A RULE TO SQUARE NUMBERS MENTALLY.

By L. C. KARPINSKI,
University of Michigan.

In the January, 1914, issue of SCHOOL SCIENCE AND MATHEMATICS rules were given to square mentally numbers between 25 and 50 and between 75 and 150. The algebraical explanation affords a highly desirable application of the formulas of algebra to arithmetical computation, while at the same time the algebraical statement extends the rule to all numbers between 25 and 150.

To square any number between 25 and 75 find the difference between the given number and 50. This difference subtract from 25 if the given number is under 50, and add it to 25 if the given number is over 50. To the resulting number annex two zeros, and add the square of the difference.

Square 47. Difference 3. Subtract from 25 and annex two zeros, 2200. Add 9, 2209 is the square.

Square 56. Difference 6. $25+6$ is 31. $3100+36$ is the square.
 Square 64. Difference 14. $25+14$ is 39. $3900+196$ is the square.

To do this mentally remember to add the two final numbers from left to right and not from right to left as in written work. Almost all rapid mental calculation proceeds from left to right.

To square any number between 75 and 100, find the difference between the number and 100. Subtract this difference from the given number and annex two zeros. To this number add the square of the difference. A similar rule holds for numbers between 100 and 150.

Square 96. Difference 4. $96-4$ is 92. $9200+16$ is the square.

Square 86. Difference 14. $86-14$ is 72. $7200+196$ is the square.

Square 114. Difference 14. $114+14 = 128$. $12800+196$ is the square.

To square any number between 10 and 20, add the units to the given number and to this sum annex a zero. Add the square of the units. It is always easy to add the two final numbers mentally from left to right.

Square 14. $14+4$ is 18. $180+16$ is the square.

Square 18. $18+8$ is 26. $260+64$ is the square.

Only one possible difficulty remains in the numbers between 20 and 25. To square any number between 20 and 30, add the

units to the number and double the result. To this annex a zero and add the square of the units.

Square 24. $24+4$ is 28. Doubled, 56. $560+16$ is the square.

Any boy who has had just a little algebra will see that these rules are all applications of the simple formulas: $(a+b)^2 = a^2+2ab+b^2$, $(a-b)^2 = a^2-2ab+b^2$, and the teacher of algebra should emphasize this kind of simple application of algebraic formulas.

$$\begin{aligned}\text{Thus} \quad (50-a)^2 &= 2500-100a+a^2. \\ (50+a)^2 &= 2500+100a+a^2. \\ (100\pm a)^2 &= 10000\pm 200a+a^2. \\ &= 100(100\pm a\pm a)+a^2. \\ (10+a)^2 &= 100+20a+a^2. \\ &= 10(10+a+a)+a^2. \\ \text{And} \quad (20+a)^2 &= 400+40a+a^2. \\ &= 20(20+a+a)+a^2.\end{aligned}$$

GRAPHICAL REPRESENTATION OF A GEOMETRICAL SERIES.

BY D. H. RICHERT,

Bethel College, Newton, Kansas.

That a graphical representation of an algebraical expression is of great aid in making clear the full meaning of the problem, is an everyday experience of the mathematics teacher.

One of the difficulties that presents itself to the average high-school student is the conception of the sum of an infinite series (a convergent series, of course).

The following graphical representation of such a series will do much to make it clear to a student that a sum exists in such a case. (Comp. M. Milankovitch, *Zeitschr. für Math. u. nat. Unterricht*, 40. Jahrgang, S. 329.)

Given the geometrical series,

$$a+ar+ar^2+ar^3+ar^4+\dots, \quad r<1,$$

to represent this series graphically.

On the line AR take the distance AB = a . Construct at B a perpendicular to AR and make it equal to ar . Construct FC perpendicular to BF at F and make it equal to ar^2 . Through A and F draw the line AP, and through B and C draw the line BP.

Between the lines AP and BP draw the broken line CGDHEK . . . making CG, DH, . . . parallel to BF, and GD, HE . . . parallel to FC. Then $\angle FAB = \angle FBC$, for $\triangle ABF$