first place there are very few, and in the majority of instances no joints whatever required in the web of a lattice girder. It has already been remarked that no stiffening irons are needed in the web if properly proportioned, and the correct sections of iron employed for the compression and tension bars. The stiffness necessary should be inherent in the weight of metal allowed for taking the strains to be undergone by each bar, and in the peculiar form and shape given to those bars. Contrast this with the fact, that 21 per cent. of the whole weight of the sides of the tubes of the Britannia Bridge is consumed in stiffening irons and covers. It is well known that the principle of the suspension bridge affords the means of supporting a given load with the least amount of material. Independently of the virtue of the principle on which it is founded, one of the reasons is that in these bridges the net and gross weight more closely approximate to one another than in any other wrought iron construction of bridge. It is not to be supposed that all covers, rivets, and the different connexions, can be ever dispensed with where wrought iron is employed, nor do we advocate a parsimonious distribution of material or the employment of a too rigid and consequently false economy, as is very frequently done; but we maintain that a great deal may yet be accomplished in the reduction of the useless and injurious weight of metal too frequently applied in girder bridges, or in other words, in making them less heavy. The Britannia Bridge at the present day has the merit of possessing the largest single span used for railway purposes; and so long as the ordinary wrought iron is employed, much larger spans will not be reached. With the improvements that are in daily progress in everything concerning the manufacture of steel and a superior class of iron, it is more than probable that at no distant period we shall be able to look for a description of iron or steel or their compounds, which will give twice or three times the strength of our common iron, with half its weight and without any increase in price. At present, did the magnitude of the design or other circumstances warrant the expenditure, we might, with an inferior description of steel, which could be obtained by a less expensive process than the usual one, construct bridges for railway purposes, not on the suspension principle, with spans of from 800 to 1000 feet, with ease and security.

MECHANICS, PHYSICS, AND CHEMISTRY.

For the Journal of the Franklin Institute.

Work, Vis Viva, and Momentum. By DE VOLSON WOOD.

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I published an article upon the subject which forms the title of this paper, in the November number of this *Journal* for 1862, since which, I have received several letters which indicate that I failed to make the subject sufficiently lucid.

As the answers to many of the difficulties suggested, possess a gene-

ral interest, and will tend still further to elucidate the subject, I have concluded to consider them in the following article.

The difficulties cannot be in the terms used, for they are merely *conventional*; but rather in a correct conception of what they represent in nature. The real office of each expression is best shown by placing them in contrast, or by showing what they represent when applied to the same moving body. This I endeavored to do in my previous article. I will now try to explain some of the difficulties which have presented themselves to others.

One writer says "Work is performed only in giving increased velocity; not in continuing the motion already acquired." This writer either has a wrong conception of *work* or else he has not fully stated the case. Work is overcoming resistance, and resistance may be constantly overcome at a uniform velocity, or even with a diminishing velocity. It may be, however, that the writer had reference to a body free to move; in which case, the work done upon the body is one of inertia; and it is true that no additional *work* is stored in the body unless there be an increase of velocity. In this sense the statement is true, but the language does not restrict it to this.

Again, one says "Vis Viva I take to represent a force." Now this is so vague I am inclined to take issue with it and say that *it does not represent a force; but rather what a force has done, or is capable of doing if it acts for a finite time.* To illustrate, suppose a body is allowed to fall freely under the influence of gravity. Now all will admit that the force of gravity near the earth is constant; but after it has fallen for a time its vis viva is Mv^2 ; where M is constant, and v variable, hence it cannot represent a constant force.

Again, suppose a locomotive is attached to a train by means of a spring scale, which indicates a constant force of 300,000 lbs. It moves the train from rest with a constantly increasing velocity, until all the surrounding resistances of air, friction, &c., equal the applied force, when the speed will become uniform.

Now observe that the force is constant and the velocity up to a certain speed is variable, how then can Mv^2 represent the force? If it be said that it represents the force over and above that necessary to overcome the resistances; then I would observe that the resistance of the air increases with the velocity so that the surplus decreases while Mv^2 increases. It does not then represent either the force applied or the surplus force.

To show what it does represent, let us suppose, to simplify, the case that all the resistances are constant, and that after the train has acquired a velocity v , the locomotive is removed, and the train is permitted to move on until it comes to rest.

Let s be the space over which it moves,

w the weight of the train,

f the force for a unit of weight, that is the force, as indicated by a spring scale, which is necessary to draw a unit of weight (say one pound) over the resistances, with a low uniform velocity,

$wf = P =$ the total force, also equal the total resistances.
Then the total work is

$$wfs = Ps.$$

Now if we conceive P to be applied to the train to move it, and conceive all resistances removed so that the train is *perfectly free* to move, it will produce a velocity v in the train at the end of the space s , *i. e.*, it takes the same space for P to produce a velocity v , without resistance that it does to destroy a velocity v , with a constant resistance of P . The law of motion in the former case is the same as for falling bodies, and in the latter that of ascending bodies.

Then since P is constant $\frac{P}{M}$ is the accelerating force and we have

$$v = \sqrt{2 \frac{P}{M} s} = \sqrt{\frac{2wfs}{M}}$$

$$\therefore MV^2 = 2Ps = 2wfs.$$

Now observing that P is the true value of the force, we see the second member is twice the force repeated s times, in other words it is twice the sum of all the forces exerted on each unit of the space, s —and the first member must *represent* the same thing.

If the resistances are variable, this becomes general by considering the force on each element of space, and adding them all together, which gives rise to the well known expression $\int P ds$.

Another correspondent says, "A sliding body has a certain vis viva; now if we double the velocity the vis viva will be fourfold; and unless we conclude that dynamical friction increases with the velocity, I do not see how these factors preserve their equality." Dynamical friction does not increase with the velocity; hence we must reconcile the facts on other grounds.

Let φ = the total amount of friction.

$\therefore \varphi s$ = the work on space s ,

φs^1 = " " s^1 ,

v = the velocity at the end of space s ,

i = angle of inclination of the plane.

The motion of the body follows the law of falling bodies;

$$\therefore s = \frac{v^2}{2gs.i.}$$

and for double the velocity

$$s^1 = \frac{(2v^2)}{2gs.i.} = \frac{4v^2}{2gs.i.}$$

$$s^1 = 4s, \text{ and } s^1 = 4\varphi s.$$

So that by doubling the velocity we quadruple the *stored work*. We may observe that the body must pass over four times the space to produce twice the velocity.

Another says, "Gravity does not add equal increments of space in each element of time, and hence doubts whether wh really expresses the work which gravity does upon a falling body." He admits "that it adds equal increments of velocity in each element of time."

His statements in regard to gravity are correct. Let us apply the latter and see where it will lead us. If a resistance P, be moved with a uniform velocity v , then would the work done in a unit of time be

$$Pv \text{ and in a time } t, \text{ it would be } PVT = PS \quad (1)$$

But in a falling body the resistance overcome is the weight, and the velocity is uniformly increasing; so that the work done during any element of time, Δt , is by (1)

$$W V \Delta t \quad (2)$$

But from a law of falling bodies,

$$V = gt \text{ or } \Delta V = g \Delta t,$$

which in (2) gives $\frac{W}{g} V \cdot \Delta V = MV \cdot \Delta V,$

This solved by the calculus gives $\int M V dV = \frac{1}{2} M V^2$, but as I wish to make it intelligible to those who do not understand the calculus, I will give the following way of finding the sum of all the products of $V \cdot \Delta V$ between 0 and V . Let any ordinate, as DE , represent the velocity, v . Since the velocity uniformly increases, the line AC will be the limit of the ordinates. Let $Ee = \Delta V$, then will the vanishing trapezoid be ultimately equal the rectangle of which DE is the base and Ee the altitude; hence $DEed = V \Delta V$. The sum of all the infinitesimal rectangles will equal the area ABC . The sum of all the altitudes of the rectangles becomes $AB = V$; hence the area equals $\frac{1}{2} V^2$ and the expression becomes $\frac{1}{2} M V^2$.

My attention has been called to several other points, but they indicate too limited knowledge of the subject, or too little thought bestowed upon it to make a reply to them profitable in this connexion.

Momentum has no reference to the *resistance* to be overcome, but to the *motion* which a body of known mass and velocity will induce in another mass free to move, when the former impinges upon the latter. As the expression indicates, it pertains only to the relations between masses and velocities of different bodies. Momentum may be considered as the measure of an impulsive force; one-half the vis viva, the *sum* of all the energies (or forces) exerted to cause a velocity v . I trust that the relation between momentum and work, in the example used in my previous article, is sufficiently explicit. I will, therefore, drop the subject until circumstances induce a further consideration of it.

For the Journal of the Franklin Institute.

The Forces ; Slave Labor ; Working Land by Steam ; A Cycloidal Cultivator.

Of mundane agencies **FORCE**—physical energy or power—is unique and supreme. All that we see, and feel, and hear, and do; light and life, forms, qualities, and motions, proceed from it. Its proper use comprises the whole economy of life. It is emphatically *the* talent given to men to profit withal. It places the destiny of every people in their own hands, for the amount at their disposal is not limited, nor are the