XXV. On the nebular hypothesis.—IV. Correlations of central force

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insulates the conducting wire therefrom. In this case, too, we have consequently the conditions of a rapidly charged and discharged condenser, giving rise to reciprocating induced sparks.

13. This conclusion can be directly proved by recurring to the earlier experiments (fig. 2) and introducing across the circuit a wire of suitable resistance to short-circuit the extra-current so that it may complete its route without having to overleap the break at the key. This is shown in diagram in fig. 6, where R is a box of resistance-coils introduced between a and a' . If the wire of which the electromagnet E is made be stout, its resistance will be a mere trifle, and a resistance of 3 or 4 ohms at R will be sufficient. In some cases a drop of pure water between the points of contact at K has also the result of obviating the extra-current spark. And in every case, when the conditions are so arranged as to obviate the appearance, on breaking contact, of the extra-current spark at K, no earth-wire, nor any conductor, will draw a spark from the electromagnet-core—a decisive proof that the sparks so drawn were due to induction by the statical charge of the accumulating extra-current, and to that alone.

XXV. *On the Nebular Hypothesis.*—IV. *Correlations of Central Force.* By PLINY EARLE CHASE, LL.D., S.P.A.S.,
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[Continued from p. 36.]

THE various accordances of planetary mass and position which have been pointed out in preceding papers, the harmonic prediction of a source of planetary disturbance at $\cdot 269$ Earth's radius vector which was confirmed by the sun-spot observations of Messrs. De La Rue, Stewart, and Loewy, the comparison of gravitating and chemical force by means of the explosive energy of H_2O , the measurement of gravitating waves, the agreement between the ratio of selenium-conductivity in sunlight to its conductivity in darkness and the ratio of Sun's light-emitting to its earth-attracting energies, and my various mechanical estimates of solar mass and distance seem to be dependent upon mathematical laws, among which the following merit special consideration.

Let f = velocity communicable, at distance r , in time t , by

any central force varying as $\frac{1}{r^2}$,

f_1 = velocity communicated by a single impulse, or in a single instant,

$f_n = nf_1$ = velocity communicated by n equal impulses
or in n instants, the distance remaining constant.

Then

α . If the pressure resulting from f is constant, it must either be exactly counterbalanced by some opposing force or forces, so as to produce relative rest, or the motion must be maintained at a constant distance from the centre, so as to produce circular revolution, or, if the opposing force results from the transformation of interfering revolutions, the two conditions may be combined so as to produce rotation. The velocity of circular revolution = $\sqrt{fr}$; the velocity of rotation = $\frac{\sqrt{fr}}{n}$.

The value of n may be found by experiment; or if the aggregating medium is homogeneous, it may easily be calculated.

β . If the pressure is varying, and so exerted as to produce radial motion directly towards the centre, the velocity acquired at any given distance ρ may be found by the equation $v_\rho = \left\{ \frac{2fr(r-\rho)}{\rho} \right\}^{\frac{1}{2}}$. At the centre, $\rho = 0$ and $v_\rho = \infty$.

γ . If the pressure leads to perpetual radial oscillations synchronous with perpetual circular oscillations under constant pressure, the range of the radial oscillations from the centre is $2r$, and the mean velocity of radial oscillation is $\frac{2}{\pi}\sqrt{fr}$. The

equation $\frac{2}{\pi}\sqrt{fr} = \left\{ \frac{fr(2r-\rho)}{\rho} \right\}^{\frac{1}{2}}$ gives $\rho = \frac{2\pi^2 r}{\pi^2 + 4} = 1.4232r$.

δ . The *vis viva* of the total force varying as $\frac{1}{r}$, the theoretical *vis viva* of average radial velocity : *vis viva* of synchronous circular velocity : : $1 : 1.4232$.

ϵ . By the dynamic theory of gases, the average radial velocity = velocity of constant volume; by the laws of thermodynamics, heat varies as *vis viva*; and we have already seen (α) that the velocity of circular revolution = velocity of constant pressure. We have thus a mathematical verification of the empirical ratio between heat of constant volume and heat of constant pressure.

ζ . If the constrained synchronous rotation of particles in a spheroid, and the free revolution of exterior particles under the influence of any central force whatever, are due to the same primitive impulses (α), the velocity of those impulses is the limit of velocity towards or from which both motions tend. Let, therefore, $v_f = \sqrt{fr}$ = velocity of free revolution, varying as $\sqrt{\frac{1}{r}}$, $v_{//} = \frac{\sqrt{fr}}{n}$ = constrained velocity at same point, or

velocity of superficial equatorial rotation varying as $\frac{1}{r}$; then $v_0 = [v_{\parallel}] = [v_{\perp}] = nv_{\parallel} = n^2 v_{\perp} = n\sqrt{fr} =$ limit of tangential velocity, both of revolution and of rotation, under a reduction of spheroidal volume to an equatorial radius $= \frac{r}{n^2}$. Under such reduction all the particles in the equatorial planes would have the velocity of free revolution, or of perfect fluidity, provided the sphere were homogeneous.

η. If contraction were to go still further, the centrifugal force of rotation would give the nucleal particles orbits of increasing ellipticity. Those orbits would finally become linear, leading to complete dissociation when the mean linear velocity $\frac{2}{\pi}[v_{\perp}]$ was sufficient to produce a constant motion with the parabolic perifocal velocity $[\sqrt{2fr}]$. These velocities coincide at $\frac{2r}{n^2\pi^2}$ when $\frac{n^2\pi^2}{2} \times \frac{2}{\pi} \times \frac{\sqrt{fr}}{n} = \frac{n\pi}{\sqrt{2}} \times \sqrt{2fr} = n\pi\sqrt{fr}$, thus

showing that the limiting velocity between complete dissociation and incipient aggregation is $\pi \times$ the limiting velocity between complete aggregation and incipient dissociation ($n\sqrt{fr}$; see ζ).

θ. Let $t =$ time of describing $\frac{1}{2}r$ in virtual approach to the centre, under the action of a central force f ; $=$ time of describing r in circular revolution or motion under constant pressure; $fr = \sqrt{fr}$; $\pi t =$ time of free semicircular oscillation; $n\pi t =$ time of constrained semirotation; $n\pi ft = n\pi\sqrt{fr} =$ velocity acquired in the time of semirotation. Therefore the velocity of any central impulses which are capable of producing cosmical aggregation, free revolution, and nucleal rotation = velocity produced by constant equatorial pressure acting for a half-rotation.

ι. If the central force is solar gravity, the limiting velocity ($n\pi\sqrt{fr}$) of uniform impulses which would account for solar aggregation, solar rotation, and planetary revolution is *the velocity of light*. For Earth's distance from Sun being 214·86 solar radii, the velocity of light is $(214\cdot86r \div 497\cdot825) = \cdot4316r$ per sec. If we make this velocity $= n\pi\sqrt{fr}$, and find the value of $\sqrt{fr} = \sqrt{gr} = 2\pi r \times 214\cdot86 \div (365\cdot2564 \times 86400) = \cdot00062705r$ per sec., $n = \cdot4316 \div \pi\sqrt{fr} = 219\cdot0894$, and $v_{\perp} = \sqrt{fr} \div n = \cdot0000028622r$ per sec. The accordance of this value with other estimates (μ) is as follows, the unit being $r \div 10000000000$:—

Spörer.	Nebular.	Light.	Schwabe.
29533	29192	28622	28511.

κ . In nebular condensation the atmospheric radius varies as $t^{\frac{2}{3}}$, while the nuclear radius $\propto t^{\frac{1}{3}}$. The nuclear radius therefore varies as the $\frac{2}{3}$ power of the atmospheric radius. In ordinary considerations of the nebular hypothesis, planetary aggregation has been regarded as atmospheric under a velocity varying as $\sqrt{\frac{1}{x}}$; but there are many traceable evidences of

nuclear influence under a velocity varying as $\frac{1}{r}$. Obermayer's law (Trans. Vienna Acad.; 'Nature,' June 1, 1876, p. 119), that the friction-coefficient of permanent gases varies as the $\frac{1}{2}$ power of that of the coercible gases and as the absolute temperature, seems to be a particular case of the nuclear and dissociating influences of central force.

λ. The formation of the Jovian and Telluric belts, as indicated by the limiting velocity of their chief planets, shows the combined action of nuclear and atmospheric influences. At the equator $g = 32\frac{1}{2}$ ft. ; the time of $\frac{1}{2}$ rotation = 43200 sec. ; $\therefore \pi r \sqrt{fr} = 43200 \times 32\frac{1}{2}$ ft. = 262·5 m. per sec. The limiting velocity for Jupiter is nearly, if not precisely the same ; but it cannot be so accurately determined on account of greater uncertainty in regard to the elements of calculation. The solar modulus of light at the principal centre of gravity of our system ($\odot \text{ 4}$) is 505400 solar radii ; the nuclear formative is $\therefore \sqrt{505400} \times$ the atmospheric velocity ; when the nuclear velocity (v) was the velocity of light ($\cdot 4316r$), the atmospheric velocity was $\cdot 4316r \div \sqrt{505400} = \cdot 0006071r$. If $\cdot 0006071r = 262\cdot 5$ m., Sun's distance ($214\cdot 86r$) = 92,902,000 miles.

μ. Earth was the seat of theoretical primitive nuclear rupture, the velocity of primitive rotation within the nucleus varying as r , and communicating a similarly varying *vis viva* of superficial rotation to the shrinking nucleus. This gives a present theoretical time of solar rotation = $365\cdot256 \text{ days} \div \sqrt{214\cdot86} = 24\cdot912 \text{ days}$. The estimates from observation vary between Spörer's (24·624 days) and Schwabe's (25·507 days).

v. According to the same data, the present height of possible solar atmosphere, or the radius of equal centripetal and centrifugal solar force, should be $(214\cdot86)^{\frac{2}{3}}r = 35\cdot873r$; the time of planetary revolution at Sun $31558150 \text{ sec.} \div (214\cdot86)^{\frac{2}{3}} = 10020\cdot242 \text{ sec.}$; the present limit of uniform planetary velocity $2\pi r \div 10020\cdot242 = \cdot00062705 r \text{ per sec.}$; the present solar modulus of light $(\cdot4316 \div \cdot00062705)^2 r = 473755\cdot65 r$; the present modulus-atmosphere $35\cdot873 \times \text{modulus} = 16995141 r$.