Auroral and Sun-spot Frequencies Contrasted

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with alternating and direct currents. The alloy selected was ferro-nickel, as it was expected that this would show the effect strongly, as the constituents give a strong thermo-electric E.M.F. A current was sent through a long ferro-nickel wire doubled back on itself, and this current was measured by a Kelvin balance, while the voltage on the ends of the wire was measured by a Kelvin electrostatic voltmeter (reading to 1 in 1000). No difference could be detected between the apparent resistance with direct current and with alternating current of frequencies up to 80 per second (the usual correction for contact difference being found and applied).

Mr. W. DUDDELL suggested that the Author should proceed with his experiments using very much higher frequencies.

XXX. *Auroral and Sun-spot Frequencies Contrasted.* By C. CHREE, *Sc.D., LL.D., F.R.S.**

§ 1. DURING several recent investigations I have had occasion to contrast the annual variation in years of many and in years of few sun-spots of elements such as the diurnal range of the magnetic declination, or the frequency of occurrence of magnetic storms. The formula first advanced by Wolf

$$R = a + bS \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

as connecting R, the range in the mean diurnal inequality of declination throughout the year, with S the corresponding sun-spot frequency—a and b being constants—can be applied with considerable accuracy to the range in individual months of the year, and to magnetic inclination, horizontal force and vertical force, as well as declination. But taking any one element, a and b are different for the different months of the year, and b/a is in general decidedly larger for winter than for summer.

Suppose, now, that dashed letters refer to a winter, undashed to a summer month, and that suffixes 1 and 2 relate

* Read November 23, 1906.

to two years in which sun-spots are respectively many and few. Then for the ratio of the ranges in the summer and the winter month concerned, we have in the year of many sun-spots,

$$R_1/R_1' = (a/a')\left(1+\frac{b}{a}S_1\right) \div \left(1+\frac{b'}{a'}S_1\right),$$

supposing for simplicity the sun-spot frequency the same for the two months. On the same hypothesis, we have for the corresponding ratio in the year when sun-spots are few,

$$R_2/R_2' = (a/a')\left(1+\frac{b}{a}S_2\right) \div \left(1+\frac{b'}{a'}S_2\right).$$

From these two equations we at once deduce

$$\frac{(R_2/R_2')-(R_1/R_1')}{R_1/R_1'} = \left(\frac{b'}{a'}-\frac{b}{a}\right)(S_1-S_2) \div \left\{\left(1+\frac{b}{a}S_1\right)\left(1+\frac{b'}{a'}S_2\right)\right\}.$$

As already mentioned, observation shows that b'/a' exceeds b/a, and by hypothesis S_1-S_2 is positive, thus

$$R_2/R_2' > R_1/R_1'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In temperate latitudes, whether sun-spots be many or few, the diurnal range of any magnetic element is larger in summer than in winter, *i.e.* R_1 exceeds R_1' and R_2 exceeds R_2'. Thus (2) shows that *relatively* considered the diurnal range is more variable throughout the year when sun-spots are few than when they are many. In other words, if the annual change of the diurnal range be illustrated by a curve whose ordinates represent the ratios borne by the ranges in individual months to their arithmetic mean for the twelve months, the maximum and minimum ordinates differ more when the year selected is one of few than when it is one of many sun-spots.

§ 2. Again, taking a list of the more considerable magnetic disturbances recorded at Greenwich from 1848 to 1903, as given by Mr. W. Maunder, I obtained the following figures * for the relative frequency at different seasons of the year, treating separately the fourteen years of largest (S max.)

* Phil. Mag. Sept. 1905, p. 314.

and the fifteen years of smallest (S min.) sun-spot frequency:

Winter. November to February.		Equinox. March, April, Sept. Oct.		Summer. May to August.	
S max.	S min.	S max.	S min.	S max.	S min.
35	28	38	48	27	24

The figures denote percentages of the totals for the whole year. The average *absolute* numbers of storms per annum were

Whole period.	S max. years.	S min. years.
13·0	18·4	8·5

With increased sun-spot frequency, the absolute number of storms increased in all three seasons of the year; but relatively considered the increase was least in the equinoctial months—the season when magnetic storms are most numerous at Greenwich—and the tendency obviously was towards a more uniform distribution throughout the year. The phenomenon is thus analogous to that described above in the case of the regular diurnal range.

§ 3. In temperate latitudes, as is well known, magnetic storms of any considerable intensity are usually associated with auroras. It was thus of interest to determine whether auroral frequency showed phenomena corresponding to those just described in Terrestrial Magnetism. It has long been known that auroral frequency, as observed in temperate latitudes, varies in a general way with sun-spot frequency. The results obtained, however, have not shown a very exact correspondence between the years of maximum and minimum in the two classes of phenomena. In the case of sun-spot frequency—except as regards data for recent years—practically the only source available has been the data published by Wolf and Wolfer, which extend back to 1749. It is improbable that the unit in Wolfer's latest table * represents an absolutely unchanging value throughout the whole period, but great care has been taken to make the table as homogeneous as possible, and the epochs it gives for the occurrence of sun-spot maximum and minimum are presumably, in at least the great majority of cases, very approximately correct.

* *Met. Zeit.* 1902, vol. xix. p. 195.

Auroral data are exposed to many more uncertainties. The observed frequency varies enormously at different parts of the Earth, and the number of auroras recorded in any specified area is largely dependent on the provision made for observing and recording them. Conspicuous auroras are unlikely to escape notice in populous countries where they are rare occurrences, but in Arctic latitudes where auroras are common many doubtless fail to be recorded. With the increase of population and the development of means of communication characteristic of the last 100 years, there has no doubt been a tendency to an increase in the proportion of auroras which come to be recorded.

Thus auroral frequency is a quantity which is certainly not expressed in terms of an invariable unit; and the various tables which have been published show irregularities due to temporary and local causes, whose disturbing influence it is practically impossible to assess. In the following investigations the methods adopted aim at reducing to a minimum the effect of the various uncertainties.

§ 4. The sun-spot frequencies made use of are derived exclusively from Wolfer's table, which gives data for each individual month during the 153 years 1749 to 1901. The auroral frequencies are from two sources, viz.: "Catalog der in Norwegen bis Juni 1878 beobachteten Nordlichter zusammengestellt von Sophus Tromholt, herausgegeben von J. Fr. Schroeter" (Kristiania, 1902), and Joseph Lovering's "On the Periodicity of the Aurora Borealis" (Mem. American Academy, New Series, vol. x. 1868).

Of the several tables in the former work, that employed is Table E, pp. 414–417, which gives auroral frequencies derived from the whole of Scandinavia from July 1761 to June 1878. In this table Schroeter has combined Tromholt's results for Norway with those of Rubenson for Sweden. In the original the yearly totals are for years commencing in July. In order, however, to obtain results more strictly comparable with Wolfer's mean annual sun-spot frequencies, I have calculated from the Scandinavian monthly totals data for years commencing in January. In addition to yearly totals—from July to June—for the whole of Scandinavia, Tromholt's Table E gives yearly totals for five subdivisions

of the country, numbered I. to V. according to latitude. I. includes all districts north of 68°·5, II. extends from 68°·5 to 65°, III. from 65° to 61°·5, IV. from 61°·5 to 58°·5, while V. includes the extreme south of Scandinavia from 58°·5 to 55°.

For a considerable time subsequent to 1761, observations from district I. were very few, a fact due probably more to lack of observers than anything else. This possesses some importance for the following reason. The annual variation in auroral frequency is largely dependent on the fact that aurora is seldom vivid enough to be visible until the sun is several degrees below the horizon. In high latitudes there is daylight throughout the whole 24 hours near midsummer, and no daylight near midwinter, and the auroral frequency in these regions, as was pointed out many years ago by Lovering, has a single maximum near midwinter, and a single minimum answering to a total absence of aurora near midsummer *. This state of matters is at least approached in district I., and to a lesser extent in district II. (*cf.* loc. cit. Table G, p. 420). Further south in Scandinavia the annual variation is similar to that in England, showing two maxima near the equinoxes, a principal minimum at midsummer, and a secondary minimum at midwinter. The mean annual variation deduced for the whole of Scandinavia will clearly depend to some extent on how far the several districts contribute to the general result. Assuming an increasing relative contribution from district I., if annual variations be calculated from two different periods, one may not unreasonably expect the later period to show the equinoctial maxima less prominently and the midsummer minimum more prominently than the earlier period. This is one of my reasons for contrasting one period of a special type with two of an opposite type, the one earlier the other later.

Lovering gives annual variations for a number of separate stations. Most of these, however, are based on too few years' observations to suit the present enquiry.

Of the data for separate stations or districts, I propose to use only those for New York State (l.c. p. 181). These extend over 26 years, including 1205 separate observations,

* *Cf.* A. Angot's 'Les Aurores Polaires,' Paris, 1895, p. 125.

and represent a latitude much lower than that of Scandinavia. The other data employed are from Lovering's General Catalogue, pp. 195–200. This comes down to 1864, and extends to earlier than the 14th century. Though enumerating nearly 10,000 auroras it is probably, judging by Tromholt's figures, very far from complete. The data are doubtless of a very heterogeneous character, and the same precautions appeared necessary as in the case of the Scandinavian data.

Recently Prof. Schuster * has gone very fully into the existence of periodic variations in Wolfer's sun-spot frequencies, and has concluded that there is evidence of the existence not merely of the ordinarily recognized period of 11·125 years, but of others which like it are submultiples of a period whose most probable value is 33·375 years. This is one of the reasons why I have dealt with three successive 33-year periods, viz. 1761–1793, 1794–1826, and 1827–1859.

§ 5. Before comparing sun-spots and auroras, I would draw attention to some features of Table I., which gives mean sun-spot frequencies for the several months of the year, as calculated by me from Wolfer's table for a number of specified combinations of years. It has been remarked by Mr. Ellis † —who based his remarks on Wolfer's data *for groups of 3 months* (February to April, &c.)—that sun-spot frequency shows no real annual period. The results in the first line of Table I. cannot be said to be decisive against the existence of a small annual term, though inconsistent with the existence of a large term having this period. The extent of the difference between the mean results for the several months is more easily realized in Table II., which expresses the monthly values in Table I. as percentages of the arithmetic mean for the 12 months. One would, I think, hardly have anticipated, in means based on 153 years' data, the difference of 6½ per cent. shown between the values for January and May. On the other hand, if there were a true annual period, one would expect the percentage figures in Table II. for the three

* Proc. Roy. Soc. A, vol. lxxvii. p. 145.

† Monthly Notices R. A. S. vol. lx. p. 142.

TABLE I.—Wolf and Wolfer's Sun-spot Frequencies.

		Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
153 years	1749 to 1901.....	44·41	46·67	45·40	46·13	47·41	46·02	45·46	45·83	45·91	46 52	46·36	46·23	46·04
33 „	1761 to 1793.....	63·94	64·58	57·93	63·80	70·92	65·70	63·79	62·56	68·21	66·97	71·11	67·46	65·59
„ „	1794 to 1826.....	20·32	21·67	20·76	19·57	18·67	20·21	21·19	19·68	17·73	20·99	19·43	23·13	20·28
„ „	1827 to 1859.....	55 53	56·95	56·95	56·28	54·76	53·09	50·94	55·67	57·38	60·96	55·43	58·86	56·07
39 years S max.	1760 to 1894.....	81·76	85·78	84·13	89·24	89·66	89·21	88·35	89 03	88·04	90·93	90·23	90·08	88·05
„ „ S min.	1754 to 1890.....	13·63	14·36	14·44	12·09	13·06	11·34	9·87	10·26	9·63	12·45	12·95	12·12	12·19
33 years S max.	1760 to 1872.....	83 76	88·16	88·36	90·52	92·98	92·21	90·75	91·79	92·57	94·84	94·44	94·42	91·25
„ „ S min.	1754 to 1867.....	14·67	16·01	15·87	13·29	14·11	12·21	10·52	10·65	9·55	13·64	13·72	13·38	13·14

TABLE II.—Sun-spot Frequencies. Percentages of mean year.

		Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Winter.	Summer.
153 years	1749 to 1901.....	96·5	101·4	98·6	100·2	103·0	100·0	98·8	99·6	99·7	101·1	100·7	100·4	99·8	100·2
33 „	1761 to 1793.....	97·5	98·5	88·3	97·3	108·1	100·2	97·3	95·4	104·0	102·1	108·4	102·9	99·6	100·4
„ „	1794 to 1826.....	100·2	106·8	102·4	96·5	92·1	99·7	104·5	97·0	87·4	103·5	95·8	114·0	103·8	96·2
„ „	1827 to 1859.....	99·0	101·6	101·6	100·4	97·7	94·7	90·9	99·3	102·3	108·7	98·9	105·0	102·5	97·5
39 years S max.	1760 to 1894.....	92·9	97·4	95·6	101·4	101·8	101·3	100·3	101·1	100·0	103·3	102·5	102·3	99·0	101·0
„ „ S min.	1754 to 1890.....	111·9	117·9	118·5	99·2	107·2	93·1	81·0	84·2	79·0	102·2	106·3	99·5	109·4	90·6
33 years S max.	1760 to 1872.....	91·8	96·6	96·8	99·2	101·9	101·1	99·4	100·6	101·5	103·9	103·5	103·5	99·4	100·6
„ „ S min.	1754 to 1867.....	111·7	121·9	120·8	101·2	107·4	93·0	80·1	81·1	72·7	103 8	104·5	101·9	110·7	89·3

successive 33-years periods to resemble one another more closely than they actually do.

The monthly means in these three periods show very considerable differences; in the earliest period, for instance, the difference between the March and November means amounts to 20 per cent. of the mean value for the epoch.

The central period 1794–1826 comprises that period of 33 consecutive years which gives a minimum mean sun-spot frequency. That mean is only a third of the mean appropriate to the combined periods 1761–93 and 1827–59.

As a rule, three successive years of conspicuously high frequency occur at each sun-spot maximum, and three of conspicuously low frequency at each minimum. Wolfer's table includes 13 groups of these 3 extreme years of sun-spot maximum, and 13 of sun-spot minimum. The respective means from these 39 years of maximum and 39 years of minimum form the fifth and sixth rows of Tables I. and II. The last two lines in these Tables refer to shorter groups of 33 years of sun-spot maximum and 33 of minimum which correspond more closely to the auroral data presently to be described.

§ 6. Probably the simplest way of investigating the relationship between sun-spot frequency and the magnitude of any element is to form two mean values for the element, the one corresponding to years of many, the other to years of few sun-spots, and then to assume that the difference between these means depends on the corresponding difference in sun-spot frequency. In applying this method to data from the whole period covered by Wolfer's table, one would employ the sun-spot frequencies given in the 5th and 6th rows of Table I. Doing so, we should get differences of sun-spot frequency varying from 81·76–13·63, or 68·13, in January, to 78·77 in August. The existence of so large a difference between the sun-spot data for January and August is immaterial, provided there is a direct connexion, which possesses no lag, between sun-spot frequency and the element concerned. If, however, the connexion is of a less simple character, for instance if the element depends on the sun-spot frequency for some months previously, the application of the above method would lead to an overestimate of the influence of

TABLE III.—Auroral Frequency. Annual Variation.

		Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Total Auroras.	Average per year.
Scandinavian,	1761–1877	12·40	11·78	13·18	7·82	0·93	0·04	0·26	4·17	11·87	13·52	12·23	11·80	9243	79·0
	1761–1793	10·59	10·55	12·62	9·71	1·99	0·08	0·60	6·13	14·37	13·02	10·51	9·83	2512	76·1
,,	1827–1859	13·29	11·42	12·07	7·54	0·43	0·00	0·18	3·66	11·67	14·26	12·68	12·78	2784	84·4
,,	Two periods combined. ...	11·94	10·99	12·34	8·63	1·21	0·04	0·39	4·90	13·02	13·64	11·60	11·30	5296	80·2
,,	1794–1826	13·96	14·49	14·65	4·52	0·31	0·00	0·08	1·92	9·12	13·42	14·49	13·03	1304	39·5
Lovering's General Catalogue.	Up to 1864	9·54	9·73	11·96	9·37	5·09	3·22	3·72	6·45	10·39	11·37	9·87	9·29	9766	...
,,	1761–1793	7·81	8·14	11·44	11·03	8·05	4·67	5·49	7·19	10·24	10·33	8·47	7·15	2421	73·4
,,	1827–1859	10·61	9·31	10·44	8·46	4·95	3·72	4·62	6·13	9·79	10·54	10·18	11·26	3900	118·2
,,	Two periods combined. ...	9·21	8·73	10·92	9·75	6·50	4·20	5·05	6·66	10·01	10·44	9·33	9·21	6321	95·8
,,	1794–1826	10·73	12·44	14·48	11·93	4·26	1·36	1·02	3·58	7·84	9·03	10·90	12·44	587	17·8

sun-spot frequency in January and an underestimate in August. If we use more exact methods, *e. g.* the method of least squares,—still assuming purely synchronous variation—the above source of uncertainty is less easily recognized, but it exists all the same. Considered absolutely, the difference between the mean monthly values in Table I. is greater for the 39 or the 33 years of S max. than for the corresponding group of S min. years; but relatively considered—*cf.* Table II.—the variability from month to month is greater for the S min. group. Thus in the 39 years of S min. the means for March and September differ by 39 per cent. of the mean from the 12 months. Moreover in the S min. groups of years there is a decided difference—some 20 per cent.—between the means derived from the 6 winter and the 6 summer months. There is a smaller difference, but in the same direction, between the winter and summer means from the 33-year period 1794–1826 remarkable for its low average sun-spot frequency. This is unquestionably somewhat suggestive of an appreciable real annual period in sun-spot frequency in years when sun-spots are few; but whereas in the 39 or 33 years of S min. the mean frequency is large for May and November, and small for July, in the period 1794–1826 the exact opposite is seen. Again, an appreciable excess in the winter over the summer mean also appears in the period 1827–59, when the average sun-spot frequency considerably exceeded the average from the whole 153 years.

§ 7. Proceeding to Table III., we have in the first line the annual variation of auroral frequency in Scandinavia as derived from 117 years. The monthly values represent percentages of the value for the whole year. The largest values occur in October and March, and a secondary minimum is recognizable in December. The dip in the February value arises really from the smaller number of days in that month. When referred to an equal number of days, the February frequency exceeds that in January in the ratio of 104 : 100.

In the second and third lines we have similarly annual variations from the two 33-year periods of high average sun-spot frequency, their mean appearing in the fourth line.

For the ratio, however, between the mean frequencies from the earlier and from the later of these two periods we have

From Sun-spots.	From Auroras.
100 : 85	100 : 111.

If we contrasted these two periods with one another we should thus associate an increase in auroras with a diminution in sun-spots.

Again, the annual variations from the two periods differ markedly. In the later period, as compared to the earlier, the summer frequencies fall and the winter frequencies rise.

The differences apparent between the two periods may represent a real change, but in all probability they are largely due to an increase in auroral observers, especially in the northern districts of Scandinavia. Taking the yearly data for the five districts mentioned above, we obtain for the total number of auroras observed the results given in Table IV.

TABLE IV.—Auroral Observations in Scandinavia.

	District.					All Scandinavia.
	I.	II.	III.	IV.	V.	
July to June.						
1761 to 1794	22	293	970	1453	555	2481
1794 to 1827	52	694	408	518	56	1316
1827 to 1860	827	873	352	1531	473	2811
1st & 3rd periods combined.	849	1166	1322	2984	1028	5292

An aurora is often seen in more than one district, but comparatively seldom in all. Obviously the increase in the auroral frequency in the latest as compared to the earliest of the three periods arises from the large increase of observations in the two most northern districts; and this being so, the tendency naturally is to bring the annual frequency nearer to the Arctic type with a single maximum at midwinter.

Coming now to the intermediate period in Table IV., we see that the development of district I. was mainly subsequent to 1827. Thus what we should *a priori* expect to observe in

the annual variation from this period in Table III. would be a form intermediate between those from the other 33-year periods, but approaching most closely that from the earlier period. What we actually do find is a variation differing from that of 1761–93 in the expected direction, but to an even *greater* extent than does the variation from the period 1827–59. Comparing the results from 1794–1826 in Table III. with the means from the preceding and succeeding 33-year periods we largely eliminate the influence of the change in observational conditions. The figures show that in the 33-year period characterized by few sun-spots, summer occurrences of aurora were relatively much fewer and winter occurrences more numerous than in the adjacent 33-year periods of high average sun-spot frequency. We thus have low sun-spot frequency associated with an exaggeration in the annual variation of auroras, the precise phenomenon already described in connexion with Terrestrial Magnetism.

§ 8. The mean sun-spot frequency for the two 33-year periods 1761–93 and 1827–59 combined was 60·83, whilst the corresponding auroral frequency for the year was 80·2.

Comparing these with the corresponding figures for 1794–1826, we have for a trebling of sun-spot frequency only a doubling of auroral frequency. This would suggest that if auroral frequency is connected with that of sun-spots by a formula of type (1), then the constant a does not vanish; *i. e.*, a total absence of sun-spots would not be accompanied by a total absence of auroras. A difficulty, however, arises here, which the figures in Table IV. for the several districts of Scandinavia will serve to explain.

The ratio borne to the frequency of auroras in the average year of the period 1794–1827 by the corresponding frequency for the combined periods 1761–94 and 1827–60 is roughly 2 : 1 for the whole of Scandinavia; but is 9 : 1 for district V., 3 : 1 for district IV., 3 : 2 for district III., and less than 1 : 1 for district II. Any comparison for district I. would be misleading.

These figures suggest that with decrease in sun-spot frequency the diminution in auroral frequency is enormously greater in the south than in the north of Scandinavia.

Several authorities have called attention to analogous phenomena, and the theory has even been advanced that the difference in auroral frequency in years of many and few sun-spots really arises from the alternate expansion and contraction of Fritz's isochasms * (curves of equal auroral frequency). The theory does not seem to be strongly supported, but there seems little if any doubt that a substantial difference really exists between the long period changes of auroral frequency in different regions. Results from several stations in Greenland—whose substantial accuracy seems accepted by Prof. A. Paulsen, one of the leading authorities on the subject—appear to indicate that auroral frequency is there very considerably *less* when sun-spots are many than when they are few.

§ 9. We now pass to the consideration of the figures from Lovering's general catalogue in the last five lines of Table III. As before, the monthly figures are expressed as percentages of the yearly total. The figures corresponding to the whole period covered by the table are given by Lovering † himself. Though derived from very heterogeneous data, they represent very fairly the type of annual variation which is characteristic of lower temperate latitudes. The midsummer minimum and the equinoctial maxima are less pronounced than they are even in the South of England.

Comparing the totals from the two periods 1761–93 and 1827–59, we see that the excess from the later period is even greater than it was in the case of Scandinavia. The annual variations from the two periods also differ, and in the same direction as before. From May to September the relative frequency is decidedly less, and at midwinter considerably greater, for the later period than for the earlier, and the equinoctial maxima are but indistinctly shown in the later period. As in the case of Scandinavia, northern stations may have contributed more to the means for 1827–59 than to those for 1761–93. It is clear, however, from the substantial frequencies in the summer months, that even in the period

* *Cf.* Angot, 'Les Aurores Polaires,' p. 138.

† *L. c.* pp. 200 & 216. The total for December on p. 200 should apparently be 907, and not 1007 as printed.

1827–59 data from temperate latitudes must largely have prevailed. Thus the closer approach to the Arctic or single maximum type is difficult to wholly explain, unless we admit that it is partly a real phenomenon, representing a real difference in the distribution of auroras throughout the average year of the two periods 1761–93 and 1827–59. As regards the intermediate period 1794–1826, we see that its mean annual variation differed from that of the neighbouring periods in the same direction as it did in the case of Scandinavia. The fall in the summer frequencies is very pronounced, even as compared to the period 1827–59 alone. Of the equinoctial maxima, that in March is much enhanced, but that in October seems to have vanished, unless it is represented by the maximum now shown in December.

The average year from the two periods 1761–93 and 1827–59 combined shows fully five times the auroral frequency of the average year of the period 1794–1826. This is intermediate between the ratios deduced for the Scandinavian districts IV. and V.

§ 10. Table V. shows the annual variation of auroral frequency in several pairs of groups of years characterized respectively by many and by few sun-spots. The first line gives results for the whole of Scandinavia from 30 years, made up of the 3 years of largest sun-spot frequency from each of the ten 11-year cycles covered by Tromholt's table; the second line gives the results for the corresponding 30 years of few sun-spots. Any *gradual* change in the nature of the observations should affect the two sets of results nearly equally. The third and fourth lines give corresponding data calculated from Lovering's general catalogue, also for 30 years of many and 30 years of few sun-spots.

Of the ten 11-year cycles employed in the two cases nine are identical; the Lovering data cover one cycle prior to the common nine, the Scandinavian one later. This was unfortunately necessary owing to the dates when the Lovering table ended and the complete Scandinavian table commenced. The two sets of data combined represent eleven consecutive 11-year cycles, and the sun-spot data for the 33 years of many and the 33 years of few sun-spots from these eleven cycles

TABLE V.—Auroral Annual Variation in years of many and few Sun-spots.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Total Auroras.	Average per year.
Scandinavian.														
30 years S max. 1769–1872 ...	11·81	11·08	12·58	7·98	0·67	0·10	0·27	4·57	12·01	13·44	12·91	12·58	2997	99·9
„ S min. 1764–1867 ...	12·47	12·26	14·05	7·59	1·03	0·00	0·16	3·85	12·04	13·72	12·69	10·14	1844	61·5
Lovering's General Catalogue.														
30 years S max. 1760–1861 ...	8·44	8·56	9·80	9·84	5·68	4·48	4·84	7·48	10·08	11·12	9·92	9·76	2500	83·3
„ S min. 1754–1856 ...	10·86	10·13	12·92	10·92	5·33	3·34	3·82	4·97	9·76	8·79	9·95	9·22	1649	55·0
New York State.														
9 years S max. 1828–1849 ...	6·60	7·45	7·66	11·28	5·53	7·87	8·51	10·21	11·49	9·79	7·66	5·96	470	52·2
6 „ S min. 1832–1844 ...	5·99	6·45	10·14	12·45	7·37	7·37	8·76	6·45	12·45	11·98	5·53	5·07	217	36·2

TABLE VI.—Auroral Annual Variation in years of many and few Auroras.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Total Auroras.	Average per year.
Scandinavian.														
30 years Auroral max. 1768–1872	12·98	10·72	12·78	8·07	0·71	0·09	0·30	3·99	11·52	13·97	12·66	12·21	3358	111·9
„ „ min. 1765–1866	11·00	10·17	12·67	8·19	1·09	0·00	0·19	4·80	12·99	14·97	13·63	10·30	1563	52·1
New York State.														
9 years Auroral max. 1829–1850	6·35	9·20	10·87	9·87	7·69	7·02	8·36	11·04	9·03	10·20	5·02	5·35	598	66·4
„ „ min. 1827–1851	5·26	3·86	7·02	11·23	10·18	3·16	6·67	10·88	16·49	12·98	4·91	7·37	285	31·7

are those given in the last two lines of Tables I. and II. For the purposes of the present enquiry, these sun-spot data answer sufficiently closely to either the Scandinavian or the Lovering auroral data.

The last two lines of Table V. give auroral data for New York State, derived from 3×3 years of largest sun-spot frequency and 2×3 years of least sun-spot frequency included between 1828 and 1849.

Each of the three pairs of comparative figures in Table V. shows a decided rise in the spring maximum in the years of few as compared to the years of many sun-spots.

The average monthly frequency from May to August is also less in the years of few sun-spots, very decidedly so in the case of Lovering's data; in the New York data the depression of the midwinter minimum is the more decided. On the whole, the phenomena resemble those already described, *i. e.* the annual variation is accentuated in the years of few sun-spots. The differences, however, between the selected years of many and the selected years of few sun-spots are less conspicuous than those between the 33-year periods; and such differences as exist may not unreasonably be partly ascribed to the differences in the annual variations of the sun-spot figures in the last two lines of Tables I. & II. This explanation cannot, at the same time, go very far, in view of the fact that there is no marked depression in the auroral frequency in September in the groups of 30 years of few sun-spots.

The differences between the average number of auroras in the year from the first two groups of contrasted years in Table V. are much less than the corresponding differences in Table III., notwithstanding that the sun-spot differences are larger in the case of Table V. The ratio of the mean frequency from the period or group of years of many sun-spots to the corresponding frequency from the period or group of years of few sun-spots takes the following approximate values in the several cases (p. 450).

Obviously the conclusions one would draw as to the extent of the influence of sun-spot on auroral frequency would vary immensely according to one's method of attacking the problem.

	From the 33-year periods. Tables I. and III.		From 10×3 years of S max. & S min. Tables I. and V.	
	Sun-spot ratio.	Auroral ratio.	Sun-spot ratio.	Auroral ratio.
Scandinavian data ..	3 : 1	2 : 1	7 : 1	1·6 : 1
Lovering's data.....	3 : 1	5·5 : 1	7 : 1	1·5 : 1

§ 11. Table VI. deals with the same Scandinavian and New York data as the previous tables, but instead of selecting groups of 3 years of S max. and S min., it selected groups of 3 successive years of largest and 3 successive years of least auroral frequency. One of the groups for New York State was made up of the last and the two earliest years of the period of observation. The differences between the mean sun-spot frequencies for the contrasted groups of years in Table VI. are really much less than in the corresponding cases in Table V., but the opposite is true of the mean auroral frequencies. For the ratio between the auroral frequencies in the contrasted groups of years we have 21 : 10 in Table VI., both for Scandinavia and New York State, whereas in Table V. the corresponding ratios are respectively only 16 : 10 and 15 : 10.

As to annual variation in Table VI., the group of years of few auroras shows, as compared to that of many auroras, an enhanced maximum at one or both equinoxes, and a lower minimum both in winter and summer.

Lovering's general catalogue is not considered in Table VI. owing to the erratic way in which the auroral frequencies in it vary from year to year. In some of the 11-year cycles years of many and few auroras seem to occur almost promiscuously, and the selection of 3 successive years as representative of either high or low frequency presented difficulties. Even in the comparatively homogeneous Scandinavian data, the same phenomenon occurred to a certain extent.

§ 12. The difference between the results in Tables V. and VI., and the great irregularity in the variations from year

to year of auroral as compared to sun-spot frequency, point to one of two conclusions,—either

(1) Auroral data are so heterogeneous, or so intrinsically defective, that consecutive years' results are affected by large differential errors when treated as measures of the same quantity; or

(2) Auroral frequency depends, and to no small extent, on something more than the contemporaneous value of sun-spot frequency.

As to the question of a possible lag: In the case of Scandinavia the groups of 3 years selected from consideration of sun-spot frequency for use in Table V., represented an earlier epoch than those selected from consideration of auroral frequency for use in Table VI. in 14 cases, the same epoch in 2 cases, and a later epoch in only 4 cases. This is, to say the least, not unfavourable to the view that auroral frequency tends to lag behind sun-spot frequency.

In every case we have found the annual variation in auroral frequency, monthly values denoting percentages of the total number for the year, to be more uniform when sun-spots are numerous than when they are few. The differences, however, between the frequencies in the contrasted years are in some instances not very conspicuous, and may be partly due to chance. Also, supposing it to be a fact that the annual variation in temperate latitudes becomes more accentuated as sun-spots diminish, this may mean one of two things. There may be, as sun-spots decrease, a greater relative diminution in summer than in winter of the physical phenomena which appeal to our eyes as aurora, or there may only be a general diminution in the brightness of auroras throughout the whole year. From what happens during magnetic storms, it can hardly be questioned that the cause —presumably electric discharges in the upper atmosphere— to which auroral phenomena are due is often active when aurora is invisible. It may even conceivably be in continuous operation, though incapable of appealing to the eye, however

favourable the visual conditions, until a certain minimum intensity is reached.

Only exceptionally brilliant auroras have much chance of being seen until the sun is far below the horizon so that a general reduction of intensity might well be more prejudicial to visibility at midsummer than at other seasons.

In conclusion, I should like to draw attention to the utility for investigations such as the present of trustworthy auroral observations taken on a uniform plan, desirably throughout more than one sun-spot cycle, at a considerable number of stations suitably distributed over the earth.

Discussion.

Dr. J. A. Harker asked if in making calculations upon sun-spot frequencies any account was taken of the different sizes of spots.

Dr. Chree said the Astronomer Royal measured the areas of the spots, but Wolf used a "relative number" depending upon the total number of spots observed and upon the number of groups and isolated spots. The two methods of estimating the sun-spot frequency agreed well.