the given solution of ioduret, mingled with starch solution, and stop at the moment in which the liquid, after passing through all the shades mentioned, becomes extremely clear. The quantities of chloric solution are in inverse ratio with the proportions of chlorine which they contain. The only precaution necessary in the assay, is to hold in the left hand the glass which contains the measured ioduret charged with eight or ten drops of starch solution, and twirling it round a little, pour into it from the right hand the chlorated solution contained in the crusts.

The determination of the value of an alkaline hypochlorite is effected in the same way in operating upon a recent solution of the salts made in the proportion of ten grammes to a litre of water. One condition only it is important to observe, in order that the operation be quickly and accurately performed; that is, to add to the test solution, charged with starch, a drop or two of concentrated sulphuric acid, for the purpose of disengaging chlorine, when the solution of hypochlorite is added. If this be not attended to, the operation is slow, and requires repeated additions, for to the coloring and discoloring produced by the first drops of hypochlorite in the non-acidulated proof liquor, succeeds spontaneously a fresh coloration which is soon destroyed by a few drops of hypochlorite, and this effect continues four or five times in succession, until the ioduret of potassium is decomposed.—*Ann. des Mines*, tom. iii, liv. 11.

FOR THE JOURNAL OF THE FRANKLIN INSTITUTE.

On the Diameter of Screws. By JAMES DEAN, A.M., A.A.S., &c., and late Prof. Nat. Phil. and Ast. Univ. Vt.; investigated in 1825.

Every attentive student must have been surprised, on his first examining the investigation of the mechanical powers, at finding that the *diameter* of the screw was not an element in expressing the ratio of the power to the weight necessary to produce an equilibrium; and that, other things being equal, the diameter of the cylinder may vary *ad infinitum*, without, in the least, affecting that ratio. That reducing, or increasing, the diameter to an extreme degree, the distance of the threads remaining the same, will diminish the effect of any given power, will be universally admitted without any very nice reasoning on the subject; but it seems far from useless to ascertain the *exact* diameter, which, under any given circumstances, will enable a given power to produce the greatest effect, and it is very natural to look among the effects of friction for this as well as other deviations from abstract theory. Now the ratio of the friction to the pressure which produces it, has been found by Amontons, Parent, Coulomb, &c., to be very nearly constant between the same surfaces, and commonly about one-third. And as the writer cannot learn that friction is much considered in this country as a retarding and computable force, he takes the liberty of translating from Bossut's *Mechanics*, those sections in which he treats of friction as affecting inclined planes.

269. Let a body P , be placed on an inclined plane whose length is GH , height HI , and base GI ; resolve the weight of the body, represented by the vertical line PD , into two other forces PC parallel, and PA perpendicular, to the inclined plane. It

Fig. 1.

is obvious that the force $PC = P \times \frac{HI}{GH}$,

and the force $PA = P \times \frac{GI}{GH}$. The first

of these two forces, called the relative weight of the body, tends to make it slide down the plane, while the second produces a pressure on the plane, and causes a friction of the first species, so that putting n , for the ratio of the pressure, the friction $= nP \times \frac{GI}{GH}$. If, therefore, the body be left entirely to itself, it

will not descend unless its relative weight PC , be greater than the friction, that is, unless $\frac{P \times HI}{GH} > nP \times \frac{GI}{GH}$, or $HI > n \times GI$.

Hence, a body laid on an inclined plane, and left to the action of its weight only, will not descend unless the height of the plane is greater than the product of the base multiplied by the ratio of the friction to the pressure.

270. Let a body be ready to descend, or let its relative weight be equal to the resistance of friction $HI = n \times GI$, or $n = \frac{HI}{GI}$. Thus,

when the inclination of the plane is such that a body may be ready to descend by its own relative weight, the ratio of the friction to the pressure is that of the height of the plane to its base. Knowing, therefore, either of these ratios the other is known also.

Suppose, for example, the friction to be one-third of the pressure $\frac{HI}{GI} = \frac{1}{3}$, and by trigonometry the ratio $\frac{HI}{GI}$ may be considered as that of the tangent of the angle of inclination HGI to radius, and by trigonometric tables, this last ratio being $\frac{1}{3}$, the angle HGI is about $18^\circ 26'$. So that the friction being taken at one-third of the pressure, the angle of inclination of the plane must be about $18^\circ 26'$, that the body may be on the point of descending by its own relative weight.

If, on the contrary, the angle of the inclination of the plane be given, the ratio may be found by the tables, which is the value of n . This furnishes a very simple and convenient method of ascertaining friction by experiment. It is only necessary to lay the body on a plane at first very little inclined to the horizon, and gradually increase the inclination until the body begins to descend, and the ratio of height of the plane to its base is that of the friction to the pressure.

271. We will now consider a body which some power is ready to raise along an inclined plane, by overcoming its relative weight and friction. It is easy to find the general value of that power, but we shall confine ourselves to the two cases which most frequently occur,

viz., when the direction of the power is parallel to the length of the plane, or to its base. The one method may easily be extended to every other case.

Fig. 2.

272. In the first place, let the power Q , act in a direction parallel to the length of the plane. In order that the body may begin to slide in the direction GH , the force Q , must be equal to the sum of the relative weight of the body and the friction. But in constructing, as above, the rectangle $PADC$, and always calling n , the ratio of the friction to the pressure, the force, or relative weight, $PC = P \times \frac{HI}{GH}$,

the force $PA = P \times \frac{GI}{GH}$, the friction $= nP \times \frac{GI}{GH}$; and, therefore,

$Q = P \times \frac{HI}{GH} + nP \times \frac{GI}{GH}$, a formula which exhibits the quantity by which the friction enters into the expression of the power Q .

Example.—Let the weight $P = 8000$ lbs., the angle of inclination $HGI = 30^\circ$, or $\frac{HI}{GH} = \frac{1}{2}$, $\frac{GI}{GH} = \frac{1.73}{2} = 0.866$, very near, and $n = \frac{1}{2}$, then $Q = 4000$ lbs. + 2309.4. The power Q must, therefore, be more than 6309.4, while without friction it would be only 4000 lbs.

273. In the second place, let the power Q act in the direction parallel to the base of the inclined plane. Having resolved, as above, the weight of the body into two forces PC , and PA , the one parallel, the other perpendicular, to the inclined plane, resolve in like manner

Fig. 3.

the power Q , expressed by the part PO , of its direction, into two other forces PN , and PM , the one parallel, the other perpendicular, to the length of the plane. Then the force,

$$PC = P \times \frac{HI}{GH}, \quad PA = P \times \frac{GI}{GH}, \quad PN = Q \times \frac{GI}{GH}, \quad \text{and} \quad PM = Q \times \frac{HI}{GH}.$$

The whole pressure on the inclined plane being equal to the sum of the two forces $P A$, $P M$, if n always represents the ratio of the friction to the pressure, it is plain that the friction,

$$= n \times \left(P \times \frac{G I}{G H} + Q \times \frac{H I}{G H} \right)$$

This being premised, in order that a body may be ready to slide in the direction $G H$, the force $P N$ must be equal to the sum of the force $P C$, and the friction, or

$$\frac{Q \times G I}{G H} = \frac{P \times H I}{G H} + n \times \left(\frac{P \times G I}{G H} + \frac{Q \times H I}{G H} \right),$$

whence,

$$Q = P \times \frac{H I + n \times G I}{G I - n \times H I}.$$

Without friction the value of the power would be $\frac{P \times H I}{G I}$, so that $\frac{P \times (H I + n \times G I)}{G I - n \times H I} - \frac{P \times H I}{G I}$, or $\frac{n P \times G H^2}{G I^2 - n \times G I \times H I}$, is the quantity by which the friction increases the power.

Example.—Let $P = 8000$, $H G I = 30^\circ$, or $\frac{H I}{G H} = \frac{1}{2}$, $\frac{G I}{G H} = \frac{\sqrt{3}}{2} = .866$, $n = \frac{1}{3}$. It will be found that $Q =$ almost 9021.7 lbs. Without friction the power would be only about 4618.8 lbs.

Of Friction in the Screw.

Fig. 4.

Let the inclined plane $G H I$, expressing the last value of Q in the above paragraph, as the power in the screw always acts in a direction parallel to $G I$, or perpendicular to the axis of the cylinder, be wrapped round a cylinder of such radius $C p$, that $G I$, the base of the triangle, shall be just equal to the circumference, and, of course, just reach round it; and let $G H$, the inclined plane itself, be produced in the *same direction* an indefinite number of turns around the cylinder. Here then obviously $G H$, forms one thread of a screw, while $H I$, the height of the inclined plane, is equal to the distance of the threads

parallel to the axis. Then let a nut be perforated, and formed in the interior, so as to correspond to the thread of the screw, applying itself to the surface G H, in its whole length, and perhaps two or three other turns, and let the weight P be laid on the nut, the friction is the same whether it be supported on a small, or large, surface, and let this nut be furnished with a lever C R, perpendicular to the axis, and extending to R, the point where the new power q , is applied, make an equilibrium with the weight P, and its friction in the same manner as Q does at the surface of the screw. On the principle of

the lever, $q : Q :: C p : C R$, or $q = Q \times \frac{C p}{C R}$. Let this equation be incorporated with that expressing the value of Q in the inclined plane, where the power acts in a direction parallel to the base, and we shall

have $q = P \times \frac{C p \times (H I + n \times G I)}{C R \times (G I - n \times H I)}$. This is the formula for the screw as commonly constructed; P = the weight to be raised, q = the power necessary to just not produce motion when acting at the distance C R, and in a plane perpendicular to the axis C, C p = the semi-diameter of the screw, H I = the distance of the threads measured parallel to the axis, G I = the circumference of the screw, n = the ratio of the friction to the pressure.

Now, in order to find the value of C p, when q is a minimum, the other quantities remaining constant, P and C R being omitted, as affecting all the terms equally, for conciseness' sake let C p = x , H I = h ,

and $\pi = 6.28$, &c., and the fraction will become $\frac{h x + n \pi x^2}{\pi x - h n}$, which has its least value when its differential,

$$\frac{h \pi x dx + 2 n \pi^2 x^2 dx - h^2 n dx - 2 h n^2 \pi x dx - h \pi x dx - n \pi^2 x^2 dx}{\pi x - h n^2} = 0,$$

whence $\pi^2 x^2 - 2 h n \pi x = h^2$ and πx , (the circumference of the screw,) = $h \times (n + 1 + n^2 \frac{1}{2})$ from which the diameter may be computed as usual.

From the denominator of the first fraction in the last paragraph, it appears that when $h n = \pi x$, that is, when the circumference of the screw : distance of the threads :: the friction : the pressure, the denominator becomes 0, and, therefore, the power q must become infinite, or the effect 0. When this is the case, if h , or the perpendicular height of the inclined plane, formed by one revolution of the thread, be taken = 1, or radius, then $n = \pi x$ the circumference of the cylinder, or base of the plane, or cotangent of the angle contained between any thread and a plane perpendicular to the axis. Let a represent this

angle, then $n = \cot. a = \frac{\cot. \frac{1}{2} a - 1}{2 \cot. \frac{1}{2} a}$ by an : trig: whence $\cot. \frac{1}{2} a - 1 =$

$2 n \cot. \frac{1}{2} a$, and $\cot. \frac{1}{2} a = n + 1 + n^2 \frac{1}{2}$, which is precisely the value of the circumference given by the calculus when the effect is greatest, so that the angle of inclination at which the greatest effect is produced, is exactly half of that angle which, with the same degree of friction, is the limit of all effect.

From the calculus it appears that screws are commonly made much too large in proportion to the distance of the threads, to produce any given effect with the least power, the circumference in hardly any practical case being, for that purpose, greater than twice the distance of the threads, and the diameter much less than that distance. Thus, suppose it be desired to produce a pressure equal to ten tons, with a screw whose threads are one inch asunder, lever eight feet long, and friction one-third the pressure; if the screw be six inches in diameter, the power must exceed 275 lbs., if the diameter be only two inches, any power greater than 122 lbs., will be sufficient. If the friction amount to half the pressure, the dimensions still remaining unaltered, the powers corresponding to the same two diameters, will be 398 and 168 lbs. respectively. The least powers with these two degrees of friction would be 72 lbs. and 98 lbs., and require diameters only 0.4416 in., and 0.515 in. respectively; but no substance is known, which, when reduced to even twice these dimensions, would have strength enough to resist the torsion. It appears, therefore, that a screw, as an engine of force, with any given distance of thread, ought to be made as small as the strength of the material will admit.

In order to ascertain, by computation, the middle course between wasting materials, labor, and power, on one hand, and hazarding the failure of the machine on the other, we may adopt the doctrine "that in resisting torsion the whole lateral cohesion is exerted at $\frac{1}{4}$ th the radius of the cylinder from the centre."* The notation of the formula for the screw being retained, and S being put for the lateral cohesion of a square unit of the substance used, the area of the section is $\frac{\pi x^2}{2}$,

and its strength $\frac{S \pi x^2}{2}$, then $CR : \frac{x \cdot S \pi x^2}{4 \cdot \frac{\pi x^2}{2}} : \frac{S \pi x^3}{8 CR} =$ the force of the cohesion at the end of the lever, which, being substituted for q , in the formula $\frac{S \pi x^3}{8 CR} = \frac{P \times (hx + n\pi x^2)}{CR \times (\pi x - hn)}$, and this equation being reduced, gives $x^3 - \frac{hn}{\pi} x^2 - \frac{8 P n}{S \pi} x = \frac{8 P h}{S \pi^2}$, in which, if the known quantities are properly substituted, the value of x may be safely adopted as the radius of the cylinder.

But when the exertion of great force, or the weakness of the material, renders a large cylinder indispensable, the distance of the threads should be greater also, because increasing the distance of the threads does not require, by any means, a proportional increase of power. Thus with the six-inch cylinder, above mentioned, with the friction of one-third, increasing the distance of the threads from one to two inches, requires an increase of power less than one-sixth, viz., from 275 lbs. to 319 lbs.; and with a friction of one-half the increase is only one-eighth, or from 398 lbs. to 448 lbs., which may generally be compensated by using a longer lever, while the action will be much more prompt and rapid. And less than double the power will produce the same effect, though the distance of the threads be made

* Edin. Encycl. vol. v, p. 400; art. "Carpentry."

equal to the diameter of the screw, or six times their former distance. The reason of this will be obvious, when it is considered that in this case the whole power except 37 lbs. is employed in overcoming the friction, and that at such a small angle of elevation, the pressure, and, consequently, the friction, is but slightly varied by a variation of that angle. And with the same distance of the threads, if the diameter receive successive increments, the ratios of the corresponding powers will continually approximate to the corresponding ratios of the several diameters, because the effect of friction, which absorbs most of the power, is proportional to its distance from the fulcrum.

But after all it is far from being eligible, in practice, *always* to give screws the shape above described, especially in the fastening of instruments, or machines, where the object is not to produce but to prevent motion, because if the distance of the threads be greater than n times the circumference, the friction of the screw will not prevent it from running back. On this account it may not be improper to investigate the figure of a screw so formed, that, without making the angle of ascent of the thread more acute than necessary, yet the friction shall always hold it where the power leaves it. Let it be required, therefore, so to shape a screw, that with any given degree of friction the power necessary to turn it forward shall have a ratio to the power necessary to turn it back equal to the ratio of the two given quantities $f : b$; f being, of course, larger than b . As the length of the lever to which these powers are applied, cannot effect their ratio to each other, they may be considered as applied at the surface of the cylinder, and this surface may be further considered, as extended on a vertical plane, in which case one revolution of the thread is the length of an inclined plane, the circumference of the cylinder is its base, and the distance of the threads its height; reversing the course of reasoning adopted in the first two sentences of the description of the screw. Bossut, in Art. 273, as translated above, gives the force which, acting parallel to the base, (as it always does in the screw,) just balances the relative gravity, and the friction,

$$= \frac{P \times (H I + n \times G I)}{G I - n \times H I},$$

where P = weight, $G I$ = base, $H I$ = height, and n = the ratio of the friction to the pressure. But in order to express the greatest force which, acting in the direction of the relative gravity, would not drag the body down the plane, $H I$ in the preceding equation, must be taken with the contrary sign from what it is in the equation above, because the force $P C$, or $P \frac{H I}{G H}$, or relative gravity, instead of oppos-

ing the power, assists it, and the force $P M$, or $Q \times \frac{H I}{G H}$ reversed, instead of increasing, now diminishes the pressure which produces the friction, or resisting force. With this change the greatest force which (acting parallel to the base, and from the summit towards the foot of the plane,) will not drag the body down the plane, will be found,

$$= \frac{P \times (-HI + n \times GI)}{GI + n \times HI}.$$

Let now $HI = \text{rad.} = 1$, $n = \cot. a$, as before, and $GI = \cot. z$; z being the angle at the base of the inclined plane, on which, with the given degree of friction, n , the force forward and backward will have the required ratio $f : b$. Then dividing by P , we shall have $f : b :: \cot. a \cot. z + 1 : \cot. a \cot. z - 1$
 $\frac{\cot. z - \cot. a}{\cot. z + \cot. a} :: \cot. (a - z) : \cot. (a + z) :: \sin. 2a + \sin. 2z : \sin. 2a - \sin. 2z$. Then by com. and div. $f + b : f - b :: \sin. 2a : \sin. 2z$, whence the base GI , or circumference of the screw, and thence its diameter may be easily found.

By the diameter of a screw is always meant the arithmetical mean between the greatest and least diameters of the bearing surface of the thread. The screw is also supposed to have a square thread, or at least that the section of its bearing surface with a plane passing through the axis, is a right line, and perpendicular to the axis. But when the thread is in the form of a triangular prism, or rather, when the section above mentioned makes an oblique angle with the axis, the pressure, and of course the friction is increased in the ratio of the sine of

that angle to radius, so that $\frac{n}{\text{its sine}}$ must be taken instead of n , through the whole investigation. It is supposed also that there is no pressure except on the threads of the screw; in practice, however, there is almost always a pressure at the other extremity equal to the weight raised, or effect produced, and the increase of power required by the friction resulting from this pressure, is proportional to the mean distance of its action from the axis of the screw. In most of the instruments called vices, or rather wherever the screw acts by tension between the points of action, this pressure is distributed over a circular space surrounding the body of the screw, and the friction thus remote from the axis must absorb a considerable portion of the power; but in the presses, or where it acts by compression between the points of action, it is commonly confined to the extremity of a pivot, and its retarding effect must be comparatively trifling.

Tables showing the results of the preceding investigation, the distance of the same thread after a complete revolution being the unity, CR=100 in., P=100 lbs.

TABLE I.—Showing the circumferences, diameters, radii, and angles of screws, and the proportional powers required for several different degrees of friction.

Degrees of Friction.	n	Bases and Circumference.	Diameters	Radii.	Angle a .	> Min. pwr.	Least power.
	3	$3+\sqrt{10}=6.1623$	1.9615	.9808	18° 26'	9° 13'	6.0437
	2	$2+\sqrt{5}=4.236$	1.3484	.6742	26 34	13 17	2.8559
	$1\frac{1}{3}$	$\frac{4+\sqrt{25}}{3}=3$	.9549	.4774	36 52	18 26	1.4433
	1	$1+\sqrt{2}=2.4142$	.7684	.3842	45 0	22 30	.9276
	$\frac{3}{4}$	$\frac{3+\sqrt{25}}{4}=2$	.6366	.3183	53 8	26 34	.6369
	$\frac{1}{2}$	$\frac{1+\sqrt{5}}{2}=1.618$	.515	.2575	63 26	31 43	.416672
	$\frac{1}{3}$	$\frac{1+\sqrt{10}}{3}=1.3874$	.4416	.2208	71 34	35 47	.306365
	$\frac{1}{4}$	$\frac{1+\sqrt{17}}{4}=1.2808$	.4076	.2038	75 58	37 59	.261075
	$\frac{1}{5}$	$\frac{1+\sqrt{26}}{5}=1.2198$	.3883	.1941	78 41	39 21	.2368
	$\frac{1}{6}$	$\frac{1+\sqrt{37}}{6}=1.1805$	.3758	.1879	80 32	40 16	.2218

TABLE II.—Giving the power for several diameters under each degree of friction, and showing that the diameter given by the formula and the preceding table requires the least power.

Several Degrees of Friction.	<i>n</i>	Diameters.	Powers.	Several Degrees of Friction.	<i>n</i>	Diameters.	Powers.
	3	2 1.9615 1.96 1.92	6.04582 6.04368252 6.043686 6.04636		$\frac{1}{2}$	3 2 .52 .515 .51	.96 .7161 .416689 .416672 .416691
	2	2 1.36 1.3484 1.34	2.9339 2.8561 2.8559 2.856		$\frac{1}{3}$	3 2 .4418 .4916 .4414	.68 .52 .3063647 .3063646 .3063647
	$1\frac{1}{2}$	2 .96 .9549 .95	1.9167 1.43242 1.432391 1.43242		$\frac{1}{4}$	2 .42 .4076 .4	.4261 .26115 .26107 .26110
	1	2 .78 .7684 .76	1.3785 .927765 .92762 .92770		$\frac{1}{5}$	2 .4 .3883 .38	.371 .23684 .23680 .23683
	$\frac{3}{4}$	2 .64 .6366 .63	1.0032 .63663 .63662 .63666		$\frac{1}{6}$	2 .38 .3758 .37	.3347 .221784 .221779 .221789

TABLE III.—Which, for the several degrees of friction, shows the diameters of the screws, and the angles of the threads necessary to insure a given ratio between the powers requisite to turn the screw forward and backward.

Different Degrees of Friction.

Proportion of Forces Forward and Backward.					
<i>n</i>	3 : 2	2 : 1	3 : 1	4 : 1	5 : 1
	5.285. 3° 27' 3.74 : 2.49	3.152. 5° 46'	2.073. 8° 44' 5.85 : 1.95	1.709. 10° 33'	1.525. 11° 47' 8.57 : 1.72
2	3.953. 4° 36'	2.345. 7° 44'	1.525. 11° 47'	1.245. 14° 21' 4.619 : 1.55	1.102. 16° 7'
1½	3.285. 5° 32'	1.937. 9° 20'	1.245. 14° 21'	1.004. 17° 35'	.88. 19° 54'
1	3.051. 5° 46' 1.224 : .816	1.855. 9° 44'	1.188. 15° 0' 1.732 : .577	.955. 18° 26'	.833. 20° 54' 2.235 : .447
¾	3.285. 5° 32'	1.937. 9° 20'	1.245. 14° 21'	1.004. 17° 35'	.88. 19° 54'
½	3.953. 4° 36'	2.345. 7° 44' .682 : .341	1.525. 11° 47'	1.245. 14° 21'	1.102. 16° 7'
⅓	5.285. 3° 27' .402 : .268	3.152. 5° 46'	2.073. 8° 44' .513 : .171	1.709. 10° 33'	1.525. 11° 47' .582 : .186
¼	6.750. 2° 42'	4.033. 4° 31'	2.668. 6° 48'	2.209. 8° 12'	1.978. 9° 9'
⅕	8.264. 2° 12'	4.945. 3° 41'	3.28. 5° 33'	2.721. 6° 40'	2.441. 7° 26'
⅙	9.804. 1° 52' .2 : .167	5.872. 3° 6'	4.91. 4° 40' .252 : .084	3.24. 5° 37'	2.91. 6° 15' .281 : .056