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XXXIV. *On Kerr's Magneto-optic Phenomenon in the case of Equatorial Magnetization of Iron.* By R. SISSINGH*.

1. **K**ERR† found in the years 1877 and 1878 that, on the reflexion from magnetized iron of light polarized in a plane parallel or perpendicular to the plane of incidence, there arises in the reflected ray, in addition to the component furnished by the metallic reflexion, another component polarized in a plane at right angles to this one. An exact explanation of this phenomenon requires the determination of the amplitude and phase of the magneto-optic component and its relation to the angle of incidence. This determination, in the case of iron magnetized parallel to the plane of incidence and to the reflecting surface, is the object of the present investigation, in which the results obtained are compared with the theory of H. A. Lorentz‡. This theory permits of a more simple explanation of the phenomenon§.

2. *Method of Observation.*—If the polarizer is perpendicular or parallel to the plane of incidence, and the analyser in a plane perpendicular to the polarizer, the reflected light after

* Translated from Wied. *Ann.* vol. xlii. p. 115 (1891), by James L. Howard.

† J. Kerr, *Phil. Mag.* (5) vol. iii. p. 321, 1877; (5) vol. v. p. 161 (1878).

‡ H. A. Lorentz, *Verslagen en Meded. der Acad. Amsterdam*, Reeks 2, Deel xix. (1883); *Beiblätter*, viii. p. 869 (1884).

§ For this explanation the reader may be referred to my original paper in the *Verhandelingen der Akad. von Wetenschappen* (Amsterdam), Deel xxviii.

magnetic reflexion will not be extinguished by the analyser, because the magnetic component of the light does not pass through the analyser without being weakened. By rotating one or both Nicol prisms, however, the intensity of the light can be reduced to a minimum or zero. These rotations will be called minimum- or zero-rotations, and will be designated by $\phi_{ip}^{m(o)}$, $\phi_{ia}^{m(o)}$, $\phi_{lp}^{m(o)}$, $\phi_{la}^{m(o)}$. The indices m and o denote the kind of rotation (m =minimum, o =zero), p and a indicate which Nicol prism is rotated, i and l refer to the plane of polarization of the incident light, i meaning that the light is polarized in a plane nearly parallel, and l nearly perpendicular, to the plane of incidence. Prof. van der Waals has shown how, by the help of these rotations, we can determine the amplitude μ and the phase m of the magnetic component of the light*.

For the exact determination of the quantities observed the positive direction of rotation must first be defined, and a mode of expressing differences of phase must be agreed upon. Let the observer imagine himself looking along the direction of the incident or reflected light, with his face turned towards the mirror; then the rotations of the Nicol prisms are positive if in the same direction as the motion of the hands of a watch with its face to the observer. The difference of phase of two waves of light whose oscillations are not parallel to each other is zero, if the greatest distances (of the æther molecules) from the mean position are reached at the same time for each wave. These distances, which are in a certain sense arbitrary, I have

Fig. 1.

chosen to be as given by Oy , Oa , Ob in fig. 1; Oy for incident and reflected light polarized parallel to the plane of incidence, Oa for incident and Ob for reflected light, both polarized perpendicularly to the plane of incidence. In this figure AO is the incident ray, OB the reflected ray, and SS' the mirror. Oa and Ob lie in the plane of incidence; Oy is perpendicular to the plane of incidence. Oa is at right angles to OA , and Ob to OB . The differences of phase are

* Cf. P. C. Ksz, "*Over de terugkaatsing van het Licht door magneten*", Dissertation, Amsterdam, 1884; *Beiblätter*, ix. p. 275 (1885).

always retardations, reckoned with reference to the reflected wave polarized parallel to the plane of incidence.

In accordance with the convention adopted by all previous observers, I have called the magnetization positive if the north poles of the magnetic molecules are directed towards that side of the normal to the mirror on which the polarizer lies.

3. We can now deduce the formulæ which serve to determine the amplitude μ and the phase m of the magnetic component of the light, in the case of a bundle of incident rays polarized parallel to the plane of incidence. In these formulæ f and h are the amplitudes of the reflected light polarized parallel and perpendicularly to the plane of incidence respectively, when the incident ray has amplitude equal to unity and is polarized in the same plane. Φ is the relative difference of phase of these two waves in the case of metallic reflexion.

Let the polarizer be set parallel to the plane of incidence ; and let the amplitude of the light incident on the mirror be unity. Now turn the polarizer through a small angle, ϕ_{ip} . The rays falling on the mirror are then :—

A ray polarized parallel to the plane of incidence, amplitude 1 ;

A ray polarized perpendicularly to the plane of incidence, amplitude ϕ_{ip} .

By reflexion, we obtain :—

A ray polarized parallel to the plane of incidence, amplitude f , retardation of phase zero ;

A ray polarized perpendicularly to the plane of incidence, amplitude $h\phi_{ip}$, retardation of phase Φ ; and

A ray polarized perpendicularly to the plane of incidence, amplitude μ_i , retardation of phase m_i .*.

If now the analyser be turned through a small angle ϕ_{ia} out of the position in which the reflected light was quenched before the polarizer was turned and the mirror magnetized, the analyser transmits a bundle of rays with

amplitude	$-f\phi_{ia}$,	retardation of phase	0,
„	$h\phi_{ip}$,	„	„ Φ ,
„	μ_i ,	„	„ m_i .

* The indices attached to μ and m indicate that the plane of polarization of the incident ray is nearly parallel (i) or nearly perpendicular (l) to the plane of incidence.

The resulting intensity is

$$\{-f\phi_{ia} + h\phi_{ip} \cos \Phi + \mu_i \cos m_i\}^2 + \{h\phi_{ip} \sin \Phi + \mu_i \sin m_i\}^2.$$

This will be a minimum when ϕ_{ia} , ϕ_{ip} have the values ϕ_{ia}^m , ϕ_{ip}^m , given by the following equations:—

$$\left. \begin{aligned} -f\phi_{ia}^m + h\phi_{ip} \cos \Phi + \mu_i \cos m_i &= 0, \\ h\phi_{ip}^m - f\phi_{ia} \cos \Phi + \mu_i \cos (\Phi - m_i) &= 0, \end{aligned} \right\}. \quad (1)$$

These equations can also be determined by a geometrical construction for the resultant amplitude of the waves which traverse the analyser.

If the incident ray is polarized perpendicularly to the plane of incidence, we obtain in the same manner:—

$$\left. \begin{aligned} -f\phi_{ip}^m + h\phi_{ia} \cos \Phi + \mu_l \cos m_l &= 0, \\ h\phi_{ia}^m - f\phi_{ip} \cos \Phi + \mu_l \cos (\Phi - m_l) &= 0, \end{aligned} \right\}. \quad (2)$$

In the above deduction quantities of the order $(\phi_{ip})^2$, $\mu\phi_{ip}$, &c. are neglected, because the rotations ϕ are only a few minutes of angle, and μ is less than 0.001*.

4. If, on reversal of the magnetization, the amplitude μ of the magnetic component of the light also changes its sign without any alteration of the phase m , it follows from the formulæ (1) and (2) that the minimum positions of the Nicol prisms are only symmetrical with respect to the plane of incidence and a plane perpendicular to it, in case the incident or reflected ray is polarized in one of these planes. ϕ_{ip} , ϕ_{ia} , ϕ_{lp} , ϕ_{la} must be equal to zero. If we call the angles between the minimum positions of the Nicol prisms for positive and negative magnetization ψ_{ip}^m &c., it follows from (1) and (2), with the above assumption:—

$$\left. \begin{aligned} \tan m_i &= -\cot \Phi - \frac{1}{\sin \Phi} \frac{h\psi_{ip}^m}{f\psi_{ia}^m}; & \mu_i &= + \frac{f\psi_{ia}^m}{2 \cos m_i}; \\ \tan m_l &= -\cot \Phi - \frac{1}{\sin \Phi} \frac{h\psi_{ia}^m}{f\psi_{ip}^m}; & \mu_l &= + \frac{f\psi_{ip}^m}{2 \cos m_l}; \end{aligned} \right\}. \quad (3)$$

* ϕ_{ip} , ϕ_{ia} , ϕ_{lp} , ϕ_{la} indicate the small angles between the plane of polarization of the Nicol prisms and the plane of incidence or a plane at right angles to it. The indices i and l mean that the incident light is polarized nearly parallel or normal to the plane of incidence. The analyser is set in a plane nearly perpendicular to the plane of incidence.

The intensity of the light emerging from the analyser after the minimum rotation ϕ_{ia}^m is, referring back to the first formula of all, § 3,

$$(h\phi_{ip} \sin \Phi + \mu_i \sin m_i)^2.$$

If we turn the analyser out of the minimum position through an angle ϵ , the intensity alters by $f^2\epsilon^2$. As a slight alteration of intensity can be the more easily observed the smaller the original intensity, it follows that the accuracy with which the minimum rotations can be observed is greatest at small and large angles of incidence, and is smallest in the neighbourhood of the polarizing angle, where the total intensity is greatest. This accuracy is not the same for positive and negative magnetization, excepting when ϕ_{ip} , ϕ_{ia} , ϕ_{ip} , ϕ_{ia} are equal to zero.

5. The positions of the Nicol prisms in which the light reflected from the magnetized mirror is extinguished by the analyser, that is the "zero rotations," are deduced from the formulæ for the intensity obtained in § 3.

These give :—

$$\left. \begin{aligned} \cot m_i &= -\frac{f\phi_{ia}^o}{h\phi_{ip}^o} \frac{1}{\sin \Phi} + \cot \Phi, & \mu_i &= -\frac{h\phi_{ip}^o \sin \Phi}{\sin m_i} \\ \cot m_l &= -\frac{f\phi_{ip}^o}{h\phi_{ia}^o} \frac{1}{\sin \Phi} + \cot \Phi, & \mu_l &= -\frac{h\phi_{ia}^o \sin \Phi}{\sin m_l} \end{aligned} \right\} \quad (4)$$

If the amplitude μ , and not the phase m , of the magnetic component of the light alters its sign with the direction of magnetization, we can replace ϕ_{ia}^o &c. in equations (4) above by the angles ψ_{ia}^o . . . , between the zero-positions of the Nicol prisms for positive and negative magnetization, these latter angles being twice as great as the former. We thus obtain a factor 2 in the numerator of the equations for μ_i and μ_l . Now it can easily be proved that the zero-rotations can be most accurately observed when $\sin \Phi$ (Φ =difference of phase on reflexion from the metal) is greatest; that is, in the vicinity of the polarizing angle. At the polarizing angle the zero and minimum rotations are equal to each other*.

6. *Description of the Apparatus.*—The mirrors were ground on iron rings which were encircled by seven layers of windings of copper wire 1.5 to 2 millim. thick. Fig. 2 gives a

* For complete proofs of these statements the original memoir must be consulted.

section through this ring by a plane normal to the reflecting surface.

The coils do not project over the surface of the mirror. The magnetizing current was furnished by 15 Bunsen-cells, in 5 sets each containing 3 cells in parallel, the sets being joined up in series; the magnetization was measured by Rowland's method*.

In the centre of the mirror, for a magnetizing current of 15 amperes its value was 1400 C.G.S. units per unit volume. The mirror was nearly saturated; the magnetism increased 8 per cent. for a rise of current from 7.5 to 22 amperes, and only 2 per cent. between 15 and 22 amperes. These rings had the advantage that small masses of iron could be strongly magnetized, and a spectrometer used for the investigation. Two mirrors were experimented upon. The diameter of the rings was 10 centim., the thickness 0.6 centim., the length of the mirror 3 centim., and its breadth 2.8 centim.

The optical constants were determined by a Babinet's compensator†.

Mirror.	Polarizing Angle (J).	Principal Azimuth (II).
I. . . .	77° 23.5' ‡	26° 34'
II.. . .	76° 30.5' ‡	26° 44'

7. In the observations a parallel plane-polarized bundle of rays is reflected from the mirror, and the intensity of the reflected light is brought to zero or a minimum by rotating the analyser. In order to make measurements with very small or with normal incidence, the collimator, which gives the parallel bundle of rays, and the polarizer are separated from the spectrometer. These three parts—collimator, polarizer, and spectrometer—are so arranged that they can be moved round the edge of a circular platform.

* Rowland, Phil. Mag. (4) xlv. p. 140, 1873.

† For the adjustment of the compensator and the method of using it, cf. Sissingh, Dissertation, Leyden, 1885, and *Arch. Néer.* xx. p. 171.

‡ There is a considerable difference between the polarizing-angles of the two mirrors ground and polished in the same manner. If the two mirrors were not ground and polished in exactly the same way, I often observed differences in J and II. I hope to investigate this more fully in the future.

The polarizer can be moved along a groove at right angles to this, and is inserted on a pillar provided with sights. This apparatus permits the polarizer to be rotated about a vertical or horizontal axis, and to move in a vertical direction. In this way we are able to bring the axis of the polarizer into the same line as the collimator axis, and to put the spectrometer with its axis perpendicular to this line. The rings containing the mirrors are, on account of their weight, not placed on the spectrometer table, but rest on a bridge above it. The rings can thus be rotated about a vertical axis and displaced in a horizontal direction, by means of two movable sledges at right angles to each other. This allows the centre of the mirror to be placed in the axis of the incident ray, and also permits of an exact adjustment of the mirror for all angles of incidence. For angles of incidence smaller than 24° , the analyser is also separated from the spectrometer and placed on a cross-piece which can be moved relatively to the circular platform, just as in the case of the polarizer. As the rotations to be measured only amount to a few minutes of angle, it is absolutely necessary that the different parts should be accurately adjusted and securely fastened in their proper places.

8. In order to measure accurately the small rotations, the Nicol prisms are placed in a parallel beam of light*. The collimator slit is focused by a telescope situated behind the analyser. At each point of the image of this slit we receive in this manner only those rays which pass in the same direction through the Nicol prisms, and which have therefore in these prisms the same plane of polarization. When the prisms are crossed, a dark band is observed which is displaced parallel to itself along the image of the slit on moving the polarizer or analyser. If the axes of the collimator and Nicol prisms are parallel to each other, the band occupies the centre of the image of the slit when the Nicols are crossed. All the rotations of the plane of polarization are referred to this position of the dark band. In this way we have only to adjust the position of a dark band, instead of, as in the older method, estimating when the intensity of the light is a minimum. For convenience of adjustment the centre of the slit has a fine thread stretched across it. The band is sharply defined if the intensity of the light is not too small. Further, the axes of the Nicol prisms must coincide as nearly as possible. The polarizer is a Lippich prism with plane faces which are in-

* Sissingh, Dissertation, Leyden, 1885; also *Arch. Néer.* xx. p. 178. Lippich, *Wien. Ber.* lxxxv. p. 268 (1882).

clined to each other at a small angle, viz. about 1 minute *. The tube in which the prism is fixed is attached by means of three pressure- and three tension-screws to the rim of a second tube, which is placed at the centre of the graduated circle.

Following Gauss' method, and observing the images of the telescope spider-lines reflected from the faces of the prism, the axis of the latter can be set parallel to the collimator axis. The error of this setting was less than 1' of angle. The position of the polarizer is such that its axis coincides with that of the incident ray of light. The axis of the incident ray is found by means of a cap with a fine hole through its centre which fits over the collimator lens.

The analyser is a Nicol prism with its faces inclined to each other. The dark band was most clearly seen with such a prism. The condition for the proper position of the analyser was that the ray of light reflected from the face of the Nicol should describe a circle round the axis of the incident ray when the prism was rotated. The axis of rotation of the analyser was also made to coincide with that of the ray falling upon it.

This care in the adjustment of the Nicol prisms is absolutely necessary, in order to observe the dark band at every angle of incidence, and especially so if the angle of incidence is small. The new method of observation puts us in a position to determine small rotations with a degree of accuracy hitherto unattainable †.

9. As the zero- and minimum-rotations only amount to a few minutes of angle in the case of magnetic equatorial reflexion, and the small circles of the Nicol prisms are divided in degrees and have no vernier, these rotations were measured by a mirror and scale method. At a distance of 2.5 metres from the polarizer were placed a telescope and scale, the former receiving the image of the scale from mirrors attached to the polarizer. These mirrors were placed on the faces of a four-sided hollow prism surrounding the polarizer-

* Lippich, *Wien. Ber.* xci. p. 1079, 1885.

† This method, which was used by me previously in the direct determination of the principal positions of Nicol prisms in which the planes of polarization are parallel and perpendicular to the plane of incidence, is now applied to the measurement of small rotations of the plane of polarization by magnetic reflexion. In this manner the principal positions are determined with a far greater accuracy than in the indirect method hitherto almost universally adopted. Cf. Sissingh, *Dissertation*, p. 28; *Arch. Néer.* xx. p. 8; Righi, *Ann. de Chim. et de Phys.* (6) iv. pp. 446, 456 (1885); ix. pp. 127-131 (1886); x. p. 208 (1887). Cf also § 13.

tube. The normals of the mirrors were nearly in the same plane, and made angles of nearly 90° with each other*. The polarizer was rotated by means of a long arm. In the analyser a similar arm is fixed to a ring surrounding the analyser-circle and capable of being clamped by a screw. This ring carries the mirror used in reading the rotations. Since only one scale and two telescopes, placed near together, were used for these measurements, there was added, in case the angle of incidence was not too great or too small, an auxiliary mirror, in order to permit of the analyser-rotations being measured in this manner. The scales were divided in millimetres, and 1 minute of angle corresponded to 1.4 to 1.8 millim.†

Fig. 3.

The constancy of the results obtained by this method of reading was also very satisfactory : it has been used since 1886.

10. In these experiments sunlight or electric light was used. The latter was furnished by a Siemens lamp which

* This arrangement was chosen because the polarizer is only used in the positions in which the plane of polarization is nearly parallel or perpendicular to the plane of incidence.

† In order to reduce the scale-readings to minutes, the readings are taken with the Nicols adjusted to two consecutive degrees on the divided circle.

could be displaced by the observer in a vertical direction in order to bring the crater of the positive pole always opposite the slit of the collimator.

The mirror-magnet was demagnetized by alternate currents whose absolute mean value was approximately zero. By inserting a resistance constructed out of electric-light carbons, the current could be reduced to 2.3 amperes. The ring was then inserted in a shunt-circuit, by which means the strength of the current through it could be reduced to 0.02 ampere. This arrangement was carried out by means of a switchboard, and is shown in fig. 3 with commutator, amperemeter, and two plug-switches. The key of the commutator turns round the point 9 and connects together either 1 and 2 and 6 and 7, or else 1 and 3 and 4 and 5. The magnetizing current passes from A to B, *e* (or *f*), C, plug *d*₂, D, E, 1, 2, ring, 3, 6, 7, 8, F, plug *b*₂, G. If the ring is in the shunt-circuit the main current passes along A, B, *f*, C, plug *c*₂, the shunt H, K, L, G, M, N, 1, 2, ring, 3, 6, 7, 8, F, P, plug *c*₁, T, R.

11. *Test of the Method of Observation.*—If the amplitude μ , and not the phase m , of the magnetic component of the light alters its sign with the direction of magnetization, we need not determine the angle ϕ , but only ψ (§§ 4, 5). The determination of the angle ϕ requires a knowledge of the principal positions of the Nicol prisms, in which the planes of polarization are parallel or perpendicular to the plane of incidence. This would always necessitate a demagnetization of the mirror on account of permanent magnetism. First of all we shall show that on demagnetizing, as in § 10, the principal positions of the Nicol prisms are the same, whatever sign the smallest magnetizing current of 0.02 ampere may have. The permanent magnetism therefore exerts no influence on the reflexion.

Demagnetization by Currents in alternate directions of +15 amp. to +0.02 amp.	
Polarizer.	Analyser.
144.7	304.0
143.8	304.7
144.0	303.9
145.1	304.0
145.0	305.3
145.3	305.1
145.0	304.5
145.1	303.8
144.75	304.4

The Mirror demagnetized afresh. Alternate Currents of +15 amp. to -0.02 amp.	
Polarizer.	Analyser.
144.9	304.9
144.5	303.3
144.9	305.8
145.2	304.8
144.5	303.9
144.7	303.8
145.4	304.4
144.7	303.8
144.8	304.3

The value of a minute of angle is 1.56 scale-division for the polarizer and 1.66 for the analyser*.

After the first demagnetization the Nicol prisms were left in their last position. On repeating the process of demagnetization, for the second experiment, it was seen that the dark band had not changed its position. This, together with the agreement of the mean values of the scale-readings, proves that the demagnetization was fully accomplished by the method adopted. The results of a few other sets of experiments were as follows:—

Demagnetizing Current to	Principal Positions.		Minimum Positions.	
	Polarizer.	Analyser.	Polarizer.	Analyser.
+0.02 amp.	144.75	304.4		
−0.02 „	144.8	304.3		
+2.3 „			142.5	308.0
−2.3 „			147.3	300.5
+0.02 „	145.0	304.5		

These results verify the above conclusion. By a demagnetization stopping at 2.3 amperes the minimum rotations of the polarizer on account of permanent magnetism were -2.3 and $+2.45$, of the analyser $+3.6$, -3.9 ; or, expressed in minutes, -1.5 and 1.6 , $+2.2$ and -2.3 .

These observations were made with an angle of incidence of $51^{\circ} 22'$, because at this angle the minimum rotations can be observed with greater accuracy (*cf.* § 4), while the influence of permanent magnetism is not too small, its effect being a maximum for an angle of incidence of between 60° and 70° (see § 16).

12. A second proof of the full demagnetization of the mirror is furnished by the agreement between the principal positions of the Nicol prisms obtained with the demagnetized mirror and with a silver mirror respectively. The two observations must be made immediately following each other. The table of the spectrometer carrying the silver mirror comes in the place of the bridge above the spectrometer and carrying the iron mirror. However great be the care taken in this operation to keep the mirror in the proper position, the adjustment and centering of the mirror have always to be tested, and if necessary a new adjustment must be made. In place

* In the remainder of the paper these will be denoted by $1_p' = 1.56$, $1_a' = 1.66$.

of the analyser we have then the telescope of the spectrometer. Sometimes the polarizer was also removed, because the deviation of the light, although very small (1'), hinders the proper centering and adjustment of the mirror. It is not easy in these operations to avoid every alteration of the mirrors used to read off the rotations.

The observations gave, for the principal positions of the Nicol prisms:—

With Demagnetized Iron Mirror.		With the Silver Mirror.	
Polarizer.	Analyser.	Polarizer.	Analyser.
388·8	279·9	388·1	279·2
387·0	280·2	388·4	282·2
389·4	281·4		
<hr/>	<hr/>	<hr/>	<hr/>
388·4	280·5	388·25	280·7 *

A second series of observations, in which the scale-readings were not quite the same, gave†:—

Iron Mirror demagnetized.		Silver Mirror.	
Polarizer.	Analyser.	Polarizer.	Analyser.
387·3	283·7	387·2	284·6
388·0	283·3	387·4	283·6
<hr/>	<hr/>	387·2	283·3
387·6	283·5	<hr/>	<hr/>
		387·3	283·8

Since in these observations $1_p' = 1·33$, $1_a' = 1·23$, we may, taking into consideration the difficulties in the experiment, conclude that the principal positions of the Nicol prisms are the same for the silver mirror and the demagnetized iron mirror.

13. Now that we have established that the demagnetization is complete, we may proceed to show that the amplitude μ of the magnetic component changes its sign with the direction of magnetization, but that the phase m is not changed by this means. From formulæ (1) and (2), § 3, and (4), § 5, it is shown that this is the case if the minimum- and zero-rotations for positive and negative magnetization differ from each other only in their sign and not in absolute value.

For an angle of incidence of $51^\circ 22'$ the minimum rotations were determined both for positive and negative magnetization—that is to say, after each series of observations with any given direction of magnetization a demagnetization of the mirror took place. An exact determination of the minimum rotations, however, required the collimator-slit to be reduced by a cross slit to a very short length. If this was not done

* Each of these numbers is the mean of 8 settings.

† As the absolute value of the scale-divisions does not influence these results, no notice was taken of its alteration.

it, was always observed, on rotating the polarizer or analyser in order to again obtain a minimum intensity of light in the centre of the field of the analyser (that is, the centre of the image of the slit), after magnetizing the mirror, that the greatest minimum was not in the centre of the field but somewhat above or below this position. This minimum rendered an exact adjustment, so that the intensity of light was least in the centre of the field, almost impossible; and therefore by means of the cross slit only the centre of the collimator-slit was left.

The minimum rotations ϕ_i^m and ϕ_l^m were determined for the two positions in which the polarizer was parallel and perpendicular respectively to the plane of incidence; and for each position of the polarizer the analyser was read in two positions 180° apart.

The following series of observations is given in order to show what degree of accuracy is obtained; then follow the means from several series of observations:—

Polarizer perpendicular to the Plane of Incidence.

Mirror demagnetized.		Mirror again demagnetized.	
Polarizer perpendicular to Plane of Incidence.	Analyser parallel to Plane of Incidence.	Polarizer perpendicular to Plane of Incidence.	Analyser parallel to Plane of Incidence.
385.5	281.7	385.6	283.0
385.5	282.1	386.2	283.1
386.0	282.4	386.8	283.5
386.0	283.2	386.4	283.7
385.8	283.3	386.6	284.2
385.5	282.8	385.7	282.7
385.7	282.7	386.7	283.7
385.5	282.6	385.7	283.0
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385.7	282.6	386.2	283.3

Mean of these Observations.

Polarizer perpendicular to Plane of Incidence, 385.9.

Analyser parallel to Plane of Incidence, 283.0.

Polarizer perpendicular to Plane of Incidence.		Analyser parallel to Plane of Incidence.	
Minimum positions of the Analyser for Magnetization		Minimum positions of the Polarizer for Magnetization	
+	—	+	—
278.0	288.2	382.4	389.6
277.6	288.1	381.8	389.4
278.8	288.8	381.9	390.4
278.1	289.2	382.0	390.1
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278.1	288.6	382.0	389.9

It follows from these that

$$\begin{aligned}\phi_{la}^m \text{ for } + \text{ magnetization} &= -4\cdot9, \\ \phi_{la}^m \text{ " } - &= +5\cdot6, \\ \phi_{lp}^m \text{ " } + &= -3\cdot9, \\ \phi_{lp}^m \text{ " } - &= +3\cdot9.\end{aligned}$$

The mirror was then demagnetized again, and the observations gave:—For polarizer perpendicular to plane of incidence, 385·8; analyser parallel to plane of incidence, 283·5. These numbers agree exactly with those obtained above.

The mean values of the rotations for positive and negative magnetization are :—

ψ_{lp}^m		ψ_{la}^m		ϕ_{lp}^m		ϕ_{la}^m	
Magn. +	Magn. —	Magn. +	Magn. —	Magn. +	Magn. —	Magn. +	Magn. —
+5·2	—4·2	+3·6	—4·0	—3·9	+3·95	—4·9	+5·6
5·3	5·3	3·7	3·5	4·5	3·75	4·6	4·6
5·5	4·9	3·9	3·7	4·6	3·4	5·3	5·1
4·7	5·6	4·5	4·5	3·4	4·4	5·4	5·0
—	—	—	—	—	—	—	—
+5·2	—5·0	+3·9	—3·9	—4·1	+3·9	—5·0	+5·1

As the scale-divisions were $1_p' = +1\cdot30$, $1_a' = -1\cdot27$, we obtain, on reducing these minimum rotations to minutes of angle:—

ϕ_{lp}^m		ϕ_{la}^m		ϕ_{lp}^m		ϕ_{la}^m	
Magn. +	Magn. —	Magn. +	Magn. —	Magn. +	Magn. —	Magn. +	Magn. —
+4·0	—3·8	—3·1	+3·1	—3·2	—3·0	+4·0	—4·0

On looking over the whole series of observations it was seen that the deviations from the mean value amounted in each set to about $\frac{1}{3}'$; so that the final mean values will have a slightly greater accuracy than this. Within the limits of error the

values of the minimum rotation agree for positive and negative magnetization. The mean error in the principal positions of the Nicol prisms in which the planes of polarization are parallel and perpendicular respectively to the plane of incidence has about the same value. As already previously indicated in § 8, this direct method of determining the principal positions gives far more accurate results than the indirect methods usually employed hitherto.

14. Since we may obtain the amplitude μ and the phase m of the magneto-optic component both from the minimum-rotations and the zero-rotations, we must now show that the results agree. For this purpose the zero-rotations were also determined for an angle of incidence of $51^\circ 22'$, because with large and small angles of incidence the accuracy of the observations is considerably decreased, and the minimum-rotations, which are then capable of being determined with the greatest approach to accuracy, are still sufficiently well defined. As we have already shown that only the amplitude μ and not the phase m of the magnetic component of the light changes its sign with the direction of magnetization, we only require to measure the angle ψ between the zero-positions of the Nicol prisms for positive and negative magnetization (§ 5). In the following tables the values of μ and m are given for positive magnetization. The observed rotations were:—

$2\phi_{ip}^0$	$2\phi_{ia}^0$	$2\phi_{ip}^0$	$2\phi_{ia}^0$
$-0'3$	$-6'4$	$-6'6$	$-6'75$
$-0'6$	$-7'3$	$-6'9$	$-0'4$
$-0'45$	$-6'85$	$-6'8$	$-0'35$

A discussion of the different series of observations shows that the mean error in these numbers is smaller than $0'5$. From the minimum- and zero-rotations the following values of μ and m were obtained †:—

$2\phi_{ip}^0$	$2\phi_{ia}^0$	$2\phi_{ip}^0$	$2\phi_{ia}^0$	μ_i	μ_r	m_i	m_r
$-0'4$	$-6'8$	$-6'8$	$-0'3$	$+0.79 \times 10^{-3}$	$+0.81 \times 10^{-3}$	$180^\circ - 1^\circ 9'$	$180^\circ - 0^\circ 52'$
$-1'2*$	$-6'6*$	$-6'8*$	$-0'5*$	$+0.69*$	$+0.78*$	$180^\circ - 3^\circ 40'*$	$180^\circ - 1^\circ 17'5*$
$2\phi_{ip}^m$	$2\phi_{ia}^m$	$2\phi_{ip}^m$	$2\phi_{ia}^m$				
$7'9$	$-6'2$	$-6'8$	$8'0$	0.754	0.756	$180^\circ + 5^\circ 35'$	$180^\circ + 7^\circ 21'$
$7'8*$	$-6'2*$	$-6'4*$	$7'8$	$0.750*$	0.765	$180^\circ + 3^\circ 47'*$	$180^\circ + 0^\circ 37'$

† The formulæ (3) and (4), § 4 and § 5, leave the sign of the amplitude μ undetermined, and the quadrant in which the phase m lies depends upon this. In the table μ_i and μ_r are always taken as positive.

In order to determine the influence of a small alteration in the observed quantities upon the calculated numbers, the values of μ and m were reckoned for the rotations denoted by an asterisk and chosen at random. From the observations and the calculated values, we may conclude that

$$\phi_{ip} = \phi_{ia}, \quad \phi_{ip} = \phi_{ia},$$

either for zero- or minimum-rotations, and also that

$$\mu_i = \mu_v \quad m_i = m_v.$$

Two of the zero-rotations, ϕ_{ip}^0 and ϕ_{ia}^0 , are nearly equal to zero for this angle of incidence. The light reflected from the magnetized mirror can therefore be extinguished by a rotation of one of the two Nicol prisms. The influence of the magnetization is at this angle of incidence almost equivalent to a rotation of the plane of polarization. The other two zero-rotations, ϕ_{ia}^0 and ϕ_{ip}^0 , must therefore differ but slightly from the corresponding minimum-rotations, which is the case.

It is clear from the tabulated observations that, within the limits of error, the minimum- and zero-rotations give the same results. The degree of accuracy of the amplitudes μ reckoned from ϕ^m , and of the phases m reckoned from ϕ^0 , is fairly high. The error is not greater than 1 per cent.

15. In the observations one has to be very careful that the reflexion takes place exactly at the centre of the mirror, since the magnetic spectrum showed that only in this position are the lines of force parallel to the reflecting surface. If by means of an auxiliary reflector, inclined at an angle of 45° to the normal to the mirror, we allowed the light to fall perpendicularly on the mirror, the magnetization always produced some effect unless the reflexion took place exactly at the centre of the mirror. This can only arise from the component of magnetization normal to the mirror. However small was the inclination to the mirror of the lines of force outside its centre, the sign of the magnetization could always be determined by the displacement of the band in the field of the analyser. In order to eliminate entirely the influence of a normal component of the magnetization, the section of the incident pencil of light was reduced to a few square centimetres.

16. *Synopsis of Results. Comparison with Theory.*—The zero- and minimum-rotations were determined for various angles of incidence with both mirrors. These angles of incidence were so chosen that the difference in phase of the metallic reflexion varied from one to the next by $\frac{1}{16}$ of a wave-length, except for angles less than 36° , when the differences were $\frac{1}{2}$ wave-lengths. The angle of incidence $61^\circ 30'$ was chosen in order to more accurately determine the maximum value of the amplitude μ . The observed and calculated quantities are collected in the following Table :—

Magnetic Reflexion.—Lines of Force parallel to the reflecting surface.

Magnetization per unit volume=1400 C.G.S. units.

Mirror I. $J=77^\circ 23'5''$; $H=26^\circ 34'$.
 „ II. $J=76^\circ 30'5''$; $H=26^\circ 44'$. Yellow light.

Angle of Incidence.	Mirror.	Kind of Adjustment. $\theta = \text{zero,}$ $m = \text{min.}$	Observed Rotations.				$10^3 \times \mu_r$	$10^3 \times \mu_r$	$m_i - 180^\circ$	$m_l - 180^\circ$
			$2\phi_{ip}$	$2\phi_{ia}$	$2\phi_{ip}$	$2\phi_{ia}$				
86° 0'	II.	m	-1.8	-1.7	-1.7	-1.8	+0.58	+0.28	28° 26'	29° 26'
82 30	II.	0	4.0	-4.9	-5.0	-3.7	0.50	0.56	25 39	23 4.5
76 30.5	II.	0	2.7	-5.2	-4.8	2.6	0.74	0.69	14 23	15 14
71 25	II.	0	2.2	-5.9	-5.2	2.2	0.86	0.77	9 29	10 37
61 30	I.	0	-0.4	-5.9	-6.3	-0.2	0.81	0.83	2 22	1 6
51 22	I.	0	-0.9	-6.8	-6.5	-0.3	0.75	0.77	2 24	-0 44
	I.	0	-0.2	-6.8	-7.0	-0.6	0.82	0.80	-0 35	-1 25
	II.	0	-0.4	-6.8	-6.8	-0.3	0.79	0.81	-1 9	-0 52
	II.	m	7.9	-6.2	-6.2	-8.0	0.75	0.75	[5 35	7 21]
36 10	I.	m	8.0	-6.5	-6.5	8.2	0.79	0.80	[1 15	5 5]
	II.	0	-3.7	-8.9	-6.9	-3.0	0.66	0.60	5 31	6 11
	I.	0			-5.6	6.3		0.50		5 33
	I.	m	6.1	-3.7	-3.8	3.9	0.68	0.66	[14 7	2 59]
24 16.5	II.	m	3.9	-4.0			0.41	0.45		
12 0	I.	m	3.9*	-1.9			0.61*	0.26		
6 0	II.	m	2.0		-1.9	2.0	0.26	0.26		
					-2.0*	2.1		0.54*		
			1.0		-0.96	-0.96	0.14	0.11		
			1.0*	-0.997*			0.10*			
			1.0*	-0.95*			1.18*			

The numbers marked with an asterisk are values chosen at random, and serve to show the influence of a small alteration in the observed rotations upon the calculated amplitudes and phases of the magnetic component of the light. A glance at the Table will show that the mean error in μ is 0.03×10^{-3} , and in m about 1° . For angles of incidence smaller than 51° , the relative error in the phases m obtained from the values of ϕ^m is great; while for $i=24^\circ$ or smaller, a small alteration in the differences of ϕ_{ip}^m and ϕ_{ia}^m , or ϕ_{ip}^m and ϕ_{ia}^m alters the amplitude μ very considerably.

The results obtained show that, within the limits of experimental error, $\phi_{ip}^{m \text{ or } o} = \phi_{ia}^{m \text{ or } o}$, and $\phi_{ip}^{m \text{ or } o} = \phi_{ia}^{m \text{ or } o}$. It follows from these relations that $\mu_i = \mu_l$; $m_i = m_l$. All previous observers have found the same result. We must, however, have regard to the assumptions with respect to signs made in order to completely specify μ and m^* .

In the neighbourhood of the angle of incidence $i=60^\circ$, the zero-rotations ϕ_{ip}^o and ϕ_{ia}^o are equal to zero. The magnetization then produces only a rotation of the plane of polarization. For an angle of incidence of about 80° the minimum-rotations are zero, *i. e.* $\phi_{ip}^m = \phi_{ia}^m = 0$. The reversal of sign of these rotations can be seen in the Table, since at the principal angle of incidence $J=76^\circ 30' 5$, the minimum- and zero-rotations are equal to each other. For this angle of incidence, there is a difference of 90° in the retardations of phase of the magneto-optic component and that component of the metallic reflexion which is polarized perpendicularly to the plane of incidence.

If a pencil of light polarized parallel to the plane of incidence falls on a magnetized iron mirror, the reflected light is in general elliptically polarized, but in the neighbourhood of $i=60^\circ$ it is plane-polarized. If the incident ray is polarized perpendicularly to the plane of incidence, we have at $i=80^\circ$ the peculiarity that the axes of the ellipse which represent the paths of the æther molecules, are parallel and perpendicular respectively to the plane of incidence†.

* Righi found, for example, that $\phi_{ip}^m = -\phi_{ia}^m$; $\phi_{ia}^m = -\phi_{ip}^m$. By an alteration of the direction which we have called the positive direction of rotation of one of the Nicol prisms, we can make the signs agree with our results.

† Righi, *Ann. de Chim. et de Phys.* (6) x. pp. 212, 219 (1887) assumes that the influence of magnetization consists in producing a rotation of the plane of polarization together with ellipticity of the reflected light. For an angle of incidence $i=60^\circ$, the ellipticity is zero, and for $i=80^\circ$ the rotation is zero. Righi calls the first angle "incidence singulière," and the second, the principal angle of incidence of magnetized iron. From the observations given above, it is obvious that this can only apply to a particular plane of polarization of the incident light.

17. Prof. H. A. Lorentz has shown how the Hall phenomenon leads to an explanation of magnetic reflexion, if we assume that every movement of electricity in a magnetic field is accompanied by a transverse electromotive force. In this theory it is assumed that the constant of magnetic polarization is the same for air and iron; that is to say, the magnetic iron molecules cannot direct themselves in accordance with the rapidly alternating magnetic forces of visible electric vibrations (light). I have put forward an experimental proof of this, inasmuch as it has been shown that reflexion from iron does not differ in the least from reflexion at the surface of a non-magnetic metal*. The theory further considers the behaviour of ordinary electric currents and of visible electric vibrations to be the same. This, however, has not yet been experimentally proved. But I have not been able to detect any difference in the constants of reflexion of iron at 120° C., and at ordinary temperatures, although the fairly large influence of temperature on electrical resistance would lead us to expect such a result according to this theory†.

It is, however, very important to compare the results of observation with theory. In this way we obtain experimental indications of the behaviour of visible electric vibrations in metals‡.

If we have regard to the difference between the positive direction of vibration, which we have assumed in order to completely specify the phase-differences, and that which is derived from theoretical investigations, we find from the electromagnetic theory of magnetic reflexion § that

$$\mu_i = \mu_t = \frac{\sigma^3 \sin \alpha}{\rho p_1^2 p_4 \cos^3 \alpha} \cdot \frac{2\pi}{T} \epsilon_1 h N.$$

$$m_i = m_t = 3\tau - \omega - \frac{\pi}{2} - 2\delta_1 - \delta_4 - (\delta_2 - \delta_1).$$

α is the angle of incidence, σ , ρ , τ , ω are auxiliary quan-

* Sissingh, Dissertation, p. 129; also *Arch. Néer.* xx. p. 213 (1886).

† Sissingh, *loc. cit.* p. 133; *loc. cit.* p. 215 (1886).

‡ Sissingh, *loc. cit.* p. 139; *loc. cit.* p. 218 (1886). Kundt has questioned the accuracy of the above conclusion. It is, however, borne out by the small influence of temperature which Du Bois found in the case of normal polar magnetic reflexion (*Wied. Ann.* xxxix. p. 35, 1890).

§ H. A. Lorentz, *Versl. en Meded. der Acad. Amsterdam*, Reeks 2, Deel xix. (1883); van Loghem, *Theorie der terugkaatsing van het Licht door magneten*, Leyden, 1883, Dissertation; also *Beibl.* viii. p. 869 (1884).

tities from the theory of metallic reflexion*. The remaining quantities are determined by the equations :—

$$\tan \delta_1 = \frac{m \sin (\tau + \omega)}{1 + m \cos (\tau + \omega)},$$

$$\tan \delta_2 = \frac{m \sin (\tau + \omega)}{-1 + m \cos (\tau + \omega)},$$

$$\tan \delta_4 = \frac{m \sin (\tau + \omega)}{\tan^2 \alpha + m \cos (\tau + \omega)}.$$

$$p_1^2 = 1 + m^2 + 2m \cos (\tau + \omega),$$

$$p_4^2 = \tan^4 \alpha + m^2 + 2 \tan^2 \alpha \cdot m \cos (\tau + \omega),$$

$$m = \frac{\sigma \rho}{\cos \alpha} \dagger.$$

From these formulæ μ and m have been calculated for different angles of incidence. The theoretical value of the amplitude contains the factor $\frac{2\pi}{T} \epsilon_1 h N$, in which N is the magnetic force, T the time of oscillation of the electrical wave-motion (the incident light-wave), ϵ_1 is the constant of dielectric polarization of air, and h is the magnitude of the Hall transverse electromotive force. This latter is equal to hN times the component of the electric current by which it is produced, in consequence of the magnetic force N . On account of the unknown quantity ϵ_1 , this factor, which in the remainder of the paper we shall call A , cannot be calculated. Therefore, instead of the theoretical amplitudes, only numbers proportional to them have been calculated. In the following Table only the means of μ_i and μ_v , m_i and m_v are given, since we have already shown (§ 16) that these quantities are equal to each other within the limits of error.

* Cf. § 19.

† A discussion of the theoretical considerations will give the signs of the quantities involved in these formulæ and the quadrants to which τ , ω , and the various δ 's belong. Cf. *Verhandel. Acad. Amsterdam*, Deel xxviii. (1890).

Equatorial Magnetization per unit volume=1400 C.G.S. units.

Mirror I. $J=77^{\circ} 23'.5$; $H=26^{\circ} 34'$.
 „ II. $J=76^{\circ} 30'.5$; $H=26^{\circ} 44'$. Yellow light.

Mirror.	Angle of Incidence.	Amplitude of the Magnetic Component of Light.		Obs. Ampl. Calc. Ampl. $\times \frac{10^{-3}}{A}$.	Phase of the Magnetic Component of Light.		Observed - Calculated Phase.
		Calc. $\times A$.	Obs. $\times 10^{-3}$.		Calc.	Obs.	
II.	$86^{\circ} 0'$	0.226	0.284	1.26	$-54^{\circ} 16.5'$	180° $+29^{\circ} 26'$	180° $+83^{\circ} 42.5'$
II.	$82^{\circ} 30'$	0.358	0.530	1.48	$-62^{\circ} 22'$	$+24^{\circ} 22'$	$+86^{\circ} 44'$
II.	$76^{\circ} 30'.5$	0.493	0.715	1.45	$-69^{\circ} 51.5'$	$+14^{\circ} 49'$	$+84^{\circ} 40.5'$
II.	$71^{\circ} 52'$	0.548	0.815	1.48	$-73^{\circ} 44.5'$	$+10^{\circ} 3'$	$+83^{\circ} 47.5'$
I.	$61^{\circ} 30'$	0.598	0.820	1.37	$-81^{\circ} 36.5'$	$+1^{\circ} 49'$	$+83^{\circ} 35.5'$
II.	$51^{\circ} 22'$	0.545	0.760	1.39	$-85^{\circ} 55'$	$-1^{\circ} 0'$	$+86^{\circ} 55'$
II.	$36^{\circ} 10'$	0.426	0.630	1.48	$-90^{\circ} 1.5'$	$-5^{\circ} 51'$	$+84^{\circ} 49'$
II.	$24^{\circ} 16.5'$	0.266	0.430	1.62	$-91^{\circ} 56'$		
II.	$12^{\circ} 0'$	0.152	0.260	1.69	$-93^{\circ} 2.5'$		
II.	$6^{\circ} 0'$	0.074	0.125	1.69	$-93^{\circ} 1.5'$		

Since in the factor A , Hall's constant h for iron is positive, and since this depends upon the choice of the system of coordinates, and the positive directions of the magnetic force N in these observations and in the theory of magnetic reflexion are opposite to each other, the signs of the calculated and observed amplitudes do not agree with each other, because they are both referred to positive magnetization. The sign of the amplitude of a wave-motion can, however, be changed by making the phase 180° large or smaller. In the rest of the paper the phases have been so changed in order to give the amplitudes the proper signs. The differences given in the last column will then be 180° greater or smaller.

The observations showed that $\mu_i = \mu_r$; $m_i = m_r$. This relation follows also from theory. As we can only write down proportional values for the theoretical amplitudes, the observed and calculated amplitudes agree with each other within the limits of experimental error. But there is a constant difference between the observed and calculated phases. The deviations from the mean value lie within the limits of error.

Perhaps this difference of nearly 90° points to a difference in the behaviour of ordinary electric currents and the periodic motions of the electric theory of light. As we have already seen, this can be shown by other methods*.

18. Kerr, who discovered the influence of magnetization on reflexion at the surface of iron, gives only the sign, and not the magnitude, of minimum-rotations. His results agree with my observations. In his case, too, there was a change of sign at an angle slightly greater than the principal angle of incidence. Kerr found a maximum of the minimum-rotation in the neighbourhood of 60° . In my observations the maximum amplitude of the magnetic component of the light was reached between 60° and 70° , but the maxima of rotation and amplitude do not coincide. It is worthy of note that Kerr has also used the motion of the dark band in the field of the analyser as the most accurate means of adjustment. With respect to the phase of the magneto-optic component, Kerr mentions that it approximates more nearly to zero than to 90° in the case of angles of incidence less than the principal angle. According to my observations, this is the case for every angle of incidence for which the phase can be accurately determined.

The signs and magnitudes of the minimum-rotations of the analyser given by Kundt† agree with mine, with the exception of some slight differences of magnitude.

Righi's investigation‡ appeared in the year 1887, after I had commenced these measurements. I have calculated the amplitudes and phases of the magnetic component of the light from the numerous angles of incidence given by Righi (Table opposite). The principal angle of incidence and azimuth were assumed the same as that for my mirror No. II.

Where two values of μ and m are given for the same angle of incidence, the first values are calculated from the minimum- and the second from the zero-rotations. We cannot conclude from these observations that the observed and calculated amplitudes are proportional to each other, much less that the difference between the calculated and observed phases is constant. The rotations must be measured with greater accuracy before we can draw the conclusion. To reach this degree of accuracy it is absolutely necessary to test the method of observation. The influence of errors of observation masks these relations. Since Righi did not calculate any

* Although the theory of magnetic reflexion which Fitzgerald has given really applies only to transparent media, yet the amplitudes reckoned from it are also proportional to the observed values.

† Kundt, Wied. *Ann.* xxiii. p. 244 (1884).

‡ Righi, *Ann. de Chim. et de Phys.* (6) x. p. 200 (1887).

The Amplitudes μ and Phases m calculated from Righi's Observations.

Angle of incidence.	$2\phi_{ip}^m$	$2\phi_{ic}^m$	$2\phi_{ip}^m$	$2\phi_{ic}^m$	$2\phi_{ip}^0$	$2\phi_{ic}^0$	$2\phi_{\mu c}^0$	$2\phi_{ip}^0$	$10^3 \mu_r$	$10^3 \mu_i$	m_r 180°	m_i 180°	$\frac{\mu_{obs.}}{calc.}$	$m_{obs.} - m_{calc.}$
85 0	-3.0	-3.0	-3.0	-3.0					0.48	0.48	$+29 \frac{26}{26}$	$+29 \frac{26}{26}$	1.78	85 58.5
82 0					4.5	-4.5	-5.5	4.0	0.31	0.26	+64 10	+67 30	0.77	128 49.5
80 0			-6.0	-1.0						0.87		-17 2	2.04	82 35.0
78 54	0	-6.7	-6.7	0	5.0	-6.5	-6.5	4.5	0.95	0.90	-14 46	-14 46	2.07	81 39.0
									-0.53	-0.41	-40 18	-48 22	1.05	111 13.0
75 0			-7.0	4.5						0.96		-10 22	1.87	90 9.0
70 0	6.7	-7.9	-8.0	6.5					1.04	1.05	-2 36	-3 53	1.90	75 48.5
60 0	11	-9.0	-9.0	11	-2.0	-9.0	-8.7	-1.3	1.13	1.13	-1 33	-1 33	1.90	83 37.0
									1.17	1.18	-5 24	-3 29	1.97	86 33.0
50 0	10	-9.0	-9.0	11	-6.0	-13	-13	-6	2.81	2.58	-67 20	-67 20	5.08	9 8.5
									2.00	2.00	-10 38	-10 38	3.70	75 45.0
40 0	7.9	-6.7	-7.0	8.0					0.78	0.81	7 6	-1 14	1.74	95 35.0
30 0	6.6	-6.6	-7.0	7.6					0.90	1.01	-34 19	-45 50	2.63	51 31.5
20 0	4.0	-5.0	-5.0	4.0					2.64	2.64	-77 57	-77 57	1.03	
10 0	2.5	-2.5	-2.7	2.3					0.34	3.00	-37 43	90-5° 36'	2.47	

amplitudes or phases he attaches more weight to the rotations themselves, and determines fully the changes of sign. Moreover, the principal angle of incidence of magnetized iron and the singular angle of incidence are introduced. These hold good, as we have already seen in § 16, only for one particular plane of polarization of the incident light.

19. The observations recorded in this paper furnish an accurate representation of magnetic reflexion, which, on comparison with theory, assumes a simple form. In order to demonstrate this clearly, I have given a table of the constant quantities, and the relations between these and the observed quantities, which suffice to fully determine metallic and magnetic reflexion.

Ordinary Metallic Reflexion.

Jamin's Constants :—

J=Principal Angle of Incidence.

H=Principal Azimuth.

Magnetic Reflexion.

Constants of Equatorial Reflexion :—

$$\Theta = 1.5 \times 10^{-3}; S = 85^\circ *.$$

Quantities used in presenting the Theories :—

Φ =Difference of phase between the light polarized parallel and perpendicular to the plane of incidence.

f =Amplitude of the reflected component, polarized parallel to the plane of incidence.

h =Amplitude of the reflected component, polarized perpendicularly to the plane of incidence.

Cauchy's Formulæ for calculating the ratios of amplitudes and the differences of phase for an angle of incidence α :—

$$\Phi = \sin(\tau + \omega) \tan \left\{ 2 \arctan \frac{\sin^2 \alpha}{\sigma \rho \cos \alpha} \right\}.$$

$$\cos \left\{ 2 \arctan \frac{h}{f} \right\}$$

$$= \cos(\tau + \omega) \sin \left\{ 2 \arctan \frac{\sin^2 \alpha}{\sigma \rho \cos \alpha} \right\}.$$

Δ =Phase of the magnetic component of the light.

μ =Amplitude of the magnetic component of the light.

Formula for calculating the amplitude, the form of which is borrowed from theory :—

$$\mu_i = \mu_t = \frac{\sigma^3 \sin \alpha}{\rho p_1^2 p_4 \cos^3 \alpha} \Theta.$$

Formula for calculating the difference of phase :—

$$\Delta = \Delta_0 + S.$$

$$\Delta_0 = 3\tau - \omega + \frac{\pi}{2} - 2\delta_1 - \delta_4 - (\delta_2 - \delta_1).$$

* Θ is the ratio of the observed to the calculated amplitude, and S is the constant difference between the observed and calculated phases.

Auxiliary Quantities determined by

J and H:— σ, τ^* .

Auxiliary Quantities determined by

J, H, α :— $\rho, \omega^\dagger$.

Observed Quantities:—

$\Phi, \frac{h}{f}$ = Azimuth of the polarization produced.

Auxiliary Quantities determined by

J, H, α :— $\delta_1, \delta_2, \delta_4, p_1, p_4^\ddagger$.

Observed Quantities:—

$\phi_{ip}^{(o)}, \phi_{ia}^{(o)}, \phi_{lp}^{(o)}, \phi_{la}^{(o)}$.

Relations between the observed and calculated values:—See §§ 4, 5.

* σ and τ are determined by

$$2H = \tau + \omega; \quad -\cos 2J = \frac{\tan(\tau - \omega_2)}{\tan(\tau + \omega_2)}; \quad \rho = \tan J \sqrt{\frac{\cos 2H}{\cos(\tau - \omega_2)}}.$$

† ρ and ω are determined by

$$\cot 2\omega = \frac{\sigma^2}{\sin^2 \alpha \sin 2\tau} - \cot 2\tau; \quad \rho^2 \sin 2\omega = \frac{\sin^2 \alpha \sin 2\tau}{\sigma^2}.$$

‡ The formulæ for calculating these values are given in § 17.

20. Although the relations of the amplitude and phase to the angle of incidence are determined by the above observations, further investigation is necessary to ascertain the cause of the very marked dispersion first found by Righi, and which I also confirmed during my observations*. Further, the theory just enunciated gives a simple relation between the amplitudes of the magnetic components of the light on polar and equatorial reflexion, so that it can be further verified by this means.

This research was carried out in the Physical Laboratory of the University of Leyden. I have to thank the Director, Prof. H. Kamerlingh Onnes, most heartily for the interest which he showed during its progress, and for the many accessories which were placed at my disposal.

XXXV. *On the Change in the Absorption-Spectrum of Cobalt Glass produced by Heat.* By Sir JOHN CONROY, Bt., M.A., Fellow and Bedford Lecturer of Balliol College, and Millard Lecturer of Trinity College, Oxford †.

SIR DAVID BREWSTER made some experiments on the influence of heat on the absorbing power of coloured media, and states in his 'Treatise on Optics' (edit. 1853,

* It is remarkable that the dark band in the analyser never showed the least trace of colour.

† Communicated by the Physical Society: read February 13, 1891.