The Magnetization of Iron in Bulk

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was completely vaporised. He asked if the ions were produced in the mixture or in the gas. Had the authors tried the effect of screening the mixture from the electroscope?

Dr. C. CHREE remarked on the fact that the authors employed formulæ of the type $I=at^n$, t representing absolute temperature, under circumstances in which n was a large positive number, in one case as big as 100. To fit such a formula I must be practically either infinitely small or infinitely large, except over a very limited range of temperature. He should like some further details as to the method of arriving at formulæ of a type so remarkable physically, and as to the range of temperature wherein they were supposed to hold.

Dr. WILLOWS, replying to Mr. Skinner, said that the effect was still produced when the mixture was covered with a metal plate, but they had not tried screening with mica. In reply to Dr. Chree, he said that their formula held over a range of 100°C. , and that Dr. H. A. Wilson had arrived at a similar formula which applied over a range from 200°C. to 1000°C.

XXX. *The Magnetization of Iron in Bulk.* By W. M. THORNTON, D.Sc. (Vict.). (Communicated by Sir OLIVER LODGE, F.R.S.) *

- § 1. A method of testing the permeability of iron in bulk, with experiments on large rings.
- § 2. The influence of core-currents upon the growth of the magnetizing current. Comparison of coils with solid and laminated cores.
- § 3. Calculation of subsidence or growth of current in coil with large solid core of circular section. Evidence of sudden molecular movement as the iron approaches saturation.

THE permeability of specimens of iron has long been known to depend upon their size, shape, and previous history. The usual method of determining magnetic properties by tests on small samples cut or specially cast from a larger mass cannot be entirely relied on to give the same

* Read June 24, 1904.

values that would be obtained if it were possible to apply tests to the material in bulk. It is especially necessary, in order to have uniform results, even from a batch of samples of the same metal, that they should have had the same annealing or have been subjected to the same process and time of cooling. If they could be conveniently compared, a 20 per cent. difference between the behaviour of a small specimen and of the actual magnet frame would be found to be not uncommon, and the permissible variation between castings of the same group is sometimes specified of this magnitude.

One method of testing has been developed* free from most objections but not entirely free, because the amount of metal examined is a very small portion of the whole. It seems, in fact, that accuracy cannot be guaranteed unless the specimens are large enough to be considered as "in bulk," and are tested as a whole.

The two methods at present in use with the smaller specimens are the ballistic and the magnetometric. The latter is only suitable for long thin rods, the former for cores which, either by reason of the low inductance or high resistance of their windings, have small time-constants, so that the total inductive change following any alteration of the exciting current is practically instantaneous.

Some years ago the writer had occasion to measure the slow magnetic movements which occur on making or short-circuiting the exciting current of large dynamo-electric machines†. To do this a recording millivoltmeter was connected to an exploring coil surrounding the core. Any change in magnetization was then registered by the induced voltage in this secondary circuit, and the absolute accuracy of the result was made to depend only on the standardization of a voltage and the speed of the record. For let a voltage e be induced during time t in a coil with n turns. The magnetic movement in the core which has produced this is then

$$N = \int \frac{e}{n} \cdot dt,$$

* Drysdale, *Journal Inst. Elect. Engrs.* vol. xxxi. p. 283.

† *Ibid* 'The Electrical Engineer,' vol. xxix. p. 523 ff.

And if $\bar{e}$ be the mean voltage during the total time T of change

$$N = \frac{\bar{e}T}{n},$$

corresponding, where a is the cross-sectional area of the core, to a change in flux density of

$$B = \frac{\bar{e}T}{na}.$$

The immediate object of these tests was to measure directly the leakage coefficients of large machines. Incidentally the permeabilities of various parts of the magnetic circuits were also determined. It was then suggested by Mr. C. C. Hawkins that since these were values for iron in bulk, the same method might be used for testing large rings specially wound for the purpose.

The lower limit to the size of ring depends only on the point at which the iron can be considered to be in bulk. So far as this alone is concerned a mass of anything over 100 lb. would probably be sufficient, in the shape of a yoke-ring of the usual proportions; and it is evident that to avoid expense the smallest size should be used, provided that the method of testing to be employed can be applied with accuracy and convenience.

In the experiments previously referred to the magnet core was of cast steel 15 inches diameter, the whole frame weighing several tons. This circuit had an inductance of about 50 henries and a resistance of 10 ohms, the time-constant therefore being 5 seconds.

The method of testing adopted in this case was to make the exciting circuit, allow the current to become steady, and then to short-circuit the windings, recording photographically the voltage induced in the exploring coil. It was found that the methods were uniformly accurate, and in applying the test to sample rings a time-constant of this magnitude was aimed at.

Two rings were kindly prepared by Messrs. W. H. Allen, Son & Co., one of cast iron the other of cast steel*, both being the yoke-rings of standard machines with the brackets

* *Vide* Journal Inst. of Elect. Engrs. vol. xxxiii. p. 539.

and facings turned off. The former was wound with 1000, the latter with 777 turns of copper wire to carry 50 amperes for a few seconds without undue heating, having respectively 2.9 and 1.9 ohms resistance cold. In each case, however, the attempt to use the "make, short-circuit" method, recording photographically, had to be abandoned on account of the difficulty experienced in obtaining satisfactory records. The motion of the spot of light was so rapid that the photograph was too faint for accurate measurement. Another method had then to be found, and recourse was eventually had to the simple device of altering the current gradually with several brief pauses, recording as before the induced voltage in the secondary or exploring coil.

A vertical wooden pipe 10 inches square by 6 feet long was filled with dilute sulphuric acid and fitted with lead end-plates to which the voltage required to give the desired current was applied. A movable lead electrode was suspended from a pulley so that it could be readily raised or lowered. The movable and lower electrodes were connected through a standardized Weston ammeter to the primary winding on the ring. With the movable plate at the bottom the winding was short-circuited, when at the top it received the full exciting voltage.

The galvanometer used in the secondary circuit was by Pitkin, of the moving-coil type with the lower suspension made bifilar. It was just aperiodic. The recording drum, 5 inches diameter by 10 inches long, was contained in a box provided with an adjustable shutter, and was at first driven by a clock within the drum. It made one revolution a minute and ran for ten minutes, the speed slowly decreasing from the start.

To avoid this drop of speed a small spindle was screwed into the end of the axle of the drum, and, passing through the end of the box, was fitted with a pulley so that by using a gramophone-clock as motor a suitable constant speed could be obtained for the record.

In applying the magnetizing current the pauses, which were about every ten amperes, were always long enough for the spot of light to remain for a moment at rest so that the curves as recorded were as follows (fig. 1).

The upper areas represent the changes of magnetic flux with increasing currents, the lower with decreasing. The area A of each curve was measured with a planimeter.

FIG. 1.—Typical Record for Half a Hysteresis Loop.

(Each area denotes the change of B for an observed change of H .)

Knowing, then, the voltage e for one centimetre deflexion and the seconds s the periphery of the drum took to move one centimetre, the area multiplied by the product es gave the total lines threading the exploring coil. The corresponding change in flux density

$$B = \frac{Aes \times 10^8}{na} = kA \text{ say ;}$$

A having in succession the values recorded. The flux density at any current is then k times the sum of the areas up to that point. The value of the magnetizing current at each pause was noted by the observer. The areas can be readily checked by allowing the current to change from zero to the maximum and back to zero without pause, when it is found that the agreement between the areas so found and the separate steps is within 1 per cent.

The difference between the areas above and below the zero line represents the residual magnetism.

The results of the tests on the two rings are given in Tables I. and II. and in fig. 2.

The observations are reduced in a manner similar to the "step by step" method of ballistic testing. The labour could be greatly reduced by the use of a quantometer*, if that most

* Vide 'The Electrician,' vol. I. p. 383.

Fig. 2.—Magnetization Curves of Large Rings.

TABLE I.—Cast Steel Ring.

H.	B inc.	μ .	B dec.	μ .	w ergs per c.c.
0	-5000	$-\infty$	5000	$+\infty$	
·628	-4000	-6370	6500	10340	
1·25	0	0	7000	5510	
2·51	+5000	1990	8000	3180	1200
5·0	7900	1580	9800	1920	2490
7·54	9500	1260	11000	1460	3300
10·05	11000	1095	12000	1194	4300
12·57	12000	955	13000	1034	4900
17·6	13600	774	14200	807·5	5900
23·6	14500	614	15000	635	6630
25·1	15000	598	15300	610	7000
37·8	16300	432	16500	436	7800
50·4	17100	340	17150	341	8500
62·8	17300	275	17350	276	
75·2	17600	233	17600	233	
109·3	18100	165·5	18100	165·5	
125·7	18500	147·3	18500	147·3	
201	19600	97·6	19600	97·6	

Volume of Cast Steel Ring 14520 cub. centim.

TABLE II.—Cast Iron Ring.

H.	B increasing.	μ .	B decreasing.	μ .	w ergs per c.c.
0	-2900	∞	2900	∞	
2·5	-1800	720	3000	1353	
3·78	-950	250	3400	900	
4·4	0	0	4000	796	
6·28	1000	159	4250	678	380
12·67	3000	239	5000	398	2800
25·14	5150	203	6100	243	5200
37·7	6500	172·5	7000	185·7	7700
50·28	7200	143	7550	150	8900
62·8	7800	124	8100	129	10,200
73·4	8300	113	8500	116	11,100
88·0	8750	100	9000	102·2	12,200
100·5	9150	91	9300	92·5	13,000
125·7	9800	78	9950	79	15,000
138·0	10100	73·5	10200	74	15,250
163·2	10600	65	10850	65·25	
188·5	11000	58·6	11000	58·5	
251·4	11900	47·3	11850	47·25	

Volume of Cast Iron Ring 32580 cub. centim.

useful instrument were generally available. Dr. Beattie having kindly lent his, the tests were repeated, obtaining agreement within 2 per cent. The method of testing was now as originally designed. The ring windings were connected to the generator through reversing and spring-break change-over switches*. The operations were then:—

1. Make the circuit and allow the current to reach a steady state.
2. Short-circuit the windings and allow the current to fall to zero.
3. Reverse the coil-connexions and repeat the process.

This was done several times for each current, the quantum-meter readings being eventually reduced as in ordinary ballistic testing. The instrument is so easily worked, and its readings are so constant when there is a strong field in its air-gap, that if it were better known it would no doubt be used for magnetic testing on a large scale in preference to any other.

For measuring large changes of magnetization, these methods, the only two which are not ballistic, represent extremes. In the first with a recording voltmeter the best results are obtained when the current is varied as slowly and uniformly as possible; in the second with the quantummeter, when the change is as rapid as the inductance of the winding and the tailing-off effect of the core currents will permit. The latter do not affect the accuracy of either method.

§ 2. The rings exhibit in a very marked manner the effects of eddy currents and of change of permeability in the core on the growth of the magnetizing current. Curves I_1 , I_2 , fig. 3, were taken, using an oscillograph as a dead-beat ammeter, and recording the deflexions photographically, as previously described, but with the drum making one revolution a second. They are remarkable on account of the pause in the rate of growth observed in each, and for the difference between the two curves. I_1 corresponds to a molecular change from $+B_r$, the residual magnetism, to $+B_m$, the maximum induction-density; I_2 from $+B_r$ to $-B_m$. The magnetic

* 'The Electrician,' vol. lii. p. 479, "On a master switch for Central Stations."

movement is therefore much greater in the second case, and the current is accordingly more retarded.

In order to study the reaction of the core currents on the magnetizing circuit, it is necessary to compare the curves of

FIG. 3.—Rise of Current Magnetizing Cast Steel Ring.

fig. 3 with those obtained when the core is laminated. For this purpose a 20-kilowatt transformer was used, having about the same weight of iron as the rings. The high-tension winding was excited from storage-cells, and on closing the

switch the current rose as shown in fig. 4. Here the curve I_1 indicates as before the rise from zero after the core has been already magnetized in the same direction; I_2 when the magnetization is being reversed. The following are the details of construction affecting the curve:—

Turns on primary winding	97
Resistance of primary winding	·75 ohm.
Reluctance of core (l/A calculated)	·15/ μ .
Calculated inductance $\mu=100$	$L=.31$ henry.
Weight of iron magnetized	510 lb.
Maximum value of H applied to core	67
Calculated time constant	·41 second.

The curves of rise were difficult to obtain photographically, the spot of light moving at about 140 cms. a second. It was necessary to greatly overdevelop the extra-rapid bromide paper used.

FIG. 4.—Rise of Current in Coil with Laminated Core.

The general expression for the voltage in an inductive circuit to which a steady voltage e is applied is

$$e = ri + \frac{dN}{dt} \quad (1)$$

Here N is the total magnetic change and equals Li . The

inductance L is, in all cases where iron is used, a variable. Hence

$$e = ri + L \frac{di}{dt} + i \frac{dL}{dt}; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and since L is proportional to μ we may write

$$e = ri + k\mu \frac{di}{dt} + ki \frac{d\mu}{dt} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or

$$k\mu \frac{di}{dt} + i \left(r + k \frac{d\mu}{dt} \right) = e. \quad . \quad . \quad . \quad . \quad (4)$$

In curve I_2 , fig. 4, the current pauses for some time after switching on, $d\mu/dt$ must therefore be exceedingly large since both i and di/dt are small. If desired it can be found, for all the other quantities in (4) are determinable.

In curve I_1 there is scarcely any initial pause, which is to be expected, since the core starts well on the way to saturation.

It is of some practical importance to notice that the horizontally shaded areas abo , aco represent the magnetism which the current induces whilst rising.

Equation (1) may be written

$$dN = (e - ri) dt;$$

so that

$$\frac{N}{r} = \int (i_0 - i) dt, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where i_0 is the final steady current.

The value $(i_0 - i)$ is the intercept between the curve and the horizontal tangent. The right-hand side therefore is the area A between the curve, the vertical axis, and this tangent.

Thus

$$N = k'A \times 10^9 = kA \text{ say.} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

k' being the volt seconds per sq. cm. of the record, known from the scale of the instrument. This is independent of whether the core is solid or laminated.

If A_1 , A_2 are the respective areas abo , aco , c the cross-sectional area of the core, and t the turns on the winding, then the change of flux density corresponding to an area is

$$B = \frac{N}{tc} = \frac{kA}{tc}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Denoting by B_1 , B_2 , the flux densities corresponding to A_1 , A_2 , the following relations are evident :—

$$\frac{B_1 + B_2}{2} = B_m, \text{ the maximum induction,}$$

$$\frac{B_2 - B_1}{2} = B_r, \text{ the residual magnetism.}$$

$$\text{Thus} \quad \frac{B_1 + B_2}{B_2 - B_1} = \frac{B_m}{B_r} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\text{Also} \quad A_1 + A_2 = B_m \cdot \frac{2tc}{k} \quad . \quad . \quad . \quad . \quad (9)$$

$$A_2 - A_1 = B_r \cdot \frac{2tc}{k} \quad . \quad . \quad . \quad . \quad (10)$$

The sum of the horizontally shaded areas therefore represents the maximum induction density reached, their difference, that is the vertically shaded area forming the loop bco , the residual magnetism. It is clear that if there were no retentivity the curves would be identical.

These relations can be illustrated from figs. 3 & 4 as follows :—

	Cast Steel Ring.	Transformer.
Area A_2 (by planimeter)	6 sq. in.	10.75 sq. in.
" A_1	3.1 "	4.67 "
($A_2 + A_1$)	9.1 "	15.42 "
($A_2 - A_1$)	2.9 "	6.08 "
Ratio B_m/B_r	3.1	2.5
Sectional area C	75.1 sq. cm.	420 sq. cm.
Turns on winding	777	97
Resistance in circuit	1.9 ohms.	5 ohms.
Scale of record	10 amp. = 1 inch.	4 amp. = 1 inch.
"	.125 sec. = 1 inch.	.05 sec. = 1 inch.
k	2.3×10^4 per sq. in.	1.0×10^4 per sq. in.
B_m (from eqn. 9)	17,900 per sq. cm.	18,900 per sq. cm.
B_r (" " 10)	5,770 "	7,560 "
H max.	225 "	67 "

Comparing the values of B_m and B_r obtained in this way with those of § 1 for the ring, it is seen that there is a fairly close agreement for the maximum induction, but about 15 per cent. difference in the residual. Of the two methods, the latter is the more likely to be correct in determining the residual flux.

These relations suggest a very simple method of measuring the induction through the windings of large electromagnets, such as the exciting circuits of large dynamo-electric machines, which may be in use but has not come to the writer's notice. It is to record the rise of current when the

FIG. 5.—Calculated and Observed Coil Currents.

magnetization is suddenly switched on, and to measure the area corresponding to abo in fig. 3. The curve will in general shape resemble curve A in fig. 5; and since most large exciting currents take from 5 to 60 seconds to reach their full value, the speed at which the record must be made is not excessive. Proceeding in this way, the magnetic leakage between the

windings and the armature can be measured by ordinary test-room instruments.

A still further simplification suggests itself in the case of machines where the time constant is above five seconds. Instead of *recording* the current rise, the times, taken by a stop-watch, for the current to reach given values in turn are plotted. The first rush cannot be measured, but after about three-quarters of a second readings can be obtained which coincide with the recorded curve. To measure the magnetism of a large generator or motor it is then only necessary to have a deadbeat ammeter and a stop-watch.

Comparing figs. 3 and 4, the first thing observed is that the magnetizing current at first rises more rapidly when the core is solid than when laminated, an effect predicted by Mr. Heaviside but not hitherto observed. This is due to the shielding effects of the eddy currents, the magnetizing influence of the coil current not reaching the central portion of the core immediately. As the magnetizing wave-front enters, there is a double effect, the flux density in the outer layers increasing with the coil current, and the area magnetized also increasing. The result is that the permeability of the core apparently increases, as shown in fig. 3. The point where this begins to fall suddenly indicates the cessation of the core currents. This takes place in about .3 second, the rate of penetration is therefore in this case about 3.3 inches a second.

When the permeability is assumed constant and no core currents flow, equation 2 takes the well-known form

$$L \frac{di}{dt} + ri = e \quad . \quad . \quad . \quad . \quad . \quad (11)$$

from which

$$i = \frac{e}{r} \left(1 - e^{-\frac{rt}{L}} \right) \quad . \quad . \quad . \quad . \quad . \quad (12)$$

If this curve be drawn with a mean value of L suitable to the circuit, the corresponding curve with core currents allowed to flow appears with a superposed wave. The solution of (4) is impossible unless μ can be expressed as a function of time; and even if this were known, there would

be a further complication in expressing the magnetic force as a function of the radius. The only course open is to find experimentally the curve of rise, and from it to deduce the changes of μ . It is seen from figs. 3 and 4 that the relation of μ to t does not lend itself to simple treatment.

The effect of the core currents may be summarized thus. When the core is of such dimensions or shape that the magnetizing wave from the circumference reaches the centre in a time equal to or less than the time-constant of the circuit, the rise is wavy; when it is greater the core currents merely prolong the time of growth without distorting the curve of rise.

§ 3. The distribution and reaction of induced currents in solid cores present the most difficult and complex problems in applied magnetism. The regulation of alternators, the heating of magnet cores due to armature reaction, the sluggishness of response exhibited by machines with solid cores which depend upon series excitation, are examples. Most of the practical cases are beyond the reach of exact analysis; but in order to see whether Mr. Heaviside's solution for the rise of magnetizing current in a coil surrounding a solid core gave any clue to the observed pause in the rate of rise when the core is very large, the following numerical illustration was worked out. The curves he gave * for a core 1 cm. radius are smooth and free from distortion. He expresses the magnetic distribution within a circular solid core, the magnetizing coil of which has just been short-circuited, by a series of terms of gradually diminishing amplitude and increasing coefficient of decrement †, viz. :

$$H_r = \frac{2H_0}{na(1+s^2n^2)} \cdot \frac{J_0(nr)}{J_1(na)} \cdot e^{Dt};$$

where H_r is the magnetic intensity at a radius r , H_0 the initial value at the boundary. The radius of the core is a ; n the successive roots of the equation

$$J_0(na) = snJ_1(na),$$

* Electrical Papers, vol. i. p. 398.

† *Loc. cit.* p. 397.

J_0 and J_1 being the usual Bessel functions. The coefficient $s = \frac{L\rho}{2\pi\mu aR}$, in which L is the inductance and R the resistance of the coil winding; so that

$$s = \frac{2A\rho T^2}{laR};$$

where A is the cross-sectional area of the core, l its mean length, ρ its specific resistance, μ its permeability, and T the turns on the winding.

$$D \text{ the damping coefficient} = \frac{-\rho n^2}{4\pi\mu}.$$

In the case of the cast steel ring $s = .6055$, assuming as a first approximation that the core is circular.

$$\text{For } a = 4.87 \text{ cms.}$$

$$l = 193 \text{ cms.}$$

$$\rho = 12,000.$$

$$T = 777.$$

$$r = 1.9 \times 10^9.$$

Thus

$$\frac{H_a}{H_0} = \frac{.2486}{1 + .3675n^2} \cdot e^{Dt}.$$

$$\text{The } n\text{'s are the roots of } \frac{J_0(z)}{J_1(z)} = .1243z.$$

This can be most readily solved by inspection and trial from tables of J_0 and J_1 . The roots so found are given in the following Table, which also gives the working of the different terms. It will be seen that the sum of the first twenty terms of the amplitude is only .9 instead of nearly unity. The reason for this is that in the expression for s the conductivity enters as an unknown figure. The best values for cast steel containing silicon give an average of 12,000 for ρ , but the composition of the steel has so great an influence that any numerical estimate is uncertain. If the value chosen were too large the roots would also be too large, and the amplitudes therefore diminished. The ratio H_0/H_a is plotted in fig. 5; and since the values of H_a and H_0 are both proportional to the coil-current, this ratio shows the theoretical

TABLE III.

$z=na.$	$n.$	$n^2.$	$(.3675n^2 + 1)$	A.	D.	.2 sec.		.5 sec.		1.0 sec.		2 sec.		3 sec.		4 sec.	
						$e^{D\epsilon}.$	$Ae^{D\epsilon}.$	$e^{D\epsilon}.$	$Ae^{D\epsilon}.$	$e^{D\epsilon}.$	$Ae^{D\epsilon}.$	$e^{D\epsilon}.$	$Ae^{D\epsilon}.$	$e^{D\epsilon}.$	$Ae^{D\epsilon}.$	$e^{D\epsilon}.$	$Ae^{D\epsilon}.$
2.13	.437	.1915	1.07	.2325	-.333	1.068	.218	1.18	.197	1.39	.1673	1.94	.12	2.717	.085	3.79	.061
4.94	1.014	1.025	1.377	.1805	-1.78	1.42	.127	2.43	.074	5.94	.0304	34.9	.005	206	.0008		
7.85	1.613	2.61	1.96	.1270	-4.53	2.45	.0517	.96	.0134	89	.0014						
10.83	2.225	4.96	2.82	.0885	-8.62	5.6	.0156	.74	.00119								
13.86	2.845	8.12	3.98	.0625	-14.1	16.75	.0037										
16.9	3.47	12.06	5.44	.0457	-20.95												
Ratio $H_e/H_o =$				.7367	..	..	.4162	..	.2856	..	.1391	..	.125	..	.0858	..	.061

The sum of the amplitudes of the first 15 terms = .87, of first 20 terms = .9.

change of the latter with time when the permeability is constant. The decay of current in a short-circuited coil is in this case the same as the growth of current when first switching on, the curve as drawn being merely inverted.

The curve B of fig. 5 is the rise of current in a coil surrounding a solid core 41 cms. diameter. The small dip observed beyond the bend must be attributed to a rapid molecular movement. The change in the curvature is sudden, and is always obtained even with repeated magnetizations in the same direction. The above analysis shows that no such dip would be obtained if the permeability were constant. It can then only be attributed to a sudden increase in the number of molecular magnets brought into alignment. The observed effect resembles the phenomenon of crystallization in a super-saturated solution, starting at the circumference. It is in fact the direct observation, only possible when dealing with large masses of iron, of the sudden slip noticed in the magnets of Ewing's hysteresis model when the field approaches saturation.

The growth and subsidence of magnetic induction in rectangular cores cannot well be calculated except on the assumption that the core currents follow rectangular paths. This is approximately true when the thickness is small compared with the length of the section, but not when it is comparable to it.

Photographs have been taken by Professor Hele-Shaw of the flow of liquid between the minor and major semiaxes of rectangles. These show clearly the transition from a circular path at the centre to an approximately rectangular path at the outer boundary. At intermediate points the curves are too complex for the action of the core currents to be calculated where the thickness is not small compared with the length, on account of the difficulty of finding an expression for the instantaneous magnetic force in terms of the distance from the centre.

DISCUSSION.

Prof. S. P. THOMPSON expressed his interest in the paper, and referring to the curves showing the rise of the magnetizing current, said he thought the explanation given by the

author was not the only possible one. He was not inclined to think that the effect was due to internal eddy currents in the core.

Mr. W. H. PRETTY said that the question of magnetization tests of iron and steel in bulk was a very important one. The curves shown by the author suggest the possibility of detecting cavities in steel castings, as such castings would doubtless give curves differing from those representing a truly solid block of the same material cast at the same time and subjected to the same after-treatment. It would be interesting to have a series of tests on the same block of steel in which each process in the manufacture is represented, *i. e.* as cast, after annealing, and after the surface-scale or skin has been removed to various depths; also after the block has been cut up into slabs or laminæ and put together again. Referring to the dip in the curves of rise of magnetization observed with large cores, Mr. Pretty said it was not unlikely due to internal mechanical stress and the accompanying strain, and that as it became possible to obtain steel more and more homogeneous such internal molecular changes would become more apparent.

Dr. THORNTON, in reply to Mr. Pretty, said that he was now experimenting upon the effect of cutting up a large core into laminæ and then building up the core again.