LXIV. Note on the measurement of small inductances and capacities, and on a standard of small inductance

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force above, as acting on the interfacial current-sheet, it is not difficult to verify that when the incidence is oblique, the incident, reflected, and refracted wave-trains exert independently on the reflecting surface their full oblique thrusts in their own directions of propagation, as is implied in Prof. Poynting's calculations referred to at the beginning.

The result here verified, that motion of a material body does not affect the pressure exerted on it by the ambient radiation, has been rejected by Prof. Poynting in a later postscript added to the memoir above referred to, on the ground that radiation shot out of a radiator A into a moving absorber B would, according to it, alter the store of momentum of the two bodies. But if the bodies are in thermal equilibrium, other compensating events are at the same time occurring, viz. the absorber B is also radiating towards A. And indeed if the temperatures of A and B are unequal, the aggregate momentum of both admittedly does change on account of their radiation.

If the present argument is right, the view which considers a ray to be a simple carrier of momentum from the one body to the other cannot therefore be maintained.

It may be noticed, in connexion with p. 584 supra, that for the same amplitude of ionic excursions in the vibrating molecule, as determined by its maximum electric moment, and for the same periodic time, it follows from Hertz's formulae for a simple radiator, and may be generalized by the theory of dimensions, that the radiation emitted per unit time is proportional to the refractive index of the surrounding medium, and therefore the equilibrium-density of the radiation in that medium is proportional to the square of the same index, in accordance with Balfour Stewart's law derived from the doctrine of equilibrium of exchanges between sources at uniform temperature.[1]


LAST year a paper was read before the Physical Society by the present writer and Mr. W. C. Clinton, on the "Measurement of Small Capacities and Inductances" †.

In that paper we described two forms of motor-driven commutator for the measurement of small capacities and

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inductances. Since that date, these appliances have been extensively used for this purpose in the Pender Electrical Laboratory at the University College.

In the measurement of small inductances lying in value between 100,000 and 10,000 centimetres, it is essential to use in connexion with the modification of the Anderson method*, described in our paper, a very sensitive galvanometer; and when small inductances of this order are being measured we have since found that the stray field from the motor employed to drive the commutator produces, by a dynamo action, a small electromotive force in the commutator which makes itself evident in the galvanometer circuit, and so gives rise to an irregularity, vitiating the results. The remedy for this, of course, is to employ an enclosed iron-clad motor, or else to place the commutator at a greater distance from the motor, connecting the two by a long shaft. This has already been done and has been found to be effective.

Meanwhile, in the course of the experiments to overcome these difficulties, the attempt was made to use a telephone in place of the galvanometer and a simple interrupted current in the battery-circuit. In the bridge arrangement described by Prof. Anderson (loc. cit.) we substituted an ordinary buzzer in the battery-circuit to interrupt the current at the rate of about 100 per second, and in the bridge-circuit an ordinary Bell telephone for the galvanometer, the commutator being abolished. Under these circumstances, it was found that an observer with sharp hearing could obtain a very good balance when a coil having small inductance was placed in one arm of the bridge, and a condenser of suitable capacity placed as described by Prof. Anderson (see fig. 1).

Mr. J. C. Shields, who has been engaged in experiments on this matter in the Pender Laboratory, found that with this arrangement he could make very quick and fairly accurate determinations of small inductances, the accuracy of the reading being determined by the limits within which a value could be assigned to \( r \) in the equation given by Prof. Anderson, viz. :

\[
L = C\left\{ r(R + S) + RQ \right\},
\]

the value of \( r \) being that of a resistance inserted in the bridge-circuit, which is varied until no sound is heard in the telephone.

In the above equation \( L \) is the inductance and \( R \) the resistance of the coil being measured, \( C \) the capacity of the condenser, and \( S \) and \( Q \) the resistances of the adjacent

and opposite bridge-arms, and \( r \) the resistance inserted in series with the telephone in the bridge-circuit.

When \( r \) was adjusted to produce silence in the telephone, it was found that variations to the extent of about 1 per cent. either way, and sometimes much less, caused the sound to reappear in the telephone, and hence gave the limits within which the inductance \( L \) could be determined.

In the experiments here described, the capacity generally employed consisted of two leyden-jars, the joint capacity of which had been determined carefully with the Fleming-Clinton commutator, and found to have the value 0.00272 microfarad. The tests of this telephone method were made by Mr. J. C. Shields on a number of coils of silk-covered copper wire, each of which consisted of one layer of the wire wound uniformly and in closely adjacent turns upon a wooden or glass circular-sectioned rod. One coil, much employed, consisted of a wooden rod about two metres in length wound over as above described with one layer of no. 32 s.w.g. wire in closely adjacent turns. The mean diameter of one circular turn of this wire was 4.096 centimetres, and the length of the solenoid or spiral wire was 200.3 centimetres, and the number of turns of wire 5000 in all, and hence
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the number of turns per centimetre of length of the spiral was 24.96.

This long coil belonged to a resonance apparatus designed by Seibt, and is hence alluded to as the long Seibt coil. The resistance of the wire on this coil was about 152 ohms, and it was connected to a Wheatstone’s bridge (as shown in fig. 1), the other arms of which are denoted by P, Q, and S.

The arrangement of apparatus used, consisted therefore of an ordinary Post-Office plug Wheatstone’s bridge having the spiral of which the inductance was to be determined connected to it. The battery-circuit contained the buzzer, and the bridge-circuit a telephone in series with a plug resistance-box, affording values for r. The condenser consisted of one or more leyden-jars or a mica condenser. The steady balance was obtained first in the usual way with a galvanometer and steady current.

The following Table gives the values of the bridge-arms, the bridge resistance r, the capacity used, and the inductance L calculated from the formula given by Anderson.

The Table contains two sets of measurements, one set marked A, made by Mr. Shields with the Fleming-Clinton commutator, and the other marked B, made with the telephone and buzzer as above described.

**Table I.—Results of Inductance Measurements of a Long Coil, having a Dimension ratio of 50 : 1.**

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R.</th>
<th>S.</th>
<th>r</th>
<th>C. in mfd.</th>
<th>L, observed in cms.</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1000</td>
<td>152.26</td>
<td>152.26</td>
<td>4280</td>
<td>0.00272</td>
<td>19,900,000</td>
</tr>
<tr>
<td>100</td>
<td>1000</td>
<td>152.31</td>
<td>152.31</td>
<td>7675</td>
<td>0.00149</td>
<td>19,400,000</td>
</tr>
<tr>
<td>100</td>
<td>1000</td>
<td>151.1</td>
<td>161.1</td>
<td>4400+50</td>
<td>0.00272</td>
<td>20,300,000</td>
</tr>
<tr>
<td>1000</td>
<td>10,000</td>
<td>151.5</td>
<td>151.5</td>
<td>3350+50</td>
<td>0.00272</td>
<td>19,200,000</td>
</tr>
<tr>
<td>100</td>
<td>10,000</td>
<td>151.5</td>
<td>151.5</td>
<td>365+5</td>
<td>0.00272</td>
<td>19,900,000</td>
</tr>
<tr>
<td>1000</td>
<td>10,000</td>
<td>152</td>
<td>152</td>
<td>24200+100</td>
<td>0.00272</td>
<td>20,100,000</td>
</tr>
<tr>
<td>100</td>
<td>1000</td>
<td>151.4</td>
<td>151.4</td>
<td>4400+50</td>
<td>0.00272</td>
<td>20,300,000</td>
</tr>
<tr>
<td>1000</td>
<td>10,000</td>
<td>151.4</td>
<td>151.4</td>
<td>3230+20</td>
<td>0.00272</td>
<td>19,200,000</td>
</tr>
<tr>
<td>100</td>
<td>1000</td>
<td>151.7</td>
<td>151.7</td>
<td>455+5</td>
<td>0.00272</td>
<td>20,300,000</td>
</tr>
<tr>
<td>1000</td>
<td>10,000</td>
<td>151.7</td>
<td>151.7</td>
<td>365+5</td>
<td>0.00272</td>
<td>19,900,000</td>
</tr>
<tr>
<td>100</td>
<td>1000</td>
<td>152</td>
<td>152</td>
<td>217+1</td>
<td>0.0256</td>
<td>20,800,000</td>
</tr>
</tbody>
</table>

Mean of A readings = 19.7 \times 10^6 cms.
Mean of B readings = 19.9 \times 10^6 cms.
Value calculated from the formula \( L = \frac{\pi D r (\pi DN)}{2} \) = 20.6 \times 10^6 cms.

By numerous observations on coils of this kind, sometimes 50 diameters long or even less, the wire being wound in a single layer and in closely adjacent turns, the writer has
found that a very simple formula enables the inductance to be calculated very approximately.

If D is the mean diameter of the axis of one circular turn of the wire forming the solenoid, and if l is the length of the solenoid, and N the total number of turns on the solenoid, then it is clear that the magnetic force per unit of current in the central portions of the interior of the coil is equal to \(4\pi N/l\), and the total self-linked flux is \(4\pi N^2 D^2/4l\). Hence, if we neglect the variation of flux at the ends and consider that all up the coil it has the same value as at the centre, the inductance \(L\) of the coil is given by the formula

\[
L = \pi^2 D^2 l^2 ;
\]

where \(t\) is written for the turns per unit of length of the solenoid \(= N/l\).

Hence the above equation may be written

\[
L = (\pi Dt)(\pi DN).
\]

The first factor \(\pi Dt\) is the length of wire wound on one unit of length of the cylindrical rod used as a core. This factor is of no dimensions and is a mere numeric.

The second factor \(\pi DN\) is the total length of wire used. Hence, we have for such a solenoid:

\[
\text{Inductance} = \left(\frac{\text{length of wire per unit length of solenoid}}{\text{total length of wire used}}\right) \times (\pi DN).
\]

Applying this rule to the above mentioned long Seibt coil, we have for the total length of wire used

\[
3\cdot1416 \times 4\cdot096 \times 5000 \text{ cms.} = 64340 \text{ cms.,}
\]

and the length wound on per centimetre of the rod is

\[
3\cdot1416 \times 4\cdot096 \times 5000 \div 200\cdot3 = 321 \text{ cms.,}
\]

and hence

\[
L = 20\cdot6 \times 10^6 \text{ cms nearly, or 20\cdot6 millihenrys.}
\]

It will be seen from Table I. that the average observed value of \(L\) for this coil, as calculated from nine bridge-readings made with the telephone method, is \(19\cdot9 \times 10^6\) cms. nearly; and hence the inductance calculated by the above rule agrees within 3\cdot5 per cent. with that obtained by actual measurement.

This rule affords a very simple and convenient guide for constructing small known inductances. All that it is necessary to do is to wind silk-covered copper wire in one layer and in closely adjacent turns on a glass rod of measured diameter, and make the length of the solenoid at least 50 times the diameter. The inductance can then be predetermined
of Small Inductances and Capacities.

to within say 2 per cent. and adjusted to be of required value by varying the length and diameter of the rod. A series of tuning inductances can in this manner be easily made, which, when associated with known capacities, give circuits having known oscillation frequencies.

When the coils have a smaller dimension-ratio, being only 6 or 7 diameters long, then the above rule always gives, as it should do, an inductance value which is too large, but even in the case of such short coils not by very many per cent. This may be seen from the inductance measurements made by the telephone method on four short coils called for distinction A, B, C, and D, which had the following dimensions and windings. The coils were made of silk-covered no. 36 or no. 38 wire wound on glass tubes.

<table>
<thead>
<tr>
<th>Coil</th>
<th>Length of Solenoid = l</th>
<th>Diameter of Solenoid = D</th>
<th>Number of Windings = N</th>
<th>Calculated Inductance = L from the formula $L = \frac{\mu_0 D t}{\pi DN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20.5 cms.</td>
<td>3.3 cms.</td>
<td>513</td>
<td>1,379,800 cms.</td>
</tr>
<tr>
<td>B</td>
<td>20.4 cms.</td>
<td>3.5 cms.</td>
<td>813</td>
<td>3,918,200 cms.</td>
</tr>
<tr>
<td>C</td>
<td>20.8 cms.</td>
<td>3.4 cms.</td>
<td>847</td>
<td>3,935,200 cms.</td>
</tr>
<tr>
<td>D</td>
<td>20.9 cms.</td>
<td>3.7 cms.</td>
<td>850</td>
<td>4,670,800 cms.</td>
</tr>
</tbody>
</table>

The inductance of these coils was measured with the bridge and telephone, using a capacity of 0.00272 microfarad and the bridge values were as in Table III. (p. 592).

The mean values of the observed inductances of each coil differ from the extreme values in some cases by less than 1 per cent., and in no case by more than 2 per cent. The mean observed values are less (as they should be) than the values calculated by the formula $L = \frac{\mu_0 D t}{\pi DN}$ by about 6 or 7 per cent.

It has always been found that when the dimension ratio is as much as 50:1 or more, then there is a close agreement between the observed and calculated value of the inductance. The above described telephone and buzzer modification of the Anderson method can be therefore used to calculate the value of the capacity used in the bridge, assuming the calculated value of the inductance of the inductive arm. Thus, if in the measurements recorded in Table I. we take the inductance
TABLE III.—Results of Inductance Measurements on Short Coils, having a Dimension ratio of 7 : 1.

(Telephone and buzzer in combination with Anderson's method.)

<table>
<thead>
<tr>
<th>Coil</th>
<th>Bridge-arms in ohms.</th>
<th>r. ohms.</th>
<th>C. mfd.s.</th>
<th>L. Inductance observed, greatest deviation from mean.</th>
<th>Mean value of L. calculated by formula.</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. Q. R. S.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>10 1000 12-23 1223</td>
<td>384±2</td>
<td>.90272</td>
<td>1,321,000 1,295,000</td>
<td>1,379,800</td>
</tr>
<tr>
<td>B</td>
<td>100 1000 12-73 14</td>
<td>164±1</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
</tr>
<tr>
<td></td>
<td>100 1000 12-00 13</td>
<td>173±1</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
</tr>
<tr>
<td></td>
<td>10 1000 12-22 1222</td>
<td>370±2</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
</tr>
<tr>
<td>C</td>
<td>100 1000 12-00 13</td>
<td>173±1</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
</tr>
<tr>
<td></td>
<td>100 1000 12-73 14</td>
<td>164±1</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
</tr>
<tr>
<td></td>
<td>10 1000 12-22 1222</td>
<td>370±2</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
</tr>
<tr>
<td>D</td>
<td>100 1000 12-00 13</td>
<td>173±1</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
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<tr>
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<td>164±1</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
</tr>
<tr>
<td></td>
<td>10 1000 12-22 1222</td>
<td>370±2</td>
<td>.90272</td>
<td>1,288,000 1,268,000</td>
<td>1,385,900</td>
</tr>
</tbody>
</table>

of the long coil to be 20.6 millihenrys, and use the bridge readings to calculate the value of the capacity of the leyden-jars, we find it to be 0.00282 mfd. The values of this capacity measured by the intermittent discharge method was found to be 0.00272 mfd. Hence, the bridge-telephone-buzzer method, which needs no special appliances other than those found in every laboratory, provides a means of determining with fair accuracy quite small capacities very easily, and can be applied to the measurement of the capacities of telegraph wires or aerials or very short lengths of cables.

The method has been used for the calibration of a variable inductance made for certain resonance experiments. This inductance standard was constructed as follows:

A cylinder of boxwood about 10 centimetres in diameter and 45 cms. in length is provided at the ends with brass plates carrying centre pins, by means of which it is suspended in bearings. This cylinder is cut with a screw of 6 threads to the inch, and in this is wound a no. 14 s.w.g. copper wire, the ends of which are attached to the end plates. Parallel with the cylinder (see fig. 2) is fixed a brass rod about 1 cm. in diameter on which slides a travelling bar, the end of which...
carries a brush which makes contact with the copper spiral at one place.

A spring presses against one of the brass end plates of the cylinder and carries a terminal. The sliding bar can be moved along to any position, its setting being determined by a scale, and the amount of inductance included between the two terminals can therefore be varied. The sliding bar can be lifted and moved quickly from one position to another, or moved slowly by turning the cylinder, in which case a gradual variation of inductance takes place.

The inductance of the whole spiral was measured by the telephone-buzzer-bridge method, and found to be 227,000 cms., and it was also measured for every 5 or 10 turns and found to be as shown in Table IV.

The value of the inductance predetermined by the formula above is 246,000 cms. for 100 turns, thus showing about nine per cent. excess over the real value. The above-mentioned formula cannot of course be applied to the case of a spiral having such a small dimension ratio as 4.5 : 1. The scale of
Measurement of Small Inductances and Capacities.

the instrument has been divided to read in microhenrys directly.

**Table IV.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>60</td>
<td>124,000</td>
</tr>
<tr>
<td>10</td>
<td>10,000</td>
<td>70</td>
<td>148,000</td>
</tr>
<tr>
<td>20</td>
<td>28,000</td>
<td>80</td>
<td>172,000</td>
</tr>
<tr>
<td>30</td>
<td>50,000</td>
<td>90</td>
<td>196,000</td>
</tr>
<tr>
<td>40</td>
<td>75,000</td>
<td>100</td>
<td>220,000</td>
</tr>
<tr>
<td>50</td>
<td>100,000</td>
<td>Total</td>
<td>227,000</td>
</tr>
</tbody>
</table>

The above observations have been set out in a curve (see fig. 3).

Such a graduated standard of inductance is useful in tuning wireless telegraph-circuits and in experiments on resonance.

By the above described method the measurement of inductances, even as small as 2 or 3 microhenrys, is reduced to an extremely simple straightforward method, capable of being carried out without any special appliances, other than those found in every testing-room and laboratory. By its aid inductance as small as 10 microhenrys can be measured with an accuracy of about 5 per cent., and inductances of the order of a millihenry with an accuracy of at least 1 per cent.

A good method of constructing a small inductance of known value is to stretch two round wires of diameter $d$ cms. parallel to each other at a distance $D$ cms. apart. If these are short-circuited at the far end by a cross bar, first at one
place and then at another nearer place closer by a distance $l$ cms., then the difference of the inductances measured in the two cases has a value $L$ such that:

$$L = 2l \left\{ 4.606 \log_{10} \frac{2D}{d} + 1 \right\} \text{ cms.}$$

This formula is easily derived from one given by Maxwell. It is a simple matter to obtain in this manner an inductance having a value say of 30,000 cms., and by its aid to test methods of measurement.

By the use of a long solenoid having an inductance predetermined by the rule given above, the method can be used for the determination of small capacities of the order of a thousandth of a microfarad. It is to be hoped, therefore, that in future those who describe experiments or appliances such as wireless telegraphy arrangements in which such small capacities or inductances are used, will cease from the practice of speaking of jars with so many "square inches or square centimetres of coated surface," and take the slight trouble to measure and record the capacity and inductances, and in this way afford the means of testing theories of the operation of the appliances.

It can hardly be said that the practical problem of measuring with great accuracy very small inductances of the order of 1 microhenry or less has been satisfactorily solved.

Probably in the case of inductances of very low resistance the best method to adopt would be to measure the fall of potential down the conductor first, with a continuous current, and then with a high-frequency sine form alternating current. Professor W. Stroud and Mr. J. H. Oates recently described a bridge method employing alternating currents, which they stated could be employed for the measurement of very small inductances.


There are many occasions on which it becomes necessary to measure a small alternating current of the order of one-hundredth of an ampere.

In taking the magnetizing currents of small transformers


† Communicated by the Physical Society; read March 25, 1904.