have to pay quite a heavy duty, so that possibly some time we may be able to manufacture more of these products here and more nearly supply our wants than is the case at present.

In concluding this lecture, I desire to express my thanks to the different dyestuff firms of this city for their kindness in supplying their latest productions, and in this way making it possible to bring to your attention some products which have not yet come into general use.

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A NEW THEORY OF THE PROPAGATION OF WAVES IN LIQUIDS.

BY L. D'AURIA.

While a wave is propagated in a liquid, the particles of the liquid which at a given instant are on any given vertical plane XX (Figs. 1 and 2), perpendicular to the direction of

the propagation, will, at the next instant, be found displaced to one side, or to the other, of such plane; and the volume of liquid thus displaced, which we indicate by δQ , will be either added to, or subtracted from, the volume Q of the wave, which at the given instant is confined between the plane XX , and the abutment to the wave propagation.

Let v , represent the velocity of propagation; γ , the weight of one cubic foot of liquid; and g , the acceleration

But the quantity

$$\frac{\int_0^h f(y) y dy}{\int_0^h f(y) dy}$$

represents nothing else but the distance Y , of the centre of gravity of the displacement δQ from the surface of the wave; consequently, the velocity of propagation of waves in liquids is expressed by the general formula

$$v = \sqrt{2gY} \quad (3)$$

In the case of sea waves raised by the action of the wind, the disturbance begins at the surface and is transmitted to

the lower particles of the liquid mass by cohesion. The law of this transmission can be fortunately deduced from the effect the resistance of river beds produce upon the velocity of streams. In fact, we know that, were it not for such resistance, the velocity of streams would be equal in all parts of the cross-section; and that the effect of such resistance is to make the velocity vary from the bottom to the surface according to a law well represented by a parabola whose axis is horizontal and somewhat below the surface, as shown in *Fig. 3*. Thus we know that the dis-

turbance of the particles at the bottom of the stream is transmitted to the other particles according to the law

$$z = \frac{(h_0 - y)^2}{p}$$

in which h_0 is the depth below the vertex of the parabola, and p the parameter of such parabola. This same law holds good also when the disturbance begins at the surface, since the cohesion of the liquid particles is independent of pressure; hence denoting, in this case, by h_0 the depth of the liquid where the horizontal displacement z ceases, we have

$$y = \frac{\int_0^{h_0} (h_0 - y)^2 y \, dy}{\int_0^{h_0} (h_0 - y)^2 \, dy} = \frac{1}{4} h_0$$

or

$$y = \frac{1}{4} \sqrt{p z_0}$$

where z_0 is the displacement of a liquid particle at the surface. Observing that $p z_0$ must be proportional to the force of the wind, and that the measure of such force can be seen in the amount of liquid raised above the surface, we can assume $p z_0$ proportional to λR , in which λ is the length, and R the height of the wave. Then we can write

$$v = c \sqrt[4]{\lambda R} \quad (4)$$

in which the coefficient c is to be determined by observation.

According to the observations taken by Dr. Scoresby, in 1848, on the Atlantic Ocean, waves averaging 26 feet in height and 560 feet in length, have a velocity of propagation of about 48 feet per second. With these data, equation (4) gives $c = 4.37$, approximately.

Again, according to other observations taken by independent observers, it is found that waves averaging 15 feet in height and 300 feet in length, have a velocity of propaga-

tion of about 36 feet per second. With these data, equation (4) gives $c = 4.39$, approximately.

The closeness of these two values of c is certainly remarkable, considering the diversity of the data presented in the two sets of observations reported above. We can, therefore, write with confidence

$$c = 4.38 \sqrt{\lambda R} \quad (5)$$

In the case of the tide wave raised by the action of the lunisolar disturbing force upon the waters of the oceans, we know that the horizontal component of such force at any given place is constant at all depths, and therefore the displacement z is also constant. Consequently, we have $V = \frac{1}{2} h$, in which h is the whole depth of the ocean; and substituting in equation (3), we find

$$v = V \sqrt{g h} \quad (6)$$

In tidal rivers, if we denote by u_0 , u_1 and u_2 , respectively the mean, the surface and the bottom velocities at any given instant in any cross-section whose depth at that instant is h , we find, assuming the parabolic law of velocities in the same vertical,

$$V = \frac{1}{4} h \frac{u_1 + u_2}{u_0}$$

and consequently

$$v = \sqrt{g h \frac{u_1 + u_2}{2 u_0}} \quad (7)$$

We may for simplicity put

$$\frac{u_1 + u_2}{2 u_0} = \mu \quad (8)$$

and write

$$v = V \sqrt{\mu g h} \quad (9)$$

There is another reduction to be applied to this formula when the depth h is variable in the same cross-section. Suppose the depth at a distance x , from one end of the cross-section, be represented by

$$y = f(x)$$

then, in order to compute the mean velocity of propagation

of the wave through such cross-section, we have to use the formula

$$v = \frac{\sqrt{g}}{b} \int_0^b [f(x)]^{\frac{1}{2}} dx$$

in which b represents the width of the cross-section from one end to the other.

Observing that the mean depth of the cross-section is expressed by

$$H = \frac{1}{b} \int_0^b f(x) dx$$

we can write

$$v = \frac{\int_0^b [f(x)]^{\frac{1}{2}} dx}{\sqrt{b} \left[\int_0^b f(x) dx \right]^{\frac{1}{2}}} \sqrt{gH}$$

If we put

$$y = f(x) = p x^n$$

we find

$$\frac{\int_0^b [f(x)]^{\frac{1}{2}} dx}{\sqrt{b} \left[\int_0^b f(x) dx \right]^{\frac{1}{2}}} = \frac{2 \sqrt{n+1}}{n+2}$$

and consequently, equation (9) becomes

$$v = \frac{2}{n+2} \sqrt{\mu(n+1)gH} \quad (10)$$

Denoting by (H_1) and (H_2) , the mean depths of high water and low water in a given length l of a tidal river, then the mean velocities of the crest and the trough of the tidal wave in such distance will be expressed, respectively, by

$$v_1 = \frac{2}{n+2} \sqrt{\mu g (n+1) (H_1)} \quad (11)$$

and

$$v_2 = \frac{2}{n+2} \sqrt{\mu g (n+1) (H_2)} \quad (12)$$

The value of μ in these expressions is not necessarily the same. Suppose, for instance, that the river basin enlarges considerably at a certain point, so as to become a kind of reservoir to the river below. If the enlargement were very great, we know that the level of water there would not be affected by tide and would stand at the same elevation as the mean level of the ocean. The effect, therefore, of any enlargement of basin in a tidal river is to more or less reduce the range of tide, and to more or less elevate the plane of low water where the enlargement occurs over the plane of low water of the river below. Hence, while the trough of the tidal wave is advancing in this portion of the river, a more or less strong current, due to the higher elevation of the plane of low water in the enlarged basin, will meet the advancing wave, and finds its escape below the surface, or rather near the bottom, where it adds strength to the bottom velocity u_2 of the ebb. Consequently the value of μ in such case, in equation (12), may be considerably larger than the value of μ in equation (11), since there is no reason for an enlargement of basin to affect the bottom velocity u_2 during high water in any cross-section of the river.

The bottom velocity u_2 during low water is certainly increased when the river discharges a considerable volume of fresh water through the cross-section, for this discharge also finds an easy escape near the bottom, on account of the advancing wave. In both cases, the tendency is, therefore, to increase the value of μ in equation (12); and if we assume that in such cases the bottom velocity does not differ much from the surface velocity, we may consider $\mu = 1$, in such equation.

In ordinary cases, the bottom velocity may be considered small compared with the surface velocity, and then we find according to equation (8), $\mu = \frac{3}{4}$.

As to the value of n , it is safe to assume that the cross-section approaches the parabolic form, when $n = \frac{1}{2}$. Then, taking $g = 32.17$, we find in ordinary cases

$$v_1 = 4.81 \sqrt{(H_1)} \quad (13)$$

$$v_2 = 4.81 \sqrt{(H_2)} \quad (14)$$

and for the cases, where the volume of fresh water discharged by the river is considerable with respect to the tidal volume, or where a considerable enlargement of basin occurs, as mentioned above, we may put (assuming $\mu = 1$),

$$\frac{1}{v_2} = 5.56 \sqrt{H_2} \quad (15)$$

Denoting by θ_1 and θ_2 the delays in minutes of high water and low water in the distance l computed in statute miles, we find, in ordinary cases, the following formula :

$$\theta_1 = \frac{18.3 \, l}{\sqrt{H_1}} \quad (16)$$

$$\theta_2 = \frac{18.3 \, l}{\sqrt{H_2}} \quad (17)$$

and, when the velocity of propagation of the trough is computed according to formula (15), we find

$$\theta_2 = \frac{15.8 \, l}{\sqrt{H_2}} \quad (18)$$

Whenever we find that the delay of low water in a given portion of a tidal river corresponds with this formula, we have an evidence that, during low water, the bottom velocity of the ebb is greater than it otherwise would be under ordinary conditions; and this increment of bottom velocity must be due either to a considerable volume of fresh water passing through the various cross-sections of the considered portion of river, or to a considerable enlargement of tidal basin above such portion of river. (These facts may be turned to some advantage in the improvement of tidal rivers.)

In order to test the accuracy of the formulæ (16) and (17), we have selected the portion of the Delaware River between Edgemoor and Fort Mifflin, in which the delays of high water and low water from nearly 400 observations are found to be 73 minutes and 81 minutes, respectively. The distance between the stations is $l = 18.7$ statute miles, and from numerous soundings and careful reductions made by the U. S. Coast and Geodetic Survey (*Rep.* 1887), is found

$(H_1) = 20.5$ feet, $(H_2) = 18.3$ feet. With these data, our formulæ furnish

$$\theta_1 = 75.5 \text{ minutes,}$$

$$\theta_2 = 80.0 \text{ minutes,}$$

results which are certainly very satisfactory.

If we take the portion of the upper Delaware above Philadelphia, from Bridesburg up to Burlington, we have a case where the volume of fresh water discharged by the river is considerable when compared to the tidal volume, and consequently for the delay of low water we may use the formula (18). Now the distance between the stations is $l = 12.5$ statute miles; and according to a recent determination made by the United States Engineers, the delays of high water and low water are 59 minutes and 55 minutes, respectively, and $(H_1) = 14.2$ feet, $(H_2) = 13.7$ feet. With these data, we find

$$\theta_1 = 60.7 \text{ minutes,}$$

$$\theta_2^1 = 53.5 \text{ minutes,}$$

also very satisfactory results.

Again, the portion of the Hudson River, from Governor's Island to Dobbs' Ferry is found below an enlargement of basin which is called Tappan Sea, and, therefore, the delay of low water must also be computed according to formula (18).

The distance between the stations is $l = 24$ statute miles and the Tide-Tables give for the delays of high water and low water 72 minutes and 60 minutes, respectively. According to a publication by the U. S. Coast and Geodetic Survey, we find $(H_1) = 38.7$ feet, $(H_2) = 34.1$ feet. With these data we get

$$\theta_1 = 70.6 \text{ minutes,}$$

$$\theta_2^1 = 65.0 \text{ minutes,}$$

which results, though not so satisfactory as before, are not at all improbable. In fact, so far as the delay of high

water is concerned the agreement between the observed and computed values is certainly very good; and the difference of five minutes which appears between the computed and the observed delay of low water only shows that the value of μ , in the present case, is somewhat larger than *one*.

PROCEEDINGS
OF THE
CHEMICAL SECTION,
OF THE
FRANKLIN INSTITUTE

[*Stated Meeting, held at the Institute, Tuesday, November 18, 1890.*]

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, November 18, 1890.

Mr. T. C. Palmer, President, in the Chair.

Members present: Dr. L. B. Hall, Dr. H. W. Jayne, Mr. H. Pemberton, Jr., Mr. C. J. Semper, Dr. S. C. Hooker, Dr. Wm. H. Greene, Prof. E. F. Smith, Mr. Reuben Haines, Mr. W. W. Macfarlane, Mr. A. A. Moore, Dr. Wm. H. Wahl, Mr. Philip S. Clarkson, and a number of visitors.

The election of Mr. Cabell Whitehead, of the Mint Bureau at Washington, as an associate member of the Section, was announced.

Mr. Pemberton nominated Mr. Richard D. Baker, 216 South Third Street, Philadelphia, as a member of the Section. The nomination was referred to the Committee on Admissions.

A letter from Mr. Heyl, the Actuary of the Institute, was read, in which certain recommendations were made relative to the course to be pursued in paying bills for journals, etc., the contracting of which had already been formally sanctioned by the Section. The views expressed met with the approval of the Section and the Secretary was directed to so inform the Actuary.

On motion of Mr. Pemberton, it was voted that the Secretary of the Institute be authorized to renew the subscriptions to the journals taken by the Section.

Prof. Smith moved that the Section subscribe for the *Bulletin of the French Chemical Society*; the motion was carried.