606. A Solution of the Binomial Equation $x^{17}=1$

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605. [K. 2. a.] *A Generalisation of a Theorem in Geometry.*

We have the theorem in Elementary Geometry, viz.: If the feet of the perpendiculars from a point P to the sides of a triangle ABC are collinear, then P lies on the circumcircle of ABC(1)

In other words, let a transversal cut the sides of a triangle ABC at L, M, N . Then, if the perpendiculars at L, M, N , to the sides on which they lie are concurrent, the point of concurrence must lie on the circumcircle of ABC . What happens if these perpendiculars are not concurrent?

Let the perpendiculars at L, M, N to the sides BC, CA, AB respectively, on which they lie, form a triangle $A'B'C'$. Then ABC and $A'B'C'$ are in perspective and also directly similar.

Now, if two plane figures are directly similar, having A, B and A', B' for two pairs of corresponding points, then their centre of similitude (or double point) D is obtained as follows: Let AA', BB' intersect at O . Then the circles $AOB, A'OB'$ cut again at D .

Thus, if for the triangles $ABC, A'B'C'$ above, O is the centre of perspective and D the double point, then D and O are the points of intersection of the circles $ABC, A'B'C'$.

Also, the angle between the two circles is equal to the angle ADA' , i.e. to the angle between AB and $A'B'$, which is a right angle.

Hence the following generalisation of (1), viz.: The perpendiculars drawn to the sides of a triangle ABC , at the points where they are cut by any transversal, form another triangle whose circumcircle cuts that of ABC orthogonally.

G. A. SRINIVASAN.

606. [A. 3.] *A Solution of the Binomial Equation $x^{17}=1$.*

May I offer an algebraic solution of this equation? I have found this form useful in dealing with the geometrical aspect of this problem:

(I) From $x^{17}=1$, we have directly that $x^{16}=\frac{1}{x}$, $\frac{1}{x^{16}}=x$, and that

$$x^{16} + \frac{1}{x^{16}} = x + \frac{1}{x};$$

consequently, putting

$$\left(x + \frac{1}{x}\right) = a, \quad \left(x^2 + \frac{1}{x^2}\right) = b, \quad \left(x^4 + \frac{1}{x^4}\right) = c, \quad \left(x^8 + \frac{1}{x^8}\right) = d,$$

we obtain the relations

$$a^2 = 2 + b, \quad b^2 = 2 + c, \quad c^2 = 2 + d, \quad d^2 = 2 + a.$$

(II) Subtracting, we get

$$a^2 - c^2 = b - d, \quad b^2 - d^2 = c - a,$$

from which, by multiplication, we get

$$(a+c) \times (b+d) = -1;$$

and putting $(a+c)=e$, then $(b+d)=-\frac{1}{e}$.

(III) Again, adding, we get

$$a^2 + c^2 = 4 + b + d, \quad b^2 + d^2 = 4 + a + c,$$

or

$$(a-c)^2 = 8 - \frac{2}{e} - e^2, \quad (b-d)^2 = 8 + 2e - \frac{1}{e^2}.$$

(IV) Hence we can write either of the first two equations in (II) in terms of e , as

$$\pm e \sqrt{8 - \frac{2}{e} - e^2} = \sqrt{8 + 2e - \frac{1}{e^2}}.$$

And rationalising, and dividing by e , we obtain the symmetrical equation

$$e^3 + 0 - 8e + 4 + \frac{8}{e} + 0 - \frac{1}{e^3} = 0.$$

Now, by substituting $f = e - \frac{1}{e}$, we get

$$f^3 + 0 - 5f + 4 = 0,$$

or

$$(f-1)(f^2+f-4)=0.$$

(V) Equating the second factor to 0, we obtain $f = \frac{-1 \pm \sqrt{17}}{2}$, from which numerical values of $e, a, b, \dots$, and eventually of x itself, may be obtained.

[Note.—The factor $(f-1)=0$ is evidently connected with the equation $x^{16}=1$, this being involved in our solution, as that $x^{16} + x^{-16} = x + x^{-1}$ is equally true in that case.]

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607. [K¹. 21. b.] *A Method of Trisecting any Angle.*

OX is a straight rod.

LOQ, OP and OT are rods which can turn about a vertical axis at O .

The rod LOQ carries a small vertical plane mirror at O .

OP carries a vertical pin.

OT carries two vertical pins along its axis.

CQ and CP are equal straight rods pivoted at C , and C can slide along OX . CQ and CP are pivoted to OQ and OP at Q and P respectively.

Thus, as C moves along OX , the rods LOQ and OP turn about O , so that

$$\hat{COQ} = \hat{COP}.$$

Suppose AOB is the angle which it is required to trisect.

OX is fixed so that AOX is a straight line.

OT is fixed along the other arm, OB , of the angle AOB .

C is moved along OX until the two pins in OT are in line with the image of the pin in OP .

Thus PO and OT are incident and reflected rays in the plane mirror.

Hence
$$\hat{POQ} = \hat{TOL} = \hat{BOL}.$$

But
$$\hat{POC} = \hat{QOC} = \hat{AOL};$$

$$\therefore \hat{POQ} = 2\hat{AOL},$$

$$\text{i.e. } \hat{BOL} = 2\hat{AOL}.$$

Hence the angle AOB is trisected.

To ensure greater accuracy, the pin on OP could be replaced by cross wires, and the two pins on OT by a small telescope.

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