SOME PHASES OF THE RAPID TRANSIT PROBLEM.

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The rapid transit problem, in congested districts, has to deal with the transportation of passengers at a high average speed with frequent stops, and even in suburban traffic where the stops are less frequent, the schedule speeds have been so increased by the ability of the electric motor to accelerate rapidly, that trains hardly reach full speed before it is necessary to apply the brakes. As running at a constant speed does not occur in rapid transit service where stops are at all frequent, it becomes of the greatest importance to carefully investigate the subject of train acceleration in order to determine the method of running a train from station to station with the least expenditure of energy.

Problems in train acceleration may be divided into two broad classes: 1.—Where the road is level and

2.—Where grades exist or where an artificial profile is made in order to take advantage of down grades at starting.

Modern passenger cars demand a dead weight of approximately 550 lbs. for each passenger carried, or, in other words, only 20 per cent. of the total weight of a loaded motor car is a paying load. When it is considered that in rapid transit service with frequent stops, over 80 per cent. of the total energy output of the motive power on level roads is required to accelerate the train, it is evident that in this class of service the rolling stock requires the greatest attention.

To avoid undue complication, the following discussion has assumed that train friction is a constant quantity at all speeds, as at the low maximum speeds—20 to 30 miles per hour—reached

in practice with frequent stops of two or more per mile, the error introduced by assuming a constant friction rate will be small and will not at all alter the conclusions arrived at.

The following constants have been assumed as representing average operating conditions:

Length of run, 2000 feet.

Length of time train is in motion, 75 seconds.

Schedule speed, 16.05 miles per hour, including 10-second stops, or 85 seconds total time.

FIG. 1.—Acceleration Curves.

Distance, 2,000 feet. Time, 75 seconds. Tractive Effort, 100 lbs. per ton.
Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

Tractive effort, 100 lbs. per ton total, to be maintained uniform during acceleration of the train.

Braking effort, 150 lbs. per ton constant throughout the period of braking.

The train friction being 15 lbs. per ton, reduces the effective accelerating force from 100 to 85 lbs. per ton, corresponding to a rate of .927 miles per hour per second.

With this data a simple acceleration curve, as in Fig. 1., may be calculated, the train reaching a maximum speed of 28 miles per hour in 30 seconds, when power is shut off and the train allowed to coast with a retardation due to friction of .1635 miles per hour per second for 31 seconds to the braking line, when it is retarded at a uniform rate of 1.635 miles per hour per second and comes to rest in 75 seconds from time of starting.

Owing to the greater efficiency of the run, the train is allowed to coast after reaching its maximum speed rather than to allow it to continue at a uniform speed with just sufficient power supplied to overcome the train friction loss.

It is obvious that the train could be accelerated at a different rate than that corresponding to 85 lbs. per ton and still make the same length of run in the same time, the length of time occupied in coasting depending upon the rate of acceleration, being a maximum with an infinite rate, that is, with the train starting with a certain initial velocity. A minimum rate of acceleration is reached when no time is left for coasting, that is, when brakes are applied as soon as power is shut off.

In Fig. 2 such a set of curves has been prepared showing a train covering a distance of 2000 feet in 75 seconds, as before, but accelerating at various rates from that corresponding to 62.8 lbs. per ton as a minimum up to an infinite rate, or starting with an initial velocity of 25.2 miles per hour.

As the area enclosed by time as abscissæ and speed as ordinates represents the distance covered, this will be a constant quantity for the fixed distance of 2000 ft. assumed, and curves of Fig. 2 are thus constructed with the same enclosed area for each rate of acceleration. The fact is plainly brought out that with a low rate of acceleration a much higher maximum speed is demanded than would be the case if the rate had been increased, and a curve may be plotted by joining the maximum speeds reached for different accelerating rates, as shown.

A friction of 15 lbs. per ton has been chosen as being that of an average train composed of a motor car and three or four trailers and weighing about 120 tons. With heavier and longer trains this rate may be reduced to as low as 7 or 8 lbs. per ton, while for a motor car alone, the rate may be as high as 30 lbs. per ton, due to friction of motors and gearing. The braking effort of 150 lbs. per ton is also chosen as representing what can be done on an average by a train equipped with air brakes and operating at half the slipping coefficient of the wheels.

It is advisable in rapid transit service to keep the maximum speed reached by the trains as low as possible, as this class of work generally calls for a short time-interval between trains

FIG. 2.—Speed and Energy Curves.

Distance, 2,000 feet. Time, 75 seconds. Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

where the utmost precautions are necessary to keep the trains a safe distance apart. In fact, the maximum speed required for a given run practically determines the time-interval between trains,

as the train headway should be at least five or six times the length of time required to bring a train to rest from its maximum speed with normal braking force applied.

A second reason for higher rates of acceleration lies in the fact that with rates of acceleration approaching the minimum no margin is left for errors of judgment of the motorman, as little or no time is left for coasting, and the rate of acceleration per ampere input is continually varying with the changing passenger load and hence with an overload it becomes difficult to maintain schedule speed.

A third and most important objection to the use of a low rate of acceleration lies in the saving of energy for the run by the use of a higher rate.

Neglecting $I^2 R$ and core losses of the motors, a set of energy input curves per ton weight of train may be plotted as in Fig. 2, assuming for simplicity that motors are series wound, operate all in multiple and are so geared that starting resistance is entirely cut out at the various maximum speeds reached in the various runs, that is, that no accelerating is done on the motor curve. A constant impressed E. M. F. is assumed, and starting resistance is supposed to be cut out proportional to the motor speed, thereby keeping the current and torque constant.

Thus it is seen that accelerating a train with the minimum rate calls for the highest maximum speed, demands nearly the least current input and also demands the greatest waste of energy in the brakes, as the speed is a maximum when brakes are applied.

The areas enclosed by the various energy time curves represent the comparative amounts of energy required for the run for the different rates of acceleration, and these are plotted in Fig. 2, which compares the energy input and the average energy consumption for the run of 2,000 feet in 75 seconds for all rates of acceleration from the minimum corresponding to 62.8 lbs. per ton up to infinity.

For convenience the average energy rates are plotted in terms of watt hours, and the curve shows that while the minimum rate of acceleration corresponding to 62.8 lbs. per ton calls for an expenditure of 83.5 watt hours per ton weight of train, this is reduced to 56.5 watt hours by accelerating with a rate corresponding to 70 lbs. per ton, to 42.7 watt hours with 100 lbs., and finally reaches a minimum value of 32.5 watt hours if the accelerating rate is pushed to infinity.

Thus the energy required for the run of 2,000 feet in 75 seconds may vary from 83.5 to 32.5 watt hours per ton of train weight, a decrease of over 60 per cent., depending upon the rate of acceleration used.

The curves in Fig. 2 are worthy of careful study, and similar curves afford a means of determining the proper rate of acceleration and hence motor equipment, gearing, etc., to use for a given set of conditions. The limiting factor in the more rapid rate of acceleration of a train is the current input required. Thus, if the rate of acceleration be carried to abnormally high values, the local demand for current becomes so great that either the loss in the feeders more than offsets the reduction in energy consumption at the train, or else the interest on the increased feeder investment is not offset by this energy reduction.

There are other limiting factors governing the rate of acceleration in the size and weight of motors, which are limited in the current they can carry without undue sparking and heating. Thus it will be found that the rate at which a train accelerates, largely determines the cost of feeders, size of motors and generators, both in regard to current and thermal capacity, and also fixes the safe headway between trains.

The error made in accelerating at or near the minimum rate is clearly brought out. For example, the current consumption per ton is the same for 62.8 lbs. per ton or 85 lbs. per ton; as, although 85 lbs. calls for the greater tractive effort, the torque per ampere is so increased by the lesser maximum speed demanded, that the current input is the same in each case, hence the feeder considerations are the same in both cases, while the energy consumption shows a reduction from 83.5 watt hours per ton with 62.8 lbs. to 47.5 watt hours, or about half, with 85 lbs. per ton.

Referring again to Fig. 2, it is obvious that a similar set of curves may be plotted for a run of 2,000 feet for any other length of time than 75 seconds, and Fig. 3 gives such a set of curves plotted for lengths of time ranging from 41 seconds, as a minimum possible, up to 210 seconds. The minimum time in which it is possible to make a run of given length is determined by the braking effort, in this case assumed to be 150 lbs. per ton, the train reaching a maximum speed of 66.5 miles per hour in zero seconds with an infinite accelerating force, and being retarded throughout the entire running time of 41 seconds at the rate of 1.635 miles per hour per second, corresponding to a force of 150 lbs. per ton braking effort.

The constants assumed in these curves are the same as before, 15 lbs. per ton friction rate, and 150 lbs. per ton braking effort applied uniformly until the train comes to rest at a distance of

FIG. 3.—Speed Curves. Distance, 2,000 feet. Train Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

2,000 feet from the start. While the braking effort determines the minimum length of time for the run, the friction rate imposes a limit upon the maximum rate of acceleration possible for

lengths of time greater than 128 seconds for 2,000 feet run. That is, a train accelerating with an infinite rate and coasting the entire length of 2,000 feet would come to rest in 128 seconds with no energy loss in the brakes, and any longer interval of time occupied in the run would require some finite rate of acceleration at a maximum. This is pointed out in Fig. 3, where for a run in 210 seconds the maximum rate of acceleration possible is .175 miles per hour per second, corresponding to 24 lbs. per ton. No train in practice would require such a long time as 210 seconds, nor would it be possible to make a run of 2,000 feet in 41 seconds, but these curves have been carried out to show the limits for a given set of conditions.

As the energy lost in braking is proportional to the square of the speed when the brakes are applied, the curve A-B, Fig. 3, being the locus of the minimum rates of acceleration, that is, with no coasting, is thus the curve of maximum input for a run of 2,000 feet for any length of time. Also the curve C-D, being the locus of the various coasting lines, that is, of no braking effort, is thus the curve representing the minimum possible input.

To compare the amounts of energy required for rates of acceleration other than the maximum and minimum, a set of curves has been prepared in Fig. 4 giving the energy consumption for a run of 2,000 feet on a level track for any rate of acceleration and for any length of running time, the constants being 15 lbs. per ton friction rate and 150 lbs. per ton braking effort. For convenience in comparison, the energy consumption is reduced to watt hours per ton mile, and speed is expressed as average speed in miles per hour while train is in motion or equaling schedule speed if the train loses no time in stopping. The dotted curve A-B is the maximum energy curve corresponding to the curve A-B of minimum rates of acceleration in Fig. 3, and the curve C-D represents the minimum amount of energy possible for the different speeds and is described by an infinite rate of acceleration.

The curves of maximum and minimum energy consumption approach each other and coincide at a speed of 33.3 miles per hour, corresponding to an energy consumption of 298 watt hours per ton mile, this value being the greatest amount of energy that can be expended on the run with 150 lbs. per ton braking effort.

All energy values are net, that is, they represent the amount of energy required to accelerate the train plus energy lost in overcoming friction, and hence take no account of any losses occurring in actual operation in the motors, rheostats, gearing, etc.

Fig. 4 shows the economy resulting from properly proportioning the accelerating rate to the schedule speed and distance traveled. For example, a train accelerating with a tractive

FIG. 4.—Speed-Energy Curves.
Distance, 2,000 feet. Friction, 15 lbs. per ton.

effort of 100 lbs. per ton and making 20 miles per hour average speed, not including stops, will require 127 watt hours per ton mile, which would be reduced to 65 watt hours per ton mile if the tractive effort had been increased to 150 lbs. per ton, or, in other words, the generator capacity would be but half as large for the same service.

The curve of 300 lbs. per ton tractive effort is interesting as it represents about the maximum speed attainable with modern

apparatus for a distance of 2000 ft. with the assumed constants of friction and braking effort. Assuming the entire weight of the car to rest upon drivers, about 300 lbs. per ton would be available for traction without slipping the wheels on an average track, so that an average speed of $26\frac{1}{2}$ miles per hour is the highest that could be obtained over a distance of 2000 ft., not allowing any time whatever for coasting.

All previous curves have been based upon the assumption that trains are allowed to coast after reaching maximum speed, and also that acceleration is carried on at a perfectly uniform rate until power is shut off, but in practice this assumption may be modified somewhat, the starting resistance being cut out before the maximum speed is reached, and the latter part of the acceleration carried on at a constantly decreasing rate upon the motor curve.

Another method would be to accelerate at a constant rate until maximum speed is reached, then continue this speed constant by supplying motors with just sufficient power to overcome train friction. This latter method of train acceleration, however, demands such a considerable increase in the amount of energy required for the run that it has not hitherto been considered.

The three methods of acceleration are illustrated in Fig. 5, showing the three forms of speed curves, *a-a* accelerating at a constant rate and coasting after maximum speed is reached until brakes are applied, *b-b* accelerating at a constant rate and continuing at full maximum speed until brakes are applied, and *c-c* accelerating at a constant rate until starting resistance is cut out and further acceleration allowed to continue at a constantly decreasing rate with constant full line potential at motor terminals until maximum speed is reached, when train coasts until brakes are applied.

Curve *a-a* reaches the highest maximum speed but wastes the least energy in the brakes, and hence is the most efficient run mechanically, curve *b-b*, the constant speed method, being the least efficient.

As these three curves were plotted from the speed torque curves of an actual motor, it is instructive to compare the watt hours consumed for each run with series parallel control, operating two motors in series, then in multiple. To this end a set of energy input curves have been plotted in Fig. 5, showing that curve *c-c* requires the least maximum energy input, while curve *a-a* requires the greatest amount.

The area enclosed by the energy time curves is a measure of the average energy consumption for each run, and their respective values reduced to watt hours are:

- a. Constant current and coasting, 147 watt hours per ton,
- b. Constant current and no coasting, 160 watt hours per ton.
- c. Constant current and acceleration on motor curve, 126 watt hours per ton.

FIG. 5.—Speed Curves.

Distance, 2,000 feet. Time, 75 seconds. Friction, 15 lbs. per ton. Braking Effort, 150 lbs. per ton.

Hence, of the three methods, curve *c*, making use of the acceleration due to a series motor curve, not only requires the least maximum current input, but also requires the least average energy input to the motors for a given run and hence is the form of curve used in the majority of actual runs. Although the energy given out by the motors in run *c* is greater than in run *a*, as evi-

denced by the higher speed at which brakes are applied, yet this extra work is done so much more efficiently, owing to the smaller starting resistance loss, that the total watt hours input becomes less.

Whatever form of acceleration curve be used, the results arrived at in Fig. 4 will not be greatly modified if similar forms of curves are compared, and, being expressed in terms of actual work done, may be used for any run with a factor expressing the efficiency of acceleration: that is, the ratio of actual work done to the total energy input.

It will be noted that all curves of Fig. 4 approach a minimum value of 30 watt hours per ton mile, that is, the minimum energy expressed in watt hours per ton mile expended for a given run will be double the friction rate. The actual factor is 1.98 and forms a very convenient method of determining the net energy consumption for any speed and train weight if the friction rate be known. Thus, assuming the light load efficiency of a railway motor, including gear loss, to be 75 per cent., a friction rate of 15 lbs. per ton would demand an input of 40 watt hours per ton mile, corresponding to an input of 1200 watts per ton weight of train at, say, a constant speed of 30 miles per hour.

A number of interesting conclusions may be made from the foregoing investigation of the operation of trains upon a level track.

1st. The rate of acceleration determines the energy consumption for a given run, and since this energy consumption decreases with increased rate of acceleration, the train should be brought up to speed as quickly as possible and allowed to coast to secure the minimum energy input.

2nd. The maximum current input during acceleration increases with the rate of acceleration, and hence limits the rate at which a train can be accelerated with a given feeder loss or feeder investment.

3rd. In order to reduce the average energy consumption and also the maximum current input to a minimum for a given run, a due amount of acceleration should take place on the motor curve after starting resistance is cut out, hence a motor should be carefully proportioned for the work it has to do.

4th. A normal amount of coasting should be permitted after power is shut off, partly to provide a margin to allow for errors of judgment of the motorman, but largely because this is the

most efficient method of accelerating a train. On no account should the maximum speed be continued by supplying the motors with just sufficient current to overcome train friction, as this method of accelerating is extremely wasteful and inefficient.

Having discussed various methods of accelerating, it is interesting to follow out the problem, and determine the actual efficiency of transporting passengers by our modern methods of travel. It has been pointed out that only from 15 to 20 per cent. of a fully loaded train consists of a paying load, and with an average load as carried throughout the day this percentage will be reduced to 10 per cent. or less, that is, nine-tenths of the energy consumed in moving this train at a constant speed is wasted. But in rapid transit work a train seldom attains a constant speed, due to the frequent stops and high schedule speed, and at least ten times the energy required to overcome friction alone must be expended in accelerating the train, only to appear as heat in the brake shoes when bringing the trains to rest. That is, considering the friction work of the train as the only useful work done, an efficiency of only 10 per cent. is reached in the average run. But only 10 per cent. of this friction work is useful in moving passengers, hence the actual passenger efficiency is reduced to less than one per cent. of the total energy delivered to the train during acceleration.

When it is considered that further losses occur in operation in the motors and their method of control, in the transmission lines and generators of an electric traction system, it will be appreciated that the present method of transportation, with its efficiency of a fraction of one per cent., opens a wide field for improvement. This applies with greater force to rapid transit service using steam locomotives as a motive power, as the dead weight carried per passenger is greater with a steam locomotive than with a motor car, and the efficiency from the coal pile is much less. Hence some means of reducing the large loss, due to accelerating the train is desirable, and this is found in the adoption of an artificial profile, as followed out in a large underground road now building, using down-grades in starting, and up-grades to retard the train when stopping.

The ideal profile would provide for a down-grade at starting sufficient to do the work of acceleration and with an up-grade to do the entire braking required, thus leaving only the friction energy to be supplied by the motive power. Such a road would

operate at 100 per cent. efficiency, neglecting the dead weight of train carried and considering total friction work as useful work.

Reduced to practice, this ideal grade must be modified considerably. The per cent. grade used is limited partly by the rapidity of acceleration that may be imparted to the train without discomfort to the passengers and partly by the available tractive effort of the motive power, which must be sufficient to haul the train up the grade in case of necessity.

A second modification occurs in the necessity of having stations placed on a level track, and since a train has an appreciable length, it must travel its own length before the last car is off the level and the full effect of the grade is felt. Thus, during the first period of acceleration the rate is comparatively small and must be furnished by the motors, resulting in a material reduction in the energy gain in the theoretically ideal profile. An artificial profile can only be secured on elevated or underground roads, and a double track road necessitates two separate overhead structures and the underground road must consist of two separate tunnels with unlike profiles.

Assuming two stations to be on the same level, there are two forms of profile that may be assumed with a given maximum difference in levels, as shown in Fig. 6. Profile A consists of a down-grade with a level track at each terminus equal to the length of train operating over the road, while profile B consists of a down-grade and up-grade equal in length and percentage with a level track connecting them, and also a level track at each station equal to a train length.

In practice a perfectly symmetrical profile, as in B, will not be possible, as the level tract connecting grades must be replaced by a slight grade to provide for drainage in tunnel roads, this grade preferably opposing the direction of movement of the train.

In order to ascertain the behavior of a train upon the two forms of profiles and to determine the most efficient form of grade, a series of curves have been plotted upon the following assumptions:

Length of run total, 2,000 feet.

Train friction, 15 lbs. per ton constant.

Braking effort, 150 lbs. per ton during time brakes are set.

Running time, 75 seconds.

Length of train, 200 feet, corresponding to an average train of four to five cars.

Tractive effort, as supplied by motors, 100 lbs. per ton. (This effort being supplied irrespective of added effort due to grades.)

FIG. 6.—Energy Curves on Artificial Profile.

Distance, 2,000 feet. Time, 75 Seconds. Starting Grade of 4%. Tractive Effort of Motors, 100 lbs. per ton. Friction, 15 lbs. per ton, Braking Effort, 150 lbs. per ton.

No running is assumed to take place on the motor curve, and train is supposed to coast after reaching maximum speed. Motors are controlled by the ordinary series-parallel method, starting with two motors in series and throwing them

into multiple, the starting resistance being supposed to be cut out uniformly so that 100 lbs. per ton motor effort is maintained constant while current is on.

Due regard is paid to the fact that the tractive effort due to grade depends upon the proportional length of train off the level track at the stations, and hence is a constantly increasing quantity until the entire train is on grade.

A grade of four per cent. has been chosen for the down-grade in A, and for both down and up grades in B; and energy consumption plotted in Fig. 6 for different lengths of grade.

Thus for a run of 2000 feet in 75 seconds, the energy input with series parallel control is 40.75 watt hours per ton for a level track, which is reduced to 27.8 watt hours in curve A with a length of 500 feet, 4 per cent. grade and 22.6 watt hours per ton in C, with the same length of 4 per cent. grade, that is, with the same vertical fall.

A length of 500 feet of 4 per cent. grade calls for a vertical fall of 20 feet, which is not excessive in practice, hence by the use of a profile similar to B, Fig. 6, the energy consumption for a given run may be reduced as much as 40 to 50 per cent. from that required on a level track.

With proper proportioning of the gear ratio of the motor allowing some acceleration on the motor curve, this saving in energy consumption could even be exceeded, so that a rapid transit road, especially a tunnel road, properly laid out with an artificial profile, could operate with a very much less energy consumption than an existing surface road.

In conclusion it may be pointed out that the electric motor is eminently adapted to rapid transit service, owing to its ability to accelerate rapidly; and for tunnel work especially, it has no rival, adding to its high efficiency of operation and its perfect immunity from smoke and gases.

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