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Then
$$y = (33 \cdot 3 + 16 \cdot 6) \frac{3}{x} + \left(\frac{33 \cdot 3 \times 32x}{2 \times 4000000} \right) \text{ ins.}$$
$$= \frac{1}{2n} + \frac{n}{25}.$$

Hence y will be a minimum when $n = \frac{5}{\sqrt{2}} = 3 \cdot 54$, and then $y = \cdot 28$ in.

For $n = 1, 2, 2 \cdot 5, 5, 10, 20$, we find $y = \cdot 54, \cdot 33, \cdot 3, \cdot 3, \cdot 45, \cdot 83$ respectively.

Hence y is not strictly constant, but the variation is extremely small. For ranges between 200 yds. and 1000 yds. the minimum value of y is $\cdot 28$ in., and the maximum value is $\cdot 45$. Hence the variation is only $\cdot 17$ in., while from 200 yds. to 2000 yds. the variation is $\cdot 55$ in. With the old pattern sight the variation from 500 to 1100 is $\cdot 5$ in. and from 600 to 1800 $1 \cdot 5$ ins.

The importance or insignificance of these variations will best be seen by observing the error involved when they are neglected. Let us assume that y is kept fixed and $= \cdot 28$ in.

Then if the man is assumed to be at a distance $x_1 = 300n_1$ ft.,

$$y = (l + 2k) \frac{r}{x_1} + la \quad \text{or} \quad \cdot 28 = \frac{50 \times 3}{x_1} + 33 \cdot 3a;$$
$$\therefore \cdot 28 = \frac{1}{2n_1} + \frac{100a}{3} \quad \text{if } x_1 = 300n_1;$$
$$\therefore a = \frac{3}{100} \left\{ \cdot 28 - \frac{1}{2n_1} \right\} = \cdot 0084 - \frac{\cdot 015}{n_1}.$$

Also the horizontal distance travelled by the shot is $\frac{2v^2a}{g}$ ft. $= 300n_2$ ft. suppose.

Then $n_2 = \frac{10000}{12} a$. But when the shot crosses the horizontal line through the muzzle it is at a height of 3 ft. Hence, before it hits the ground, it travels a distance $v \sqrt{\frac{6}{g}} = 500\sqrt{3}$ ft. $= \frac{5}{\sqrt{3}} 100$ yds.

Therefore, total distance travelled by the shot before it strikes the ground is $\left\{ \frac{10000a}{12} + \frac{5}{\sqrt{3}} \right\} 100$ yds. Hence the bullet will strike the man if

$$n_1 < \frac{10000a}{12} + \frac{5}{\sqrt{3}}, \quad \text{and} \quad \frac{10000a}{12} = 7 - \frac{12 \cdot 5}{n_1}.$$

Therefore the man will be hit if $n_1 < 7 - \frac{12 \cdot 5}{n_1} + 2 \cdot 88$.

Now, if $n_1 = 9 \cdot 88 - \frac{12 \cdot 5}{n_1}$, $n_1^2 - 9 \cdot 88n_1 + 12 \cdot 5 = 0$, whence $n_1 = 8 \cdot 4$.

Hence the man will be hit without any alteration of the sight, so long as he is not more than 840 yds. from the firer.

It has already been seen that for ranges below 200 yds. the arrangement is not accurate, so that the limits of utility are practically those required in modern warfare, *i.e.* from 200 to 800 yds. H. C. FRAMPTON.

386. [D. 6. b; V. a.] The following treatment of the fundamental exponential limit, $\text{Lt}_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$, is exceedingly simple and may be new to some readers of the *Gazette*.

If $a_1, a_2, \dots, a_n$ be any number of positive proper fractions,

$$1 > (1 - a_1)(1 - a_2) \dots (1 - a_n) > 1 - (a_1 + a_2 + \dots + a_n). \dots \dots \dots (1)$$

First taking n a positive integer,

$$\left(1 + \frac{1}{n}\right)^n = 1 + \frac{1}{\lfloor 1 \rfloor} + \frac{1 - \frac{1}{n}}{\lfloor 2 \rfloor} + \dots + \frac{\left(1 - \frac{1}{n}\right) \dots \left(1 - \frac{n-1}{n}\right)}{\lfloor n \rfloor}. \dots\dots\dots(2)$$

Using (1), we easily find

$$\frac{1}{\lfloor r \rfloor} > \frac{1}{\lfloor r \rfloor} \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \dots \left(1 - \frac{r-1}{n}\right) > \frac{1}{\lfloor r \rfloor - 2n} \cdot \frac{1}{\lfloor r-2 \rfloor}.$$

Taking the expansion (2) term by term and applying this inequality,

$$\begin{aligned} 1 &= 1 = 1, \\ \frac{1}{\lfloor 1 \rfloor} &= \frac{1}{\lfloor 1 \rfloor} = \frac{1}{\lfloor 1 \rfloor}, \\ \frac{1}{\lfloor 2 \rfloor} &> \frac{1}{\lfloor 2 \rfloor} \left(1 - \frac{1}{n}\right) = \frac{1}{\lfloor 2 \rfloor - 2n}, \\ &\vdots \\ \frac{1}{\lfloor r \rfloor} &> \frac{1}{\lfloor r \rfloor} \left(1 - \frac{1}{n}\right) \dots \left(1 - \frac{r-1}{n}\right) > \frac{1}{\lfloor r \rfloor - 2n} \cdot \frac{1}{\lfloor r-2 \rfloor}, \\ &\vdots \\ \frac{1}{\lfloor n \rfloor} &> \frac{1}{\lfloor n \rfloor} \left(1 - \frac{1}{n}\right) \dots \left(1 - \frac{n-1}{n}\right) > \frac{1}{\lfloor n \rfloor - 2n} \cdot \frac{1}{\lfloor n-2 \rfloor}; \end{aligned}$$

adding and writing $S_n = 1 + \frac{1}{\lfloor 1 \rfloor} + \frac{1}{\lfloor 2 \rfloor} + \dots + \frac{1}{\lfloor n \rfloor},$

we get
$$S_n > \left(1 + \frac{1}{n}\right)^n > S_n - \frac{1}{2n} \left(1 + \frac{1}{\lfloor 1 \rfloor} + \frac{1}{\lfloor 2 \rfloor} + \dots + \frac{1}{\lfloor n-2 \rfloor}\right) > S_n \left(1 - \frac{1}{2n}\right);$$

dividing by S_n ,
$$1 > \frac{\left(1 + \frac{1}{n}\right)^n}{S_n} > 1 - \frac{1}{2n},$$

which shows that $\text{Lt} \left(1 + \frac{1}{n}\right)^n = \text{Lt } S_n = e.$

If n is not integral, let $m < n < m+1$, where m is an integer. Then evidently

$$\left(1 + \frac{1}{m+1}\right)^m < \left(1 + \frac{1}{n}\right)^n < \left(1 + \frac{1}{m}\right)^{m+1}, \text{ etc.}$$

$\left(1 - \frac{1}{n}\right)^{-n}$ may be treated in a similar manner, or, since

$$\frac{\left(1 + \frac{1}{n}\right)^n}{\left(1 - \frac{1}{n}\right)^{-n}} = \left(1 - \frac{1}{n^2}\right)^n,$$

we have by (1),
$$1 > \frac{\left(1 + \frac{1}{n}\right)^n}{\left(1 - \frac{1}{n}\right)^{-n}} > 1 - \frac{1}{n}.$$

Thus
$$\text{Lt} \left(1 - \frac{1}{n}\right)^{-n} = \text{Lt} \left(1 + \frac{1}{n}\right)^n.$$

Of course the same arguments may be used in dealing with the more general limit of $\left(1 + \frac{x}{n}\right)^n$ or $\left(1 + \frac{1}{n}\right)^{nx}$. J. LISTER.