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The Electrical Distribution induced on an Infinite Plane Disc with a Circular Hole in it. By H. M. MACDONALD. Received and read December 12th, 1895.

The function

$$W_n = \mu^{\frac{1}{2}} \int_0^\infty e^{-\kappa z} \kappa^{\frac{1}{2}} J_n(\kappa r) d\kappa \int_a^\infty J_{n+\frac{1}{2}}(\kappa r') J_{n+\frac{1}{2}}(\mu r') r' dr'$$

is such that for points in the plane $z = 0$, $r > a$,

$$W_n = J_n(\mu r),$$

and $r < a$,

$$\frac{\partial W_n}{\partial z} = 0.$$

The inducing system may be taken to be on the positive side of the plane; then in the neighbourhood of the plane the potential V_1 due to it may be represented by

$$V_1 \equiv \Sigma \Sigma A_{n\mu} e^{\mu z} J_n(\mu r) \cos(n\phi + \alpha_\mu);$$

then V , the potential due to the induced charge, is given by

$$V = -\Sigma \Sigma A_{n\mu} W_n \cos(n\phi + \alpha_\mu),$$

whence the distribution may be obtained as in Vol. xxvi., p. 260.

Note on the Convergency of Series. By ROBERT BRYANT.

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In what follows the n^{th} term of a series is used to denote the series itself.

By Cauchy's condensation test the convergency of the series $f(n)$ is the same as that of the series

$$a^n f(a^n), \quad a^n a^{a^n} f(a^{a^n}) \dots, \quad a > 1.$$

Continue this series backwards; then the series preceding $f(n)$ is

$$\frac{f(\log_a n)}{n}.$$

Hence we have the following rule :-

Change n into $\log_a n$ and divide by n .

Putting $\log_a = l$, $\log_a \log_a = l^2$, &c.,

then $f(n)$ and $\frac{f(l n)}{n}$ are both convergent or both divergent.

The following series are therefore convergent or divergent according as $r <$ or $\nless 1$,

$$r^n, \quad \frac{r^{l n}}{n}, \quad \frac{r^{l^2 n}}{n l n}, \quad \frac{r^{l^3 n}}{n l n l^2 n}, \quad \dots, \quad \frac{r^{l^k n}}{n l n l^2 n \dots l^{k-1} n}.$$

Comparing $f(n)$ with $\frac{r^{l^k n}}{n l n l^2 n \dots l^{k-1} n}$, we see that it is convergent

if $f(n) < \frac{r^{l^k n}}{n l n l^2 n \dots l^{k-1} n}$ and $r < 1$.

Assuming n so large that $l^k n > 0$, this gives

$$n l n l^2 n \dots l^{k-1} n f(n) < r^{l^k n};$$

therefore $l[n l n l^2 n \dots l^{k-1} n f(n)] < l^k n \log_a r$;

therefore $\frac{l[n l n l^2 n \dots l^{k-1} n f(n)]}{l^k n} < \log_a r$;

therefore $\frac{l[n l n l^2 n \dots l^{k-1} n f(n)]}{l^k n} < 0$.

It is divergent if this expression > 0 .

In the case where it is $= 0$, we cannot assert anything.

This is Chrystal, *Algebra*, Vol. II., p. 112.

It is well known that the series $\frac{1}{n^{1+\alpha}}$ is convergent if α be positive, and divergent if α be negative. Hence, by the above rule, the following series are convergent or divergent according as α is positive or negative,

$$\frac{1}{n^{1+\alpha}} \quad \frac{1}{n (ln)^{1+\alpha}}, \quad \frac{1}{n ln (l^2 n)^{1+\alpha}}, \quad \dots \dots \frac{1}{n ln l^2 n \dots l^{k-1} n (l^k n)^{1+\alpha}}.$$

This is Chrystal, p. 113.

Compare $f(n)$ with $\frac{\gamma^{k n}}{n ln l^2 n \dots l^{k-1} n}$; therefore compare

$$\frac{f(n+1)}{f(n)} \text{ with } \frac{n ln l^2 n \dots l^{k-1} n}{(n+1) l (n+1) l^2 (n+1) \dots l^{k-1} (n+1)} \frac{\gamma^{k(n+1)}}{\gamma^{k(n)}};$$

or, denoting $n ln l^2 n \dots l^k n$ by $P_k(n)$, therefore compare

$$\frac{f(n+1) P_{k-1}(n+1)}{f(n) P_{k-1}(n)} \text{ with } \gamma^{k(n+1) - l^k(n)},$$

therefore $f(n)$ is convergent, if

$$\frac{f(n+1) P_{k-1}(n+1)}{f(n) P_{k-1}(n)} < 1,$$

i.e., according as $\frac{f(n+1)}{f(n)} P_{k-1}(n+1) - P_{k-1}(n)$ is negative or positive.

This is Chrystal, p. 115.

It appears to be not generally known that in Cauchy's condensation test a may be any quantity greater than unity. This was discovered by Kohn. Consequently it must be noted that the base of the logarithms above may be any quantity greater than unity. Also, from and after a certain value of n , $f(n)$ must be positive and continually diminish as n increases.