would utterly fail in this last case. The efficiency of a triple expansion engine, like that of every other engine, was dependent on a large number of conditions which might be classified under two heads, namely, those which affected the efficiency of the steam and those which affected the efficiency of the mechanism. It seemed clear from other parts of the paper that the author intended only to allude to the question of efficiency of the steam, and not to the efficiency of the engines as a whole. The range of temperature in the cylinder was the one point of the three named by Mr. Wyllie which had a direct bearing on the efficiency of the steam. With regard to steam jackets, he wished to say that if anyone cylinder was to be jacketed, it should be the low pressure cylinder, although specifications frequently stated that high pressure cylinders were to be jacketed.

Mr. D. Adamson said he had had some experience in regard to triple and quadruple action engines many years ago, but he had had no experience in regard to marine engineering. He had expressed his opinion that, instead of continuing the use of one cylinder, they ought, now that they had large engines and large shafts, to distribute the percussive force over many crank pins, and he was glad that Mr. Wyllie had brought the subject before them so prominently. He had several structures working at 250 lb. pressure. If they went beyond that, he thought a quadruple action engine would be required. There was something yet for young engineers to do in improving the economy of steam engines. He believed that dividing the force over three or four crank pins was much better than running the risk of breaking the shaft by putting the whole force on one. He was convinced by experience that a quadruple engine would beat the triple cylinder immensely. He was not alarmed at the irregularities pointed out by Professor Kennedy. The cards might be a little wrong and irregular, but they were the best means they had of getting the results. Mr. Parker's observations had demonstrated the real economy of the triple engine. He had had some experience of high pressure steam. He had 3,000 boilers working at very high pressures, nearly all of steel, without having had a single explosion. His own experience was that metals did not suffer until they got to 500°. With a comparatively pure metal they might have 550° temperature before it became dangerous, or even 600° in the case of a pure iron with no alloy. He thought there was a prospect of 200 lb pressure at least being obtained by the multiple system. To work the triple action efficiently, the system of triple cranks should be adopted.

Professor Smith, of Birmingham, said he thought that after the criticisms of Professor Kennedy, they required some further explanations as to the calculations made by the author. He was quite willing to believe in the evaporation of 9, 9½, or 9¾ lb. of water per lb. of fuel, but he was very skeptical with regard to 10 lb. and upward. He thought that the greater efficiency of the triple expansion engines was chiefly due to the possibility of using a very much higher initial pressure, but the question still remained as to whether they ought to have more than two cylinders. In order to distinguish between the efficiency of the boiler and of the engine, it was only necessary to give separately the mechanical horse power of the boiler and the horse power of the engine, to compare the one with the other, and it would generally be found that the engine horse power was greater than the mechanical horse power of the boiler.

VOUSSOIR CALCULATION AND MEASUREMENT.

By C. POWELL KARR, C.E., Consulting Architect, New York.

AN arch of masonry consists of a ring of wedge-shaped stones, called arch stones or voussoirs, pressing against each other at surfaces called bed joints, which are or ought to be perpendicular or nearly so to the soffit or internal concave surface of the arch. The outer or convex surface of the ring may be either a curved surface parallel to the soffit or a series of steps to sustain the vertical pressure of that part of the load which is superimposed upon them. The projection of

the soffit upon a vertical plane is the curved line of the arch, and is called the intrados. The upper limiting line of the arch stones, whether it be parallel to the intrados or whether it be made up of the broken horizontal-vertical line of the steps, is called the extrados.

The highest point of the intrados is the crown. The line or bed from which the arch springs is the spring line. If an arch be built in a wall connected with other work, the supporting beds of the arch are called its abutments. The supports to two successive arches are called piers. The sides of the arch are called the haunches or reins. The vertical walls included between the haunches and an imaginary line passing through the spring line parallel to a line passing through the extrados above the keystone are called the spandrels. The supporting blocks, or the two voussoirs at the spring line, are the skewbacks.

In addition to their own weight, arches usually support a permanent or moving load or surcharge of earth or masonry or both. Suppose an arch were divided vertically at the crown by a plane at right angles to the span into two equal homogeneous semi-arches. Suppose one of these semi-arches to be removed, and it were possible to ascertain the force which is just necessary to keep the remaining semi-arch in its place. This force is the thrust of the arch, and is called the horizontal thrust. It is equivalent to the mutual pressure which exists between the two semi-arches in position to keep them in equilibrium. The total thrust of the arch is the sum of the vertical and horizontal thrusts, and in an equilibrated arch this total thrust is transferred to the abutments or piers. The limits of this article will not permit of an inquiry into the theory of the arch or the principles of voussoir design, and we can only make a passing reference to them.

In a general way, the thickness of an arch is assumed to correspond with some precedent or, in the absence of such, with the results of some empirical rule; then the load which it must sustain is calculated, the location of the line of resistance is plotted graphically or calculated analytically, and the thickness of the arch is so adjusted that the centers of resistance of the voussoir joints shall fall within the middle third space of the depth of the arch ring. A most unscientific method, it is admitted, but it is the one which is generally followed. The thickness of the arch is the criterion for the depth of the keystone. Captain Woodbury's special rule for designing an arch is the safest guide. It is: Depth of arch at the key in feet is twice the actual thrust acting one-third of the length of the joint from the extrados divided by the pressure (P) per unit of surface at the upper edge of the keystone joint. Captain Woodbury also remarks that in light arches the thickness at the key is found to vary from one thirtieth to one thirty-third of the span, and we will call this his general formula. The writer's observations and study of existing structures would lead him to ask of engineers a modification of Mr. Trautwine's formula for circular and segmental as well as elliptic arches by changing the constant in the denominator to 3.5 instead of 4, as Mr. Trautwine has it. We will present a table of comparisons. My rule is—

$$D = \frac{\sqrt{\text{radius} + \frac{1}{2} \text{span}}}{3.5} + 0.2 \text{ ft.}$$

TABLE OF DEPTH OF KEYSTONES BY EMPIRICAL FORMULÆ.

Edinburgh Bridge, by Mylne. Span, 72 ft.; rad., 36 ft.; depth of key, 2.75.

By Perronet's rule..... Ft. 4.20

$$\text{Formula: } D = 13 \text{ in.} + \frac{\text{span}}{23}$$

By Trautwine's rule..... 2.32

$$\text{Formula: } D = \frac{\sqrt{\text{radius} + \frac{1}{2} \text{span}}}{4} + 0.2 \text{ ft.}$$

By Rankine's rule..... 2.07

$$\text{Formula: } D = \sqrt{0.12 \times \text{radius}}, \text{ or } D = \sqrt{0.17 \times \text{radius for arch series.}}$$

By Woodbury's rule..... 2.40

$$\text{Formula: } D = \frac{S}{30}, \text{ or } D = \frac{2 \times \text{horizontal thrust}}{P}$$

By Hurst's rule..... 2.54

$$\text{Formula: } D = C \sqrt{\text{radius}}. \text{ } C = 0.3 \text{ for stone; } 0.4 \text{ for brick.}$$

By Karr's rule..... 2.62

$$\text{Formula: } D = \frac{\sqrt{\text{radius} + \frac{1}{2} \text{span}}}{3.5} + 0.2 \text{ ft.}$$

Chestnut St. Bridge, Philadelphia. Span, 60 ft.; rise, 18 ft.; radius, 34 ft.; actual depth of key, 2.50 ft.

By Perronet..... 3.68 ft.

" Trautwine..... 2.20 "

" Rankine..... 2 "

" Woodbury..... 2 "

" Hurst..... 3.10 "

" Karr..... 2.48 "

Dean Turnpike Bridge, near Edinburgh, Scotland.

By Telford. Segmental arch; span, 90 ft.; rise, 30 ft.; radius, 48.90 ft.; actual depth of key, 3 ft.

By Perronet..... 6.05 ft.

" Trautwine..... 2.62 "

" Rankine..... 2.88 "

" Woodbury..... 3 "

" Hurst..... 2.10 "

" Karr..... 2.97 "

Wissahickon Bridge, Philadelphia. 65 ft. span; 23 ft. rise; depth of arch stones, 3 ft.; radius (by calculation), 57.42 ft. Series of arches.

By Perronet..... 3.90 ft.

" Trautwine..... 2.57 "

" Rankine..... 3.12 "

" Woodbury..... 2.17 "

" Hurst..... 2.27 "

" Karr..... 2.90 "

It will be seen, upon comparison, that the proposed modification of Mr. Trautwine's rule is much nearer, upon the whole, than his own or any of the others. In justice to Mr. Woodbury, the writer would say that the figures opposite his name are derived from a form-

ula founded upon the observations of that noted writer, which we call for distinction his general formula, and that a formula which depends for its factors upon the thrusts as determined from the actual loads which the arch must bear, or the strains it must meet, would be more in accordance with the best practice, and the truth, than to try to rely upon any empirical formula, no matter how near the truth such a formula may seem to be. Captain Woodbury's special rule, as first given, is more in accordance with this principle, as it is partially founded upon the forces in action to maintain equilibrium; but it could not be applied in our comparisons, because the loads which the various bridges have to sustain are not to be had readily, and, therefore, two of the elements of his formula could not be obtained; but in designing an arch, these, with the exception of the point of application of the horizontal thrust, are the known elements upon which the arch is to be based, and the formula can therefore be used. Whenever the word "crown" is used in this article, the nomenclature of Prof. Rankine will be adopted, viz., the "crown" is the highest point of the intrados, not of the extrados, as some writers use it. These formulæ should only be used as clues to the discovery of the depth of the keystone, and therefore of the arch itself, for the final determination should be made by means of the graphical method of determining the line of resistance, and proportioning the depth of the arch to the depth of the load it is obliged to sustain.

THE PRINCIPLES INVOLVED IN CALCULATION.

For the purposes of preliminary calculation, the only part of a voussoir we need to draw or know is its face. An examination of all voussoir calculation will discover everywhere a constant reference to and application of the relations of the different parts of a triangle to each other. It need not always be a right-angled triangle, but in the analytical method it generally is. Given two parts of a right-angled triangle, we can solve all the rest. In doing so we call to our aid the formulas obtained from plain trigonometry. In every right-angled triangle, the two sides adjacent to each other which include the right-angle between them are considered, the one side the base and the other side the perpendicular, and it does not matter which side is considered the perpendicular or the base, the side opposite the right angle is the hypotenuse. The angle included between the base and the hypotenuse is called the angle opposite the perpendicular, or the angle at the base. It is the angle which is the most constantly used in obtaining the value of the perpendicular. It is not necessary to know all the formulas which pertain to the right-angled triangle in order to solve the unknown side or angle. The formulas in general and their derivatives are as follows:

1. The perpendicular is equal to the hypotenuse into the sine of the angle at the base, or $P = H \times \text{sine of}$

$$\text{angle at the base; also, sine} = \frac{P}{H}$$

(2.) The perpendicular is equal to the base into the tangent of the angle at the base, or $P = B \times \text{tang. of}$

$$\text{angle at the base; also, tang.} = \frac{P}{B}$$

(3.) The perpendicular is equal to the square root of the hypotenuse squared minus the square of the base, or $P = \sqrt{H^2 - B^2}$

(4.) The base is equal to the hypotenuse into the cosine of the angle at the base, or $B = H \times \text{cosine of}$

$$\text{angle at the base; also, cosine} = \frac{B}{H}$$

(5.) The base is equal to the perpendicular divided

$$\text{by the tangent of the angle at the base, or } B = \frac{P}{\text{tang.}}$$

$$(6.) B = \sqrt{H^2 - P^2}$$

$$(7.) H = \sqrt{P^2 + B^2}$$

(8.) The hypotenuse is equal to the perpendicular divided by the sine of the angle at the base, or

$$H = \frac{P}{\text{sine of angle at base}}$$

(9.) The hypotenuse is equal to the secant of the angle at the base into the base, or $H = B \times \text{secant.}$

From geometry we have:

(10.) Two triangles which are mutually equiangular have their homologous sides proportional, and conversely.

(11.) Two triangles are similar when they have their homologous sides parallel each to each or perpendicular each to each.

(12.) In a right-angled triangle, if a perpendicular is drawn from the right angle to the hypotenuse:

1. The triangles on each side of the perpendicular are similar to the whole triangle and to each other.

2. The perpendicular is a mean proportional between the segments of the hypotenuse. 3. Each of the sides is a mean proportional between the hypotenuse and its segment adjacent to that side.

Wherever similarity between two triangles can be detected, the discovery is rewarded by a summary calculation. If you are not familiar with the methods of graphical calculus, borrow an idea from the economy of that beautiful science. Its fundamental basis of comparison is the triangle, and the product of lines and areas is effected by means of the properties of similar triangles. Wherever similar triangles can be brought in, either by a constructed line or otherwise, to aid in the calculation, the solution can at once be proceeded with, and its accuracy and celerity is astonishing. By arbitrarily fixing the lengths of the chords of the voussoirs, as may be done sometimes, an entire set of voussoirs may be calculated without reference to the value of more than one angle; and sometimes, under favorable conditions, even that may be dispensed with, as shown in our first and simplest calculation. Endeavor, therefore, to divide the voussoir faces in such a manner by parallel lines or other constructions so as to establish symmetrical relations, so that the quantities involved can be thrown into a proportion, and the unknown term solved at once. In the solutions given, we have in every case used the values of natural sines, tangents, and cosines which are the values of an arc whose radius is unity, as fewer mistakes by the novice are likely to occur than in

using the logarithmic sines, etc., unless the latter are in constant use. When the value of the radius is a whole number, the natural method is the shortest.

The student will at once perceive from the calculations and the drawings that no allowance has been made for joints. Whatever joint thickness has been decided upon, such thickness is appended in a footnote to the drawing, and the stonemason must make his own reductions accordingly. Thus, if the instructions in the footnote read, "All joints to be three-eighths of an inch at most," the stonemason must subtract three-sixteenths of an inch from each face except the soffit face, in order to get the exact dimensions of the face. Where the section of a shaft intersects a tunnel, and voussoirs have to be constructed to fit into the curve of intersection, the voussoir stones will be of such a shape, having two curved faces not parallel to each other, that it will be impossible to convey to the stonemason by means of elevations or sections an adequate idea of the contour of the voussoir. In such a case resort must be had to isometric perspective views from various standpoints, and in extremely difficult cases resort must be had to miniature models of wood

each voussoir on the bed joint, taken on a line passing through the diametral section, such line being a continuation of the shaft diameter as shown in the drawing. At the start we are informed that the circumference of the shaft is to be divided into four equal parts by the intersection of the radial joints. This fact at once determines the uniformity in size of all four voussoirs. Then we know, from having the size of one stone, further calculation will be unnecessary. The method of procedure is as follows: 1st, to find the length of the chord subtended by the arc. The circumference is divided into four equal parts by the radial joints; lines joining these points, as shown in Fig. 1, will form the chords of the arcs, and also the side of the square which may be inscribed in the circle. This geometrical fact enables us to find the chord at once. To find the chord referred to the radius, we multiply the radius by 1.4142; or, referred to the diameter, by 0.7071. These factors are thus obtained: Suppose we have a circle whose radius is unity; inscribe a square within that circle. The diagonals of the square will intersect at the center of the circle, and are diameters of the circle. The semi-diagonals form two equal sides of a

right-angled triangle, and are radii of the circle. The chord of the intercepted arc or one side of the inscribed square is the hypotenuse of this triangle. These remarks apply equally to each of the four triangles. On the principle of geometry that the square on the hypotenuse is equal to the sum of the squares described on the two other sides (see formula 7), we can solve the hypotenuse of any one of these right-angled triangles. Each chord of the intercepted arc is a hypotenuse. Let the radius be 1, then $1^2 + 1^2 = 2$, and $\sqrt{2} = 1.4142$, which is therefore the chord expressed in feet of the intercepted arcs. E. D. Thus we see that the factor of multiplication referred to the radius is 1.4142; or, referred to the diameter, which is twice the radius, is 0.7071. Upon inspecting the geometric figures enclosed by the chords and a line parallel to one of them through the outside edges of the radial joints (as shown in Fig. 1— xz), we perceive the triangles so enclosed are homologous, therefore the side of the smaller triangle; $c'z' : c'z :: z'x' : z'x$ (see equation 10), or $2' : 3'3'' :: 2'9\frac{1}{16}'' : x$; $4 \times 0.7071 = 2.8284 = 2'9\frac{1}{16}''$ hence $x = \frac{2.8284 \times 3.25}{2} = 4.5961' = 4'7\frac{5}{32}''$. The triangle xcz is an isosceles triangle, the angle xcz is a right angle, the line cy bisects the side xz and the right angle xcz ; therefore, the angle ycz is equal to ycx , both angles are 45° , and hence the side cy is $= yz$; but yz is equal to $\frac{1}{2}$ of xz or to $2.2978'$. By construction, the distance from the center of the circle to the outside edge is equal to $2'0'' + 1'3''$ or $3'3'' = 3.25$ feet; therefore the external unknown distance in the corners is equal to 3.25 feet minus 2.2978 feet, equal to 0.9522 feet, equal to $11\frac{3}{4}''$ inches.

FIG. 1.
PLAN OF HORIZONTAL VOUSOIRS.

made to the scale preferably of one inch equals one foot.

In order to present the subject in a simple and, at the same time, comprehensive manner, we have selected for our illustrations from actual construction a plain set of horizontal voussoirs, on account of their simplicity; then a set of typical vertical voussoirs, which are the most frequently met with. They are by no means to be taken as a model of design, for they are too heavy for such a small span, but, in their outline and the method of their construction, they exemplify the principles involved in an arch of greater span and more intricate construction. We have concluded the calculations by referring to a special case of horizontal voussoirs from actual construction, which involve some of the difficulties sometimes met with in voussoir calculation. Where it is possible to draw an arch to a scale of three inches equal one foot, and by using a beam compass, great accuracy and delicacy of line work may be done, so that all the distances by construction can be scaled off on a decimal scale, the thousandths part of a foot can be read off, and no analytical work need be entered into if the drawing is accurately made; but this method is seldom pursued.

In Fig. 1 we have a plan of four horizontal voussoirs designed for the connection of a watertower shaft with its sewer outlet. The faces shown in the drawing form the bed joints for the superincumbent masonry, and

Summary of process: 1st. Ascertain the length of the chord. 2d. Form a proportion of the sides of the two homologous triangles to obtain the outer unknown side. 3d. Bisection of the calculated side to obtain the altitude of the large triangle. 4th. Subtraction of the altitude from the total distance of the outermost edge of the stone to the center of the shaft, to find the last unknown dimension required.

In Fig. 4 we show the elevation of a typical full centered arch. The arch is divided into five voussoirs

FIG. 2.
ISOMETRIC PERSPECTIVE
SKETCH OF STONE NO 2.

therefore are not seen upon the completion of the well shaft masonry. The curved faces above are visible; see Fig. 2. In deciding upon the lateral dimensions of such stones, it is necessary to know what the horizontal pressure from without and within will be. The weight supported by the stone must also be ascertained. The vertical depth may or may not correspond with the depth of the adjoining course of masonry, as its depth may be governed by other conditions, which should be studied. The elements of voussoir design stand upon debatable ground, and the thickness of any given arch under every possible load is still an unknown quantity. We shall confine our remarks in this article to calculation and measurement, to pointing out the desirable pathway of method and simplicity to students and beginners, and we do not pretend to make an exhaustive exposition of the subject.

In the drawing submitted (Fig. 1), the data given are: 1st. The diameter of the shaft; 2d, the length of the radial joints and their number; and 3d, the width of

having equal chords. As equal chords subtend equal arcs, and as equal arcs measure equal angles at the center, it follows that the angle 180° subtended by the full centered arch is divided into five equal angles, or 36° each.

In planning the dimensions of a set of voussoirs to an arch whose intrados and extrados have been determined by a previous calculation, the chords of an even number of arcs are arbitrarily assumed at some convenient readily measured distance, and are made equal throughout, their angular measure is subtracted from the total angular measure of the arch, and the remainder becomes the angular measure of the keystone, provided it is greater than the angular measure of the chord of one of the even number of arch stones; or the keystone chord is arbitrarily fixed, either by some empirical rule or by careful calculation from the conditions imposed upon the structure, and the remaining arcs divided into an even number of arcs where such a division will not conflict with the conditions of the struc-

STONE-A AND REVERSE
OF STONE E.
FIG. 5.

thank you to establish your horizontal joints of your voussoirs first and plan your courses to correspond, so as to avoid difficult stone cutting and poor stone laying.

We have assumed a simple case in order that the demonstration may be the more readily understood, and have avoided the use of logarithms in order to make the calculation plain to those who have had no time to learn their use and become familiar with their application. The principles involved are applicable to the solution of the most difficult problems in stone cutting.

The order of procedure will be as follows: 1st, to find the chord common to all the arcs, thus making necessary but one calculation; 2d, to ascertain all of the horizontal joints consecutively, beginning at the spring line of the arch; 3d, to determine the vertical joints in regular order, beginning at the spring line.

STONE-B AND REVERSE
OF STONE-D.
FIG. 6.

1st. To find the chord of stone A (see Fig. 4), denoted by db . We see by the drawing that the line ac , drawn perpendicular to the chord, bisects that chord; from the principle in geometry that the diameter which is perpendicular to a chord bisects that chord, and forms two right-angled triangles acd and acb . The side dc or cb is equal to the side da or ba multiplied by the natural sine of the angle opposite cd or cb (see formula 1). The angle opposite either of these sides, by reason of bisection, is equal to 18° , as the angle at the center of the triangle daa is 36° ; hence $dc = da \times \text{nat. sine of } \angle acd$, and $bc = dc$; $dac = 18^\circ$; $ad = 2'0''$. The natural sine of 18° is 0.309017 . These quantities being substituted in the last equation, we find $dc = 0.6180$ feet; but $db = dc + cb$, therefore bd is equal to 1.2360 feet, equal to $1'2\frac{3}{4}''$. E. D.

This chord therefore is the chord of each of the five arcs by the conditions given.

2d. In the triangle acf , which is a right-angled triangle, we are to ascertain the distance ae . If we

know ae , we can find eh , which is equal to fg , by subtracting ae from ah , a distance which is given, i. e., $2' 9\frac{1}{2}"$.

ae is the base of the right-angled triangle $ae f$, and we solve it by the formula that the base is equal to the hypotenuse into the cosine of the angle at the base opposite the perpendicular (see formula 4), or $ae = af \times \text{nat. cosine of } fae$; $af = 2' 9\frac{1}{2}" = 2.7604$ feet. The natural cosine of fae or 36° is 0.809017 ; by substitution of these values in the last equation, we have $ef = 2.2332$ feet. Therefore ah minus ae , or 2.7604 feet minus 2.2332 feet, is equal to 0.5272 ft. = $6\frac{3}{4}$ inches or the horizontal upper joint of A and E. $\bullet$ E. D.

To obtain the horizontal joint of stones B and D, we perform a similar calculation with the triangle akm . The central angle is 72° , obtained by adding to its own the preceding chord angle, the dotted lines being placed in the opposite quadrant, so as to avoid confusion of lines. Having obtained ak (by formula 4), which by the way is just one half of the horizontal joint of the keystone, C, we subtract ak from ae , and obtain the horizontal joints of B and D. $\bullet$ E. D.

3d. To determine the vertical joint of A, we resort

KEY-STONE-C.
Fig. 7.

to the triangle $ae f$. This joint or fe is equal to gh , and is determined by the formula (1) that the perpendicular of a right-angled triangle is equal to the hypotenuse into the sine of the angle at the base opposite the required side. Therefore, $fe = fa \times \text{nat. sine of } 36^\circ$, as we have seen before; $fae = 36^\circ$, $fa = 2.7604$ ft.; natural sine of 36° is 0.587785 ; by substitution and multiplication we have $fa = 1.6225$ ft. = 1 ft. $7\frac{1}{2}$ in. $\bullet$ E. D.

To determine the vertical joint of B and D, we solve in the same way the triangle akm with respect to the side km ; by formula 1, $km = am$ multiplied by natural sine of 72° , equal to 2.6252 ft.; if from km or 2.6252 ft. we subtract fe or 1.6225 ft., we will have 1.0027 ft. = 1 ft. and $\frac{3}{8}$ in. or the vertical joint of B or D. $\bullet$ E. D.

Knowing the distance km and the distance an (2.7604 ft.), in order to find the vertical joints of the keystone, it is only necessary to subtract km from an , and we have 0.1352 ft. or $1\frac{5}{8}$ in. $\bullet$ E. D.

As the corresponding vertical and horizontal joints of stones D and E are respectively equal to those of B and A, it is unnecessary to solve them separately, and all the facial joints therefore are calculated.

As a check upon the calculation, it is wise to reverse the operation, that is, assume what was just proved to calculate what was first assumed. Thus, assume the vertical joints and chords, and calculate the radial joints and horizontals. By this proposition, in the triangle $ae f$, we know the angle at the center to be 36° , also the vertical distance fe or 1.6225 ft., to find the side af . Now $af = fe$ divided by the natural sine of fae , $1.6225 \div 0.587785 = 2.7604$; but ba or the radius is = to $2' 0"$, $bf = af$ minus ab , therefore $bf = 0.7604$ ft., or $9\frac{1}{8}$ in.

In Fig. 14 we have a plan of a set of horizontal voussoirs such as are seen in the water towers of gate houses. The set shown here are designed for the intersection of a horizontal outlet to the tower; the cross section of the outlet at the point of intersection is rectangular, therefore the upper surfaces and bed joints are parallel to the horizontal course joints. The rising waters from

the tower pour over the edges of the intersection into the chamber beyond, not shown in the drawing. Stones Nos. 4 and 10 are the foundations of a pier located in the center of the outlet to divide the inflowing water into two currents. The unfinished part of the circle is the brick wall of the tower. In the calculations necessary to obtain the dimensions of the voussoirs, the procedure was as follows:

The foundation of the central pier in the water chamber beyond the well curb was to be 11 feet 10 inches long, point to point, and to be three feet wide. This foundation was divided into two stones, Nos. 4 and 10. In showing the method pursued to calculate the dimensions of stones Nos. 8 and 12, it will be shown how the center of the tower was located with reference to the facades of the building. Conceiving the center of the tower to be properly located, we assume that the diameter of the circle continued passes through the center line of the pier, as indicated in the drawing. The arc intercepted by the sides of the pier is therefore meas-

ured by a chord of three feet. This stone becomes the keystone of the arch. From each side of the keystone two chords of three feet each are laid off by scale, and the remaining chords, which belong to stones Nos. 1 and 7, are calculated. They are equal to each other.

To ascertain the chords to these stones, we must find the angles at the center subtended by them. We have all the premises necessary to do this. If from the angle subtended by the diameter, which is 180° , we subtract the sum of the angles at the center of all the known chords, we will obtain the sum of the angles at the center subtended by the two equal unknown chords.

Take any one of the known chords, as the keystone, for instance. The radius drawn perpendicular to the chord will bisect it and the angle it subtends. Conceive the bisection to have taken place by a radial line drawn as indicated, connect the intersections of the chord and the arc with the center of the circle, also the point of bisection, and we have a right-angled triangle (abc) whose perpendicular (cb) we know, by reason of bisection, to be 1 ft. 6 in., and its hypotenuse (ac), or radius of the circle, to be, by construction, 6 ft. 6 in.

Solving this triangle with respect to the angle at the center (cab), which is opposite the perpendicular, this angle is one half the total angle subtended by the chord, and we have from the formula: The natural sine of the angle opposite the perpendicular is equal to the perpendicular divided by the hypotenuse. (See last part of formula 1.) The perpendicular is 1.5 ft.; the hypotenuse is 6.5 ft. Substituting these values in the formula, and performing the indicated operations, we find the natural sine to be 0.230769 . By referring to a table of natural sines, the corresponding angle is found to be $13^\circ 20' 32.44''$. Doubling this angle, and discarding the decimal part of the seconds by adding one to the whole number, we have $26^\circ 41' 5''$ as the angular measure of the chord of the keystone. We multiply this result by 5, as there are five equal chords, each subtended by this angle, to give us the sum of the angles at the center subtended by the sum of the five equal chords; thus $(26^\circ 41' 5'') \times 5 = 133^\circ 25' 25''$. Subtracting this sum from the total angular measure of the semicircle, or 180° , we have $46^\circ 34' 35''$ as the angular measure of the sum of the two unknown chords; but these two chords are equal, therefore one of them is measured by one-half this angle, or $23^\circ 17' 17.5''$. To find the chord corresponding to this angle, we reverse the process of finding the angle from the chord. By bisecting the angle at the center, and form-

ing a right-angled triangle, one side of which is one-half the unknown chord, we have a triangle whose hypotenuse and the angle opposite the half chord ($11^\circ 38' 38.75''$) we know, to find one-half of the chord which is the perpendicular of the triangle. The formula used is: The perpendicular of a right-angled triangle is equal to the sine of the angle at the base opposite the perpendicular multiplied by the hypotenuse. (See formula 1.) The natural sine of $11^\circ 38' 38.75''$ is 0.2018318 . The hypotenuse is 6.5 ft. By substitution of these values, we obtain 1.3119 ft. for the half

chord; doubling it, we obtain 2.6238 ft., or $2' 7\frac{3}{4}"$ for the chords of stones Nos. 1 and 7. $\bullet$ E. D.

Now that we know all the chords, and all of the radial joints are known by construction, we take the same course of procedure as indicated in the calculations of the vertical voussoirs, to obtain the vertical and horizontal joints to stones Nos. 1, 2, 6, and 7.

To find the vertical joints of Nos. 3 and 5, we subtract, found by previous calculation, but results only given here, the sum of the vertical joints of stones Nos. 2 and 3 from the distance of the horizontal joint of 3 or 5 to the springing line of the arch. To find the distance ae , we have $ae = ab + be$; ab can be found from the right-angled triangle abc ; $ab = ac \times \text{cosine of } bac$ (by formula 4), or $ba = \sqrt{ac^2 - cb^2}$ (by formula 6); be we know to be $2' 0"$. The vertical joint of No. 3 or 5 is then found by subtracting from ae the sum of the vertical joints of Nos. 2 and 1. $\bullet$ E. D.

To locate the center of the tower with reference to the main walls of the building, the following method was pursued:

Let pp' be a portion of the face of the interior wall of the gate house chamber, and p^2l a portion of a face

of an interior wall at right angles to pp' . The exact location of the pts. p or p^2 are unknown; at right angles to pp' lay off $p'o' = 5' 9"$; at right angles to p^2l lay off a line l^2 equal to $6' 0"$; from the points o' and l^2 draw two lines $o'a$ and l^2a parallel respectively to pp' and p^2l and intersecting at the point a , which will be the center of the tower. With a as a center and a radius equal to $6' 6"$ describe a circle; the circle so described will be the horizontal projection of the circumference of the tower, that is, its interior circumference. Because the two lines which intersect at a are parallel to two lines which are at right angles to each other, it follows that the lines so drawn are also at right angles to each other, therefore the angle $o'al^2$ is a right angle; bisect this right angle, and prolong the line of bisection through the point b . Through the points m and f draw lines parallel to ab ; they will intersect the lines pp' and p^2l in the points p and p^2 and their distances from the line mf are required, and all the dimensions of stones Nos. 8 and 12, except the distances marked $2' 0"$, which are given. The distance ao is also required, for the distance of a given point within the building to a line passing through a at right angles to pp' is 27 ft.; then, if oa is known, the distance of p from the same given point can be determined.

The central line of the pier is made coincident with the line which bisects the right angle at a . The lines mp and $f p^2$ are tangent respectively to the circle at the points m and f , therefore $ma b$ is a right angle. The two triangles $am n$ and $ab n$ have their sides respectively parallel each to each; they are therefore

similar and equiangular, but the angle nab' , by construction, is 45° , therefore man is 45° , therefore $ma n$ is 45° . The triangle $am n$ is therefore isosceles, and $mn = am$.

We now have to find the relations existing between the two triangles $am n$ and $n o p$, in order to determine the distances ao and mp . The principles involved in determining these distances are precisely the same for determining the distances $f p^2$ and al^2 , but only the former will be worked out to show the method of reasoning involved.

$bd = ab \times bc$. If now we divide $eb \times bd$ by our unit bd , we will have $eb = ab \times bc$. Q. E. D.

From a well known principle of geometry we have the area of a triangle is equal to one-half of a rectangle having the same base and altitude. If the unit of comparison be taken as one inch on the same scale as the drawing, the length of the graphic line can at once be read off on the scale. By the aid of graphical calculus, the areas of any figures, the products of lines, the extraction of roots, the measure of angles by the lengths of their chords subtending them, and the solution of many problems, difficult analytically, can be rendered simple, time saving, and interesting.

SUMMARY OF METHODS.

When an outline of a set of voussoirs is given to an engineer or a draughtsman to calculate, it will be found expedient to determine first all the chords. In some cases where it is of great importance to work in the extrados joint of the voussoir, with the intersected horizontal course of the wall, pier, or abutment, it becomes necessary to fix the radial joints or vertical joints in such a manner that the chords exhibit great irregularity; even in such a case, it is good practice to begin with the chords.

Beginners generally attack the skewback stone of the arch first, and work it out completely, and so on in order all around; but more rapid work may be done generally by starting with the first unknown radial joint, and so on until all the radial joints are determined. Then take up all the vertical joints, beginning always with the first arch stone on the left-hand side. Then take up the horizontal joints, beginning at the left hand as before. When the vertical joints are determined, add them up decimally and fractionally, to see how closely the total known vertical distance compares with the result; similarly proceed with the summation of the horizontal joints. Generally, the fractional results will check up admirably, but the decimal results will fall short of the true figure. A counter check on the verticals can sometimes be had, in sections or pairs, by referring to the established elevations of horizontal courses; when no error can be discovered by this method, the final resort is had to a summation of excesses over the fractional values, and also to a summation of deficiencies; the balance struck by combining the two results will generally indicate an error of not greater than one sixty-fourth of an inch. To the stone showing the greatest discrepancy add or subtract the balanced error, as the case may be, and the results may then be depended upon. In thus tracing an error to its source, the student or engineer will feel more confidence in the accuracy of his work. The writer would impress upon the minds of all who desire to attain the utmost accuracy in their work, this principle: Never use a deduced result as a factor of a formula for calculating an unknown quantity—if you can help it; or, in other words, always use the figures that have been arbitrarily fixed at the start so far as possible to obtain the unknown side or angle. The reason is obvious. If you use a quantity which is the result of your own work, and it contained an error, that error would vitiate all your subsequent work. Suppose you are obliged to use the value of a given angle to determine some unknown side; suppose this side is then found and you find it more convenient to use this calculated side than the given angle in subsequent calculations; the natural tendency would be of course to use that method which will entail the least work; but in voussoir work, at least, experience will teach you that the method which will produce the most accurate results is the shortest method that can be devised. Take no chances of error, for all fine stonework calculations are generally worked out by two or more engineers at the same time, and if the results agree to the one sixty-fourth of an inch, the results are accepted. Carelessness of method will cause more distrust of one's work than a few unimportant numerical errors now and then. Whatever course of procedure you mark out in the work before you, follow it faithfully, for systematic procedure in all mathematical work is conducive to accuracy in results, and in no department of work more so than in voussoir calculation.

RECENT IMPROVEMENTS IN THE MANUFACTURE OF RIFLE BARRELS.

At the recent meeting of the British Association, Mr. Arthur Greenwood, M.I.C.E., read a paper in which he stated that the introduction of steel rifle barrels some five-and-twenty years ago, and the gradual substitution of that material in place of welded wrought iron for the barrels of military rifles and arms of precision, had caused great changes in the machinery employed in their manufacture, and that a short description of such machinery could not be without interest in this great center of firearms industry. The barrels generally used for military rifles are made from mild steel rolled solid and the bore drilled out. This is necessarily a costly operation, and numerous attempts have been made at hollow rolling to avoid the expense of drilling. To effect this, barrel blanks or short pieces of steel have been drilled and afterward rolled on a mandrel to the desired lengths, and many thousands of barrels have been made on this plan. Of late years, however, the military authorities have insisted upon barrels being first rolled solid to the desired form and afterward drilled. At the Royal Small Arms Factory at Enfield a system of continuous rolling has been established, whereby the barrel has been rolled to its exterior form at one heat by passing it through a number of rolls placed one after the other. A barrel is thus rolled in about half a minute, which gives a great saving in time and cost of production, and the material is not harmed by reheating, as was often the case in the old system. When the rolled barrel has become cold, it is passed through a straightening machine, which takes out any short bends or kinks, and is thus made ready for drilling. Before going to the drilling machine, the barrel goes to a machine which forces or squares up the ends, and prepares a bearing at each end of it upon which it will rotate in the barrel-drilling machine. The barrel-drilling machine has been specially designed to drill a perfectly true hole through the barrel, and to avoid straightening by means of external hammering, which is extremely harmful to the barrel. Three barrels are operated upon at one time, and drilled simultaneously from both ends. The machine is horizontal, and the

barrels rotate at from 700 to 900 revolutions per minute. The drills are of the form technically known as D or half round bits, and are fed up automatically, a heavy pressure of water being injected up the barrels, so as effectually to wash out the cuttings or swath, and thus insure the free action of the drills. About thirty barrels can thus be drilled in the working day by one attendant. This system reduces the cost of drilling to little more than an eighth of the former system employed at Enfield, the results as regards excellence of work being at least equally good. The barrels are then rough or quick bored, and then filled up, or fine bored, in horizontal machines by means of square bits. This system is well known, and has undergone little change during recent years. The exterior of the barrel is next turned, and ingenious arrangements have been made to secure the exterior being turned true or concentric with the bore. The system at present at work at Enfield effects this with admirable precision, a bush, or temporary bearing, being fixed to the barrel concentric with the bore. This bush revolves in a stay, and steadies the barrel until spots or bearings have been turned upon it, and from these spots the barrel is rough turned. It is afterward finished in a copy-turning lathe to the desired form, the result being a barrel with its exterior practically concentric with its bore, which is absolutely necessary for good shooting. In rifling, little change has been made in the system, but much attention has lately been given to reduce the

cost of this delicate operation. The automatic rifling machine is coming generally into use, and now, with the automatic machine which rotates the barrel after each cut, gives the feed to the cutters, and finally rings a bell to summon the attendant when the barrel is completed. One man can attend to a number of machines, producing as many as sixty or seventy barrels per working day, whereas on the old system one attendant was required for each machine, producing on an average twenty barrels per day.

MASSICK & CROOKES' HOT BLAST FIRE BRICK STOVE.

We give views of this hot blast fire brick stove, as erected at the works of the Askham & Mousell Iron Company, Limited, at Askham-in-Furness. This stove is the outcome of many years of experience obtained in the management and working of some of the largest of the Cumberland hematite iron works, and has been designed in order to avoid some of the many objections and difficulties that arise in the working of other patent hot blast stoves. It is, we know, the universal opinion of all ironmasters of any considerable experience that it is vain to look for much increase in the prices of pig iron again. There may be, and no doubt there will be, fluctuations from time to time; but the good old days when pigs sold at 140s. per ton have gone never to return, and it is useless to mourn after

MASSICK & CROOKES' HOT BLAST FIRE BRICK STOVE.