IV.—ON THE EFFECT OF THE GLACIAL PERIOD IN CHANGING THE UNDERGROUND TEMPERATURE GRADIENT.

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IN his recent work on "Controverted Questions of Geology" (p. 159), Professor Prestwich remarks that the intense cold of the Glacial period may still be perceptible in the underground temperature gradient; that "to a certain depth the rate of cooling is now abnormally slow, owing to the excessive refrigeration the crust then underwent." The suggestion is a valuable one, and I propose to test it in the present paper by estimating roughly the change which the temperature gradient may have experienced since the close of the Glacial period.

In order to avoid confusion, it may be well to state precisely the meaning which is here attached to the term "temperature gradient." If the temperature increase 1° F. for every 50 feet of vertical descent, the gradient is taken to be 1° per 50 feet or $\frac{1}{50}^{\circ}$ per foot, but not 50 feet per degree. If the gradient in course of time were to change to 1° in 60 feet, it will be said to have decreased, whereas in the latter case it would be said to have increased.

The first point to be determined is the rise of mean annual temperature which has taken place since the Glacial period. The present value of the mean annual temperature in England varies between $45^{\circ}6$ (Dartmoor), and $51^{\circ}9$ (Helston), the average over the whole country being $49^{\circ}5$ F.¹ With regard to the Glacial period, we are concerned, not so much with the mean annual temperature of the air at that time, as, in glaciated districts, with that at the base of the ice-sheet; and this, it is probable, underwent but little change during the whole time the ice-sheet lasted, however intense the cold at its surface may have been. For, if the thickness of the ice exceeded a certain limit, the temperature at the bottom, owing to the rise of the isogeotherms, would be maintained at the melting-point corresponding to the pressure of the overlying ice. With a mean temperature of 0° F. at the upper surface of the ice, and a gradient in the rock below of one degree in 60 feet, Mr. Fisher finds this limiting thickness to be 714 feet.² With a gradient of one degree in 50 feet, the limit would be 596 feet. This may, however, be an under estimate, for it does not take the depth of the "surface of invariable temperature" into account. Moreover, while the temperature at the depth of this surface is in rock the mean annual temperature of the locality, it may be less in the case of an ice-sheet whose temperature can never rise above the melting-point, and is therefore incapable of following the oscillations of the external temperature above that point. The effect of the omissions is not, however, very serious, involving indeed in some cases an increase in the above estimate, though not, it may be, a great one.

¹ This is the average of the annual means at 57 stations given by Dr. A. Buchan in his great work on "Atmospheric Circulation," "Challenger" Reports, Physics Chemistry, vol. ii, pp. 196-199.

² Phil. Mag., vol. vii, 1879, p. 385.

Considering the low mean temperature assumed to exist at the surface, there is probably no occasion to alter the above estimate of the limiting thickness. But if we double it, and take it at 1200 feet, we shall be well on the safe side; and there is no doubt that for a very long period the ice-sheet in many places attained a greater thickness than this. Now the melting-point of ice is lowered 1° F. by the pressure of every 2730 feet in the vertical thickness of overlying ice, so that the temperature at the base of the ice-sheet may have been as low as 30° or even 29° , but can never under any circumstances have risen higher than 32° . We cannot err very greatly, therefore, in supposing the surface of the ice-covered ground during the whole Glacial period to have remained at a uniform temperature of $30^{\circ} \cdot 5$. The subsequent rise of temperature may thus be estimated at 19° F.

All geologists are agreed in attributing a duration of several, or many, thousand years to the Glacial period. If this be correct, the temperature at its close must for some miles have increased almost uniformly with the depth. Thus, if in the accompanying figure O represent a point on the earth's surface, OX a horizontal, and OY a vertical line, the temperature-curve OPA near the surface must have been almost a straight line, NP representing the temperature at the depth represented by ON, or, rather, the excess of temperature above $30^{\circ} \cdot 5$. If ON were taken to represent one foot, then NP would represent the temperature gradient.

Now let us suppose that, at the end of the Glacial period, the mean annual temperature at the surface rose *suddenly* by 19° ; and let the line OB represent this amount. Then, at some subsequent time, the temperature-curve would be of the form BQC, where NQ represents the temperature at the depth ON, the lines OPA, BQC joining at the point where the change of temperature ceases at

the particular time to be sensible. The excess of temperature at the depth ON above that at the surface is represented by DQ, which is obviously less than NP, so that the temperature gradient is diminished at all points to which the heating has become perceptible.

With a further lapse of time the heating effects will have penetrated to a greater depth, and the temperature-curve will have assumed the form BRE. Here, the excess of temperature at the depth ON above that of the surface has increased (DR being greater than DQ); consequently the temperature gradient has also increased, and, continuing to do so, will in time attain a value not differing very greatly from that which it had just before either the close or the commencement of the Glacial period.

Before proceeding further it should be remarked that the inaccuracy of the assumption of a *sudden* change of surface temperature must of course affect all the numerical results hereafter obtained. But, as we do not know the law according to which the rise of temperature took place after the disappearance of the ice-sheet, it seems to me allowable to make the assumption, especially as the provision of *exact* numerical results is not the purpose of this paper. Moreover, if the change were gradual but continually in one direction, the temperature-curve would be of the form BSF, and this would result in a still further decrease of the gradient, so that the change of gradient would in reality exceed that found on the assumption of a sudden rise of surface temperature.

The last element required, and that about which the greatest uncertainty exists, is the time that has elapsed since the close of the Glacial period. On account of the small amount of denudation accomplished, the tendency of opinion has recently been to reduce the estimates of the length of this interval. Prof. Prestwich even puts it at only eight or ten thousand years. The maximum figure is probably that given by Croll of about eighty thousand years.

Under the circumstances, it seems best to calculate the change of temperature gradient for a series of different intervals. In order to give definiteness to the results, I have taken the present gradient at one degree in 50 feet, and, in Table I, have expressed the value the gradient must have had at the close of the Glacial period on the assumptions stated above, for different estimates of post-Glacial time from 5000 to 40,000 years, and for different changes of the mean annual temperature from 16° to 22° .¹

Thus, if the rise of mean temperature since the Glacial period be 19° , and if the interval since its close be only 9,400 years, the

¹ The calculations are performed as follows:—If b be the rise of mean annual temperature at the surface in degrees, κ the conductivity of rock expressed in terms of its own capacity for heat, t the number of years since the change of temperature took place, then the change of gradient is $b \div \sqrt{\pi \kappa t}$ (Rev. O. Fisher, Phil. Mag., vol. xxxiv, 1892, p. 339). This value, it should be remarked, is independent of the previous gradient. Taking Lord Kelvin's value for κ , namely 400, the above formula becomes $\cdot 02821 b \div \sqrt{t}$. If $b=20$ and $t=10000$, $b \div \sqrt{t} = \frac{1}{2}$, and the change of gradient is $\cdot 00564$ degree per foot. But the gradient now is taken to be $\cdot 02$ degree per foot, so that, at the end of the Glacial period, it must have been $\cdot 02564$ degree per foot, or one degree in 39 feet.

gradient just before that close must have been one degree in 39 feet; if the interval be 15,000 years, one degree in 41 feet; and if 27,000 years, one degree in 43 feet. Or, again, at the bottom of a mine half-a-mile below the "surface of invariable temperature," i.e. about 2700 feet deep, the temperature would be 53° above that at the surface with a gradient of one degree in 50 feet, 68° above with a gradient of one degree in 39 feet, 65° with one of one degree in 41 feet, and 61° with one of one degree in 43 feet. In any case, it follows that the gradient now differs very sensibly from the value it had at the close of the Glacial period.

TABLE I.

Showing Number of Feet of Vertical Descent corresponding to a rise of 1° F. in underground temperature at the close of the Glacial period.

Number of Years since end of Glacial period.	Assumed Increase of Mean Annual Temperature at the Surface.						
	16°	17°	18°	19°	20°	21°	22°
5000	37.9	37.3	36.8	36.3	35.7	35.2	34.8
10000	40.8	40.3	39.9	39.4	39.0	38.6	38.2
15000	42.2	41.8	41.4	41.0	40.6	40.2	39.9
20000	43.1	42.7	42.4	42.0	41.7	41.3	41.0
25000	43.8	43.4	43.1	42.8	42.4	42.1	41.8
30000	44.2	43.9	43.6	43.3	43.0	42.7	42.4
35000	44.6	44.3	44.0	43.7	43.5	43.2	42.9
40000	44.9	44.7	44.4	44.1	43.8	43.6	43.3

The question whether the present gradient retains any traces of the Ice Age depends upon several conditions, such as the mean annual temperature existing before the Glacial period, and the relative lengths of the Glacial period and the subsequent interval. Obviously, this is a question to which it is difficult, perhaps impossible at present, to furnish any decisive answer. If the pre-Glacial mean temperature were the same as it is now, if the glacial conditions commenced and ended abruptly, and if, moreover, the Glacial and post-Glacial periods were of equal duration, then the gradient now would be the same as it was before the Glacial period, that is assuming the normal gradient¹ to have undergone no sensible change during the interval. Supposing the first two of the above conditions satisfied, however, it may be worth while giving a few numerical estimates of the change of gradient depending on different assumptions as to the lengths of Glacial and post-Glacial times. These will be found in Table II, which supposes that the present gradient is one degree in 50 feet, and that the mean annual temperatures were constant during both periods and differed by 19° .

As the general opinion seems to be that the duration of the Glacial period was much greater than that of the succeeding interval, it is therefore possible that some relics of the Ice Age are still to be found in the present temperature gradient.

¹ The gradient due to the temperature of solidification, the surface temperature having remained constant ever since.

A few practical applications of these results may be mentioned in conclusion.

(1) The usual method of finding the average temperature gradient over large areas may be incorrect, unless it is known that all the stations have not recently experienced the effects of a considerable change of mean annual temperature. For instance, the change of gradient in Scotland and the north of England since the Glacial period may be different from that which has taken place in France or Cornwall, and the average of the best determined gradients in these districts is probably not the correct average for the present epoch due simply to the original temperature of solidification.

TABLE II.

Duration in years of		Gradient before Glacial period.
Glacial period.	Post-Glacial period.	
20000	10000	One degree in 46·8 feet
40000	10000	„ 44·1 „
40000	20000	„ 46·9 „
60000	20000	„ 45·9 „
80000	20000	„ 45·3 „

(2) Estimates of the earth's age founded on Lord Kelvin's solution and conditions, if derived from a temperature gradient in a recently glaciated district, will err by being too great. For instance, if the duration of post-Glacial time be 10,000 years and if that of the Glacial period were four times as great, the normal temperature gradient at the present time would be nearly one degree in 44·1 feet, instead of one degree in 50 feet. The corresponding value of the earth's age would be about 75 million, instead of about 100 million, years.

(3) Lastly, supposing it in our power to make very accurate observations of underground temperature, it would be possible, by comparing gradients in glaciated districts in the two hemispheres, to determine roughly whether their respective Glacial periods ended nearly simultaneously or at widely different epochs; and thus to obtain some suggestion of the direction, cosmical or terrestrial, in which to look for the origin of the glacial cold.

V.—ON THE CHALK OF THE LONDON BASIN IN REGARD TO WATER SUPPLY.

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1. THICKNESS OF THE CHALK.

OVER the area in question the total thickness of the Chalk (where it can be measured from top to bottom, or nearly so) varies from 623 feet at Streatham to 1146 feet at Norwich, and in the latter case the topmost beds are absent; so that the full thickness,