THE MECHANICS OF THE GYROSCOPE.

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WHILE the gyroscope affords a notable illustration of the harmony among physical laws, the elementary exposition of the principles governing its action does not seem to have received the attention it deserves.

Prof. Perry's interesting work,* entitled "Spinning Tops," contains little more than rules for ascertaining what will happen under given conditions, together with striking experimental illustrations, and applications to cosmical phenomena, such as those connected with the rotation of the earth and other planets.

It is superfluous to dwell at length on the practical importance of the subject, since rotation is one of the most general motions produced in machinery. The most recent theories of molecular structure also involve the orbital motions of atoms and electrons; so that, from the atom to the solar system, we are continually confronted with problems dealing with rotations and revolutions. On the other hand, text-books afford no adequate elementary treatment of this subject. I have therefore been reduced to think out an elucidation for myself, which I venture to present here.

According to the Newtonian system of dynamics (a system which is now universally recognized and accepted), the velocity of a particle can only be increased in any given direction by the application of a force acting in that direction; conversely, its velocity in a given direction can only be diminished by the application of a force acting in an opposite direction. The magnitude of the applied force is proportional to the rate of increase or decrease of the velocity of the particle.

Let us suppose that a series of equal heavy particles are arranged around the circumference of the circle in Fig. 1. These particles may be supposed to be rigidly connected one with another, the whole being connected by massless spokes, with an axle passing through C , the center of the circle; this axle being at right angles to the plane of the paper. This arrangement constitutes an ideal flywheel, and may be considered typical of an ordinary gyroscope disk.

Let the flywheel be set in rotation in the direction indicated by the arrow. The problem before us is to determine the nature of the forces which must be applied to the rotating flywheel in order to deviate the axis of rotation. Let us suppose that the flywheel, while still rotating about its axle, is constrained in addition to turn about the line AB , at right angles to the axle. Looking in the direction AB , let the flywheel turn about that line in a clockwise direction, so that the side L moves downward through the plane of the paper, while the side R moves upward through the same plane. The particles at e and p being, at the given instant, on the axis of rotation AB , will possess no velocity of rotation about that axis. So far as concerns other particles, their velocities of rotation about AB will be proportional to their perpendicular distances from that line. Sixteen equidistant particles on the circumference of the circle have been indicated. The rotational velocities of these particles, about the line AB , will be proportional to the respective perpendiculars let fall on AB .

In a certain interval of time the disk will complete a revolution about its axle. In one-sixteenth of this interval of time, the particle a will move round the circle so as to attain the position previously occupied by the particle b . In doing so, the particle a will acquire the velocity previously possessed by the particle b , i. e., its velocity will be diminished, since b is nearer than a to the axis AB . The diminution of velocity will of course be proportional to aD , where bD is a line drawn from b perpendicular to CA . But since the velocity of the particle a , in a direction passing vertically downward through the plane of the paper, is diminished as the particle moves from a to b , this particle must have been acted upon by a force directed vertically upward through the plane of the paper, and proportional to aD . This force is indicated by a small circle containing a dot at its center. The dot indicates the pointed end of an arrow supposed to be directed vertically upward through the paper; while the diameter of the small circle is drawn proportional to aD , or to the magnitude of the force.

While the particle a moved to b , the particle b moved to c . In this time the velocity of the particle b , perpendicular to the plane of the paper, must have been diminished by an amount proportional to bE . A small circle containing a dot at its center, and of a diameter proportional to bE , indicates the magnitude and direction of the force which must have been applied to the particle as it moved from b to c .

The force which acted on the particle c as it moved to d , and that which acted on the particle d as it moved to e , are represented in a similar manner.

Owing to the rotation about the line AB , all particles on the right-hand side of the disk are moving upward through the plane of the paper; thus it follows that the particle e , in moving to the position f , must have acquired a velocity, directed vertically upward through the paper, proportional to Gf . It must, therefore, have been acted upon by a force, proportional to Gf , directed vertically upward through the paper. The

forces acting on the particles f, g, h , can be determined in a similar manner.

It is obvious that the velocity of the particle k , directed upward through the plane of the paper, is diminished as that particle moves to the position previously occupied by the particle l . Consequently, it must have been acted upon by a force, of which the magnitude is determined in the manner previously explained, acting downward through the plane of the paper. A circle, of which the diameter is proportional to this force, while the cross at its center represents

FIG. 1.

the feathered end of an arrow directed downward through the paper, indicates the magnitude and direction of the force acting on the particle k as it moved to l . The forces acting on the particles l, m, n, p, q, r, s , are determined similarly, and represented by circles containing crosses, to indicate that the forces act downward through the plane of the paper.

A glance at Fig. 1 shows that all forces acting on the part of the flywheel above the line $a k$, are directed upward through the plane of the paper; while all forces acting on the part of the flywheel below the line $a k$, are directed downward through the plane of the paper. All the forces acting above the line $a k$ might be replaced by a single resultant force, acting upward through the paper at some point on the line $C e$; while all the forces acting below the line $a k$ might be replaced by a single resultant acting downward through the paper at some point in the line $C p$. These two resultant forces, acting parallel to each other, but in opposite directions, constitute a couple, and produce a torque or turning moment about the line $a k$. Thus, in order to turn the revolving flywheel about the diameter $e p$, we must apply a torque which, if it acted on the stationary flywheel, would turn it about the perpendicular diameter $a k$. Conversely, if we apply a torque tending to turn the flywheel about a diameter $a k$, it will turn, not about $a k$ (as might have been expected), but about the perpendicular diameter $e p$.

The torque necessary to deflect the flywheel might be produced by forces acting directly upon it, as for instance, by blowing air on the upper half of the flywheel from the back, and on the lower half from the front. Generally, however, it is more convenient to act on the axle, the end above the plane of the paper

FIG. 2.

being urged in the direction CB , while the end below the plane of the paper is urged, by an equal force, in the direction CA .

Some further points should be noted. Any force acting to the right of the line AB , is equal, both in magnitude and direction, to a corresponding force acting to the left of the same line. Consequently, as the flywheel turns about the axis AB , no work will be performed by the forces producing this rotation. This follows from the circumstance that whereas one force acts in the direction of motion (so far as relates to rotation about the axis AB) the other equal force is opposed to that motion.

The actual behavior of a gyroscope can now be easily understood. The flywheel $a a$ (Fig. 2) having been set in rapid rotation in the direction indicated by the arrow r , the frame carrying it is supported from a projection n at one end, on a pivot o . Instead of falling to the ground, as it would do if it were not rotating, the gyroscope remains with its axis $b z$ horizontal; but the axis turns in a horizontal plane about the point of support o , in the direction indicated by the arrow s . The torque produced by the pull of gravity is easily seen to be that required to turn the flywheel $a a$ about a vertical diameter in the direction mentioned. The fact that the flywheel, besides rotating about a vertical axis, also revolves in a circle about the point o as center, is merely due to the circumstance that, under the conditions of the experiment, the rotation cannot occur without the revolution.

It is instructive to consider the same problem from a somewhat different standpoint. We have already determined the nature of the applied forces required to turn the ideal rotating flywheel (Fig. 1) about the axis AB , in a clockwise direction when viewed from A . We found that a torque must be applied which tends to urge the end of the axle above the plane of the paper in the direction CB , and the opposite end of the axle in the direction CA . It will now be proved that the reaction of the rotating flywheel, when it turns as above, about the axis AB , produces a torque which tends to urge the end of the axle above the plane of the paper in the direction CB , and the other end of the axle in the direction CA .

Under the given conditions, the component velocities, downward through the plane of the paper, of the particles a, b, c, d , are all being diminished; and the consequent reactions tend to turn the axle in a clockwise direction, about the line $a k$, when viewed from the side k . The component velocities, upward through the plane of the paper, of the particles e, f, g, h , are all being increased, and the consequent reactions tend to turn the axle in the same direction. It is easily seen that the reactions due to the alterations in the velocities of the particles k, l, m, n, p, q, r, s , all tend to turn the axle of the flywheel in the same direction. Thus the torque due to the reaction of the rotating flywheel when turning about the axis AB , is of the character specified above.

The precise way in which the gyroscope (Fig. 2) acts can now be readily followed. When the frame carrying the rotating flywheel $a a$ is first supported on the pivot o , the initial tendency is for the whole to descend toward the earth, under the action of gravity. But the pivot o prevents the end b of the axle from descending, so that an incipient rotation about a horizontal diameter commences. The reaction due to this rotation produces a torque which tends to turn the flywheel about a vertical diameter in the direction of the arrow s . As the flywheel is free to turn in this direction, it at once commences to do so, and in so doing generates a reacting torque opposing the incipient rotation produced by gravity. The action of gravity being opposed, the rate of (incipient) descent of the flywheel is diminished; but so long as descent continues, a torque acting in the direction of the arrow s will be produced, and this will increase the velocity of turning, thus increasing the torque which opposes the descent of the flywheel under the action of gravity. The flywheel, finally, acquires a rotational velocity in the direction of the arrow s , which produces a reacting torque just equal and opposite to that due to the pull of gravity. If friction were entirely absent, the flywheel would then cease to descend, and would continue to turn at a uniform rate in the direction of the arrow s . In this process, the work performed is that due to the incipient descent of the flywheel; this work is just sufficient to supply the kinetic energy due to the rotation of the flywheel and its supporting framework about the axis o . When the permanent condition outlined above has been attained, no further work is done in the absence of friction. If there is friction between the supporting lug n , and the pivot o , the gyroscope will slowly descend, at such a rate that the work performed by gravity is just equal to that needed to overcome the frictional drag.

In the absence of friction, it is obvious that the gyroscope turns about o as center merely by virtue of its own inertia, after the final state has been reached; in this respect the motion resembles that of a planet around the sun. The torque due to gravity, though necessary, only serves the purpose of neutralizing the reacting torque which the turning of the flywheel about a vertical diameter produces.

When the gyroscope (Fig. 2), instead of being capable of rotation in a horizontal plane about o as center, is hinged at o , so that it can move only in a vertical plane, it descends at once toward the earth, no difference being produced whether the flywheel is rotating or quiescent. In this case, since turning about a vertical axis is prevented, the gyroscope cannot generate a torque opposed to that due to the pull of gravity.

It is plain that when we attempt to turn a rotating flywheel about a diameter, no resistance is offered; but a torque is produced, tending to turn the flywheel about a perpendicular diameter; and unless this torque is neutralized by another of equal but opposite value, motion in a direction other than that required will occur.

Prof. Perry, in one experiment, inclosed a flywheel in a box, which served as a convenient means of supporting the ends of the axis of the flywheel, without risk of interfering with its rotational velocity. He remarks on the effects observed as follows:

"When I hold this box in my hands, I find that if I move it with a motion of mere translation in any direction, it feels just as it would do if its contents were at

* "Spinning Tops." By Prof. J. Perry, F.R.S., etc. 1901. (Society for Promoting Christian Knowledge, 43 Queen Victoria Street, E.C.)

This book is based on the "Operatives Lecture" delivered before the British Association in 1890. To my mind it appears that even a general audience of "operatives" would have been quite capable of grasping and appreciating far more of the elementary elucidation than was offered in connection with this fundamentally practical branch of mechanics. The field for the exercise of the powers of the scientific imagination becomes wider, in proportion as elementary methods of exposition are extended.

rest; but if I try to turn it in my hands, I find the most curious great resistance to such motion. The result is that when you hold this in your hands, its readiness to move so long as it is not turned round, and its great resistance to turning round, and its unexpected tendency to turn in a different way from that in which you try to turn it, give one the most uncanny sensations. It seems almost as if an invisible being had hold of the box and exercised forces capriciously. And, indeed, there is a spiritual being inside, what the algebraic people call an impossible quantity, what other mathematicians call 'an operator.' (Spinning Tops, pp. 21-23.)

I shall not presume to criticize the last sentence; but the mention of "great resistance to turning round" appears capable of an ambiguous interpretation. It has already been pointed out that if a torque tends to rotate the flywheel about a diameter as axis, then the flywheel will actually turn about a perpendicular diameter as axis. It has also been proved that the forces acting on the flywheel do not work; it therefore follows that no true resistance is encountered, for motion against a resistance would entail a loss of energy. A given torque produces rotation in a direction that would not be anticipated from experience with non-rotating bodies; but the actual motion produced is exactly that to be predicted from a consideration of the problem from the standpoint of Newton's laws of motion. Thus the gyroscope, while apparently defying gravity, is really acting in accordance with accepted mechanical principles.

The movement of the axis of a revolving gyroscope-disk, at right angles to the disturbing force, is a fact familiar to experimenters. That this movement, at right angles to the direction of the force, is a consequence independently and directly deducible from the accepted law of the conservation of energy, is not so generally recognized.

Let us consider the very simplest instance of a gyroscope-disk, which, of course, is nothing more than a revolving flywheel inclosed in a case or box, as one sees in some familiar experimental illustrations depicted in text-books. The case or box then serves merely the purpose of a convenient frame to support the ends of the axis of the flywheel.

Now, we know that when the case is rotated by hand, so as to deflect the axis of the flywheel, a curious sensation of resistance is felt, when the flywheel at the same time revolves on its own axis.

Imagine there to be no friction, so that in our experimental illustration the velocity of the flywheel remains uniform. Suppose, further, the case or box to be connected with some convenient mechanism which produces the required rotation, thus deviating the axis of revolution of the inclosed flywheel, or producing more complicated movements if required, this mechanism being driven by any convenient motor. The inclosed flywheel can, of course, run independently on its own axis.

Friction in the mechanism is to be neglected; it can, if necessary, be taken into account or allowed for afterward. Ideal conceptions, such as where friction is neglected, are recognized as useful for the illustration of truths, and are quite legitimate, provided the essential conclusions are afterward slightly modified so as to fit exactly with the facts existing in Nature. The "perfect gas," for instance, constitutes a familiar theoretical ideal for deducing the essential truths about actual gases.

Provided that the flywheel is not revolving on its axis, and neglecting friction, everyone will concede that no work will be performed by the motor in rotating the box. This is tolerably self-evident. But it may not, possibly, be admitted as equally obvious, that no work will be performed by the motor in effecting deviations of the axis of the revolving flywheel.

Yet it appears that the theoretical conclusion as to the absence of work in this process, together with the correlated inference that special stresses, generated during the effected deviations of the axis of the revolving flywheel, are never in the resultant opposed to the applied forces which produce the deflections of the axis—are both independently deducible, as a simple consequence of the law of the conservation of energy.

(1) When the flywheel does not revolve, and the motor, deviating the axis in diverse directions, is started, we have a flywheel possessing unchanged rotational velocity (velocity equal to zero, that is) before and after the experiment, with nothing to show in the shape of work performed by the impelling motor—neglecting, of course, friction, as agreed. And clearly, under these conditions, no work is really done by the motor.

(2) When the flywheel does revolve on its own axis, and the motor (variously deviating the axis of the rotating flywheel) is put in action, we still have unchanged conditions before and after the experiment, in that the flywheel possesses unchanged rotational velocity.

For it is an accepted fact that no mere deviation of the axis of a revolving flywheel can alter its velocity of revolution on its own axis. To recognize this fact without difficulty, we have only to imagine the axis to be indefinitely thin, or, if we like, a mathematical straight line; in which case no torque or "couple" could act on such an axis in order to accelerate or retard the connected flywheel, since a couple or torque cannot possibly act without a radius of action. And the theory of the gyroscope or revolving flywheel is, as an accepted fact, independent of the thickness of the axis of revolution.

Accordingly we have, before and after our two experiments, a flywheel revolving on its axis with unchanged velocity. The conditions prevailing before

the experiments have remained unaltered, while no more work is demanded in order merely to deviate the axis than would be demanded if the flywheel were not revolving; accordingly the inference is, that no work is performed either in experiment (1) or in experiment (2); that is, whether the flywheel revolves or not during the deviation of its axis.

How, then, can we account for the "curious resistance," apparently encountered when the axis of a revolving flywheel or gyroscope disk is deviated by hand?

Here a brief excursion into simple elements of physiology may be apt for our purpose. We may observe that, in the particular instance of the human body, a force cannot be sustained, even in the absence of accompanying motion, without the performance of work within the body. Holding out a weight at arm's length gives the appearance of external work performed, especially if (in addition) one revolve on one's own axis, without raising the weight higher. To try to burst open a closed door by merely pressing against it, may demand much work inside the human frame; but, of course, none is accomplished outside. For it may be superfluous to draw attention to the fact, that in order to perform work outside one's self, there must be not only a force, but there must be motion against this force. Motion at right angles to a force does not constitute work; since the motion is not opposed in that instance.

If we imagine a small satellite, or even meteorite, to be guided round a circular orbit by human agency alone, this would demand internal bodily work, possibly very great. But it is known that in this instance no work whatever is really achieved outside the human body. For it is a recognized principle that merely to change the path or direction of motion of a moving particle (or collection of particles) without acceleration, demands no expenditure of work or energy whatever. The same argument obviously applies to the revolving gyroscope disk or flywheel; no expenditure of work or energy is required in order to deviate its axis or plane of revolution, provided the flywheel be not accelerated in respect to its rotational velocity about its axis.

When the deviations of the axis of a revolving flywheel are effected by hand, the curious resistance experienced is clearly referable to the transverse stress which arises in the act of deflecting the axis of a revolving body. This stress being transverse (or laterally directed), cannot oppose the deflection; but it is experienced all the same by the hands of the operator, and he may assume he has done work, in the absence of due precautions; in other words, this stress may readily convey a deceptive appearance of work accomplished outside one's self.

Owing to the physiological innervation and shortening of muscles, with attendant oxidation, in merely sustaining a stress or force even without movement against it, an expenditure of work or energy inside the human frame is entailed, which may, to the sensations, seem like outside work performed.

Hence (as a brief summary) it appears to be independently inferable from the law of conservation of energy alone, that, in the resultant, all stresses observed or experienced in deviating the planes of revolution (or the axes) of flywheels, turbines, or other rotating machinery (as occurs on board a pitching or rolling ship, or elsewhere) are stresses which entail no work or waste of power on the part of the machinery concerned.

In his appreciated work on "Spinning Tops," Prof. Perry makes the following remarks (p. 24):

"When the flywheels of steam engines and dynamo machines and other quick-speed machines are rotating on board ship, you may be quite sure that they offer a greater resistance to the pitching or rolling or turning of the ship, or any other motion which tends to turn their axes in direction, than when they are not rotating."

The foregoing reasoning shows that, far from our being "quite sure" of the resistance to pitching or rolling offered by the revolving flywheels, we are led to anticipate that, in ordinary circumstances, such resistance is non-existent or negligible. As, however, Prof. Perry's statement embodies a not uncommon error, it may be well to devote a few lines to a careful consideration of what occurs. Let us suppose that the axis of a revolving flywheel lies in the direction from stem to stern of a ship. Then, if we exclude the action of friction at the bearings, rolling of the ship from side to side will produce no effect on the flywheel, and the flywheel will therefore offer no resistance.

Turning the ship will turn the flywheel about a vertical axis. The reaction of the flywheel will tend to slightly raise the bows and depress the stern of the ship, or *vice versa*. This "tilting" of the ship will be of but small extent, and will reach its maximum when the torque producing it is neutralized by the action of gravity tending to reduce the ship to its normal level. While the tilting is increasing (e. g., while the bows are ascending and the stern descending) the reaction of the flywheel will produce a very small torque opposing the turning of the ship; but so soon as the maximum tilt has been produced (which will occur almost instantaneously) all resistance to the turning of the ship vanishes. For a constant resisting torque to be produced, the ship would have to continually rotate in a vertical plane.

When the ship pitches, the flywheel rotates about a horizontal diameter perpendicular to the axis of revolution. This rotation will produce a torque tending to deflect the flywheel about a vertical axis, and, therefore, to turn the ship to one side or the other. If no

turning is produced, there can be no torque to resist the rotation due to the pitching of the ship. This case is strictly analogous to that of the gyroscope, hinged so as to be capable of rotation only in a vertical plane. If the ship actually does turn under the action of the flywheel, a torque will be produced opposing the motion due to the pitching of the vessel; compare the slow descent of the gyroscope (Fig. 2) when friction at the pivot *o* prevents it from turning freely. In this case, some opposition would be offered to the pitching of the ship; but fairly rapid turning would be needful in order to make the opposition appreciable. In actual practice, it appears clear that revolving flywheels, etc., on board ship offer no appreciable resistance either to pitching, rolling, or turning.—Technics.

ROMANCE OF THE POTATO.

In the spring of 1903 not a little surprise was felt when it became known that a novel variety of potato was being eagerly bought up for planting at twenty shillings a pound, the usual price of seed potatoes being from two pounds to four pounds a ton. A still greater surprise was awaiting at the end of the year when tubers of another variety—El Dorado—were sold at prices ranging from one hundred pounds to two hundred pounds per pound avoirdupois. As numbers of potatoes average two to the pound, the altogether remarkable sum of one hundred pounds must have been paid for many of the single tubers. It must be a puzzle to the general reader whose knowledge of this humble esculent does not perhaps extend beyond a daily renewal of acquaintanceship with the cooked article, and to whom all potatoes are alike, to understand how such things can be. It will partly explain matters if we say that during the last thirty years potato-growers have been continually on the lookout for improved varieties of the potato—varieties producing larger crops and unsusceptible, or slightly so, to potato disease. So remarkable has been the success of potato specialists, whose business it is to produce from the potato-apple new varieties, that whereas formerly six tons of tubers to the acre was considered a fair crop, in 1903 the sort called Evergood produced from fourteen to eighteen tons, and in a year when disease was more than usually prevalent the crop of that sort was quite clean. Evergood had of course made its name previous to 1903; but when the specialist who produced this variety declared that he had succeeded in obtaining a superior variety to that, which on trial produced such enormous crops as, roughly, one to one and a half hundredweight from a pound of "seed," with fifty to ninety tubers to a single plant, the desire to secure such an extraordinarily prolific potato can easily be understood. What is said to be an even superior variety came into notice during 1903. It was to be sold to the public in 1905; but a few pounds grown in England got upon the market and sold at the prices already noted. Orders are being solicited for the autumn of 1904 at the reasonable price of five guineas a pound, or for a ton the alluring figure of eleven thousand seven hundred and sixty pounds. The present value at two hundred pounds a pound avoirdupois is the startling one of four hundred and forty-eight thousand pounds per ton, and what a field of a few acres in extent planted at these prices would be worth is too stimulating to contemplate.

No doubt it will occur to those of an inquiring turn of mind to ask how potatoes costing two hundred pounds a pound for planting can be made to produce a paying crop at five guineas. The cost of production need hardly be considered, as the prices are so abnormally large as to be unaffected by rent or wages. It is clear that if only one hundredweight is secured from each pound planted the crop at five guineas a pound will amount to five hundred and fifty-eight pounds, yielding a not unhandsome profit. But astute potato-cultivators have reverted to a method tried some sixty years ago, whereby the seed-potato is made to provide a largely increased number of sets. Usually the potato-tuber is prepared for planting by cutting it in pieces each provided with one or more growing "eyes," which become the potato plant. By starting these "eyes" or shoots into growth by means of artificial heat, they may be removed when large enough and rooted in flower pots; and when these have grown somewhat the tops of the shoots may be cut off and also rooted in the same way; and so on till the advancing season renders it impossible to proceed with profit. Meanwhile the tuber produces more shoots, which in turn are treated as above; and once this method of propagation comes to an end the tubers themselves are utilized for planting. By these methods the producing power of a tuber is increased according to the number of times each shoot is increased; hence it is apparent that by a comparatively small outlay the value of the crop can be increased to an enormous extent.

No potato has even gained so much notoriety as El Dorado, raised by the famous Scottish grower, Mr. A. Findlay, of Markinch. There was considerable excitement in Peterborough market over the sale of a specimen of the famous El Dorado potato. The tuber weighed a little under half a pound, and was disposed of at the record price of eighty pounds. "The story of this potato," says the Gardeners' Magazine, "is quite romantic; the very name was a stroke of genius. The promise fulfilled by Northern Star assured for El Dorado a hearty reception; but the output of seed was so small and the competition for tubers wherewith to raise stock was so great that prices bounded up. Mr. George Massey, of Spalding, was one of the very first