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native arsenic in barytes: on a large crystal still on the specimen there is a plane between (212) and (101), developed as in fig. 11. From measurement of a smaller crystal which was removed from the specimen, the indices of this plane would appear to be (414), the plane given by Dana. On this same small crystal are two other minute planes, affording measurements which accord with (616), (15 1 15):—

	Observed.	Calculated.
010.212 . . .	64 54	64 59
212.414 . . .	11 52	11 53
414.616 . . .	4 21	4 18
616.15 1 15 . .	5 8	5 16
15 1 15.101 . .	3 13	3 34

XXVI. On a Calculating Apparatus based on Napier's Rods.
By JOHN BRIDGE, M.A.*

[Plate VI.]

THE ingenious contrivance known as Napier's rods or bones is perhaps usually regarded as of some historical interest, but as no longer of practical use. For half a century, however, after its description in *Rabdologia*† it received much attention; and it has often been revived in various forms‡. I propose here to show that, by an improvement in manipulation, it may still be made to give valuable aid by diminishing labour and increasing safety and speed in calculation.

Fig. 1 (Pl. VI.) represents the apparatus, omitting some non-essential points, diminished linearly as 4 to 1. S S B B is a board on which, along its middle, a series of Napierian tablets (I have discarded the form of rods) are to be arranged for use. These have a notch at each end, so that they may be set in their places by means of pairs of pins (figs. 1 and 2) fixed in the board, which is about '1 inch higher between the pins than elsewhere.

The tablets, which carry on one side multiples of 0, 1, 2, 3, 4, carry on the other side multiples of 9, 8, 7, 6, 5, respectively. They are kept arranged and instantly accessible in a magazine which, when not wanted, is deposited in a cavity (M) in the board.

* Communicated by the Author.

† The full title is, *Rabdologiæ seu Numerationis per Virgulas Libri duo; cum Appendice de expeditissimo Multiplicationis Promptuario, Quibus accessit et Arithmetica Localis Liber unus.* Authore et inventore Johanne Nepero. 12mo. Edinb. 1617.

‡ See *Bulletin de la Société de l'Encouragement*, 1851, p. 415.

Fig. 1. Scale. $\frac{1}{4}$.

Fig. 2. Scale. $\frac{1}{2}$.

Fig. 4. Scale. $\frac{1}{4}$.

Fig. 3. Scale. $\frac{1}{4}$.

F G (fig. 3) is a frame which is movable along the board. It carries a number of glass slides, a, b, c, d , &c. (figs. 1 and 2), movable on it in the direction of their length. These are made opaque except at the middle, where an aperture is left (fig. 4) large enough to show two adjacent digits on the tablets.

g (fig. 1) is a fixed ridge of hard wood with notches at equal intervals; and g' (fig. 3) is a corresponding groove in the frame. p, p are pins going down through the frame so that their lower ends, which are rounded, fall into the notches of g . By the help of a little pressure from the finger on one of them they serve to fix the frame at any of the positions it ought to take—namely, so that each slide is half over one tablet and half over the next (fig. 4).

2. *Multiplication.*—Suppose 74628394 to be the multiplicand and 4736 the multiplier. Fig. 2 shows the arrangement required. Tablets headed with the digits of the multiplicand are set up in *direct* order on the board, the first filling the space 0 to 1 of the scale SS; and the slides a, b, c, d are set to the lines corresponding to the digits 4, 7, 3, 6 respectively of the multiplier.

The frame is then moved step by step from right to left. At each step the sum of all the digits seen through the apertures is taken, its units' figure recorded and its tens' figure carried. When F arrives opposite 0 on the scale the whole product will have been recorded.

3. This will be seen without explanation; but for the sake of what is to follow, the process may be compared with the arranged details of the product.

	0	1	2	3	4	5	6	7	8
1	2	81	62	40	83	21	23	61	6
2	4	92	84	21	45	62	16	32	8
3	2	11	21	80	62	40	92	71	2
4	4	22	43	61	24	81	85	42	4

Let the sum of each pair of adjacent digits be called an *element*, each vertical series of elements a *column*, and each horizontal series a *row*. The columns will then correspond to the places on the scale SS, or of the digits of the multiplicand; and the rows will correspond to the *places* of the digits of the multiplier, row 1 to 1st digit, and so on. The position of an element in the product may be described by indicating its column and its row, and its *rank* is the sum of the numbers indicating its column and its row. Thus the element (6, 3) is that which is found in column 6 and row 3, and its rank is

6+3. In the process the elements of the product came out in the following order:—

With F at 12	appeared	(8, 4),
„ „ 11	„	(8, 3), (7, 4),
„ „ 10	„	(8, 2), (7, 3), (6, 4),
„ „ 9	„	(8, 1), (7, 2), (6, 3), (5, 4),
„ „ 8	„	(7, 1), (6, 2), (5, 3), (4, 4),

and so on; at each step all the elements of the product which have the rank indicated by F.

4. It is evident how the apparatus may be made to work $ap + bq + \dots$, so as to require no writing except to record the answer, the number of slides on the frame and places on the board being supposed great enough. For example, with an apparatus having the range shown in fig. 1, we can write down the sum of the squares of two numbers of 5 figures, to do which in the ordinary way would require about 75 figures more.

5. *Division*.—Let the multiplicand (divisor) be set up, as before described, on the board, and the product (dividend) written on paper, which it would be convenient to have ruled to correspond with the scale S S. Then the question is—how to get the slides a, b, c , &c. set to the successive digits of the multiplier (quotient).

The apparatus may be made to do this by following the ordinary process of division; and it is obvious what advantages that plan would have*. But it is much better to follow a method analogous to Horner's synthetic division in Algebra†, for which the apparatus is very well adapted.

In this plan the slides are carried by the frame left to right, and each takes its proper setting when it arrives at a column agreed on, usually column 1 or column 2. Until it has *passed* this column let it be called a *trial* slide, afterwards a *collecting* slide.

6. *Division, using three trial slides*.

First stage, fixing a.—Advance F to 3, so that c, b, a come over columns 0, 1, 2. Bring c, b, a down *together*, seeking somewhat less than the first three figures of the dividend, and subtract the number found‡. Put back c, b ; move F on to 4, and append a new figure from the dividend.

Second stage, fixing b.—Subtract the element shown by a

* It would be this:—Having found by inspection (at any stage of the process) what row is to be subtracted, set a slide to that, and move the frame towards the left, subtracting each element as it presents itself.

† Todhunter's 'Algebra,' p. 805.

‡ Not the sum of the elements; for the elements shown by the *trial* slides are of different ranks.

in column 3. Bring down d, c, b together, seeking somewhat less than the remainder, and subtract the number found. Put back d, c ; move F on to 5, and append a new figure from the dividend.

The plan for any stage is—

(α) Subtract the sum of all the elements shown by the collecting slides.

(β) Seek somewhat less than the remainder with the trial* slides, and subtract the number found.

(γ) Put back all the trial slides but the one nearest F, move F on a step, and append a new figure from the dividend.

At the end the quotient is to be read off backward from the slides.

If the complete remainder at any stage is required, transfer the frame to the right-hand side of the board and move it up towards the left, subtracting at each step the sum of the elements shown.

7. It will be seen that the elements of the product or dividend are taken up at successive stages in the following order (compare art. 3):—

	0	1	2	3	4	5	6	7	8
First stage .	1	1	1	F					
Second stage .	2	2	2	1	F				
Third stage .	3	3	3	2	1	F			
Fourth stage .	4	4	4	3	2	1	F		

and so on.

The ranks being found by adding the numbers indicating the column and the row, are—

At the first stage . .	1, 2, 3
„ second stage . .	2, 3, 4, 4
„ third stage . .	3, 4, 5, 5, 5

and so on.

In the above, each slide after leaving column 2 retains the position on the frame which it got there; the partial dividends have (usually) 3 figures, and the remainders to be recorded at the end of each stage 2 figures.

If it is agreed that each slide shall get its setting in column 1, two trial slides must be used, and F for the first stage must be placed at 2. This will be found the more convenient plan, unless the divisor have a small leading figure. The estimation

* The element shown by the one nearest F is of the same rank as those shown by the collecting slides.

of the coming digit is sometimes less certain; but there is usually only one figure of remainder to record, and the result of the first subtraction need not be recorded at all.

8. If a doubt occurs at any stage, it may be solved in several ways:—

(1) By pushing forward the frame for a moment and so ascertaining what would be carried from the next rank.

(2) By increasing the number of trial slides. Thus, supposing that we have made the first subtraction, and trying for the new digit we are in doubt. Put back the trial slides, append another figure from the dividend, push on the frame a step, and subtract the sum of the elements shown. Then try again, using one more trial slide than before.

For example,

	0	1	2	3	4	5	6	7	8
Fourth stage .	4	4	3	2	1	F			
			4	3	2	1	F		
Fifth stage .	5	5	5	4	3	2	1	F	

shows the plan of changing from two to three trial slides after 4 figures of quotient have been obtained.

(3) By finding the complete remainder, recording the digits of the quotients already obtained, and beginning the division again with this complete remainder as dividend.

With three trial slides it is very unlikely that any difficulty will occur before 12 figures of quotient have been obtained, and then only if the divisor have more (say) than 7 figures. The method of computing should in all cases be exactly that given in Sonnenschein and Nesbitt's 'Arithmetic,' part i. p. 105.

9. Examples of Division.

Ex. 1. In these examples the divisor is 74628394, so that the process may with patience be followed on fig. 2.

Two trial Slides.

Dividend . . .	32496883142966
Written work . .	345599775883
Quotient	435448623344

The partial dividends are to be read obliquely 34, 49, 56, &c., and in the following example, 260, 310, &c.; so that in the above only one figure was recorded at each step: and the result was very quickly verified by reversing the motion of the frame for the purpose of multiplying divisor and quotient.

Ex. 2. The reciprocal of ·74628394.*Three trial Slides.*

Dividend . . .	10000000000000
Written work . .	617594672085
" "	237752446776
Quotient . . .	1·33997255789

10. *Square Root.*—In the process for square root a new tablet as well as a new slide is introduced at each stage. Hence in the diagram it will be necessary to represent what intervals are at each stage occupied by tablets. This is done by inserting an asterisk. Also some of the elements must be doubled; these are indicated by accents.

I avoid length of description by giving only the diagram, hoping that it will be sufficient explanation of the process. The first digit is obtained in the usual way.

Plan for finding Square Root, using two trial Slides.

0	1	2	3	4	5	6	7	8
1 * 1	F							
2' * 2	*	1	F					
3' * 3	*	2	*	1	F			
	&c.							

Plan for changing from two to three trial Slides.

0	1	2	3	4	5	6	7	8
1 * 1	F							
2' * 2	*	1	F					
	*	*	2	1	F			
3' * 3'	*	3	*	2	1	F		
4' * 4'	*	4	*	3	*	2	1	F
	&c.							

After the first two stages the process is as easy as that for division.

11. Example of process for Square Root. Two trial Slides.

Given number . .	12·34567890000
Written work . .	3 19533940908
....	1 1
Square root . .	3·51364182864

The same, using *three trial slides* after the second stage:—

Given number . .	12·34567890000
Written work . .	3 19551504526
....	243162653

A simplification worth noticing in this process arises from the fact that the digits on the collecting slides, the number of which is always odd, are equal at equal distances from the middle.

12. Since writing the above I have found analogies to the processes described. Lieut.-Col. Oakes, in the Journal of the Institute of Actuaries, Jan. 1863, gives "*A method of multiplication which may be practised mentally,*" the knowledge of which from some other source led me to construct the apparatus. The other processes came, as it were, by the spontaneous action of the machine. Methods having a partial correspondence with them are given by Dr. Peacock in his "Arithmetic" in the *Encyc. Met.* as Arabian or Persian methods; also in his 'Algebra,' vol. i. pp. 100-105.

Napier's *Promptuarium*, described in the Appendix to his *Rabdologia*, multiplies by a process arithmetically identical with that given; but the construction and manipulation of his apparatus are very troublesome.

Hampstead, Jan. 2, 1880.

XXVII. *On the Forms of the Vibrations of twitched and stroked Strings.* By F. LINDEMANN*.

1. DURING my lectures on Fourier's series and their applications it struck me that various objections might be raised against the theories of twitched and of stroked strings as they are given by Helmholtz†. This occasioned the following investigations, which are intended to establish those theories more rigorously.

In regard to the twitched string, the series

$$y = \frac{2bL^2}{\pi^2 a(L-a)} \sum \frac{1}{n^2} \sin \frac{n\pi a}{L} \sin \frac{n\pi x}{L} \cos \frac{2n\pi t}{T}, \quad (1)$$

derived in the well-known manner from the general theory, cannot be regarded as the integral of the fundamental differential equation

$$\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2}, \text{ where } \alpha = \frac{2L}{T}, \quad (2)$$

merely because the latter is satisfied by every single term of the series. In the present case the admissibility of this view

* Translated from a separate impression, communicated by the Author, from the *Berichte über die Verhandlungen der naturforschenden Gesellschaft zu Freiburg i. B.* vol. vii. no. 4, pp. 500-532.

† *Die Lehre von den Tonempfindungen*, Brunswick, 1863, Beilage II. and V. These supplements remain unaltered in the subsequent editions.