(2) The names of local Societies, or persons who are willing to arrange for a photographic survey for geological purposes in their district.

The information afforded me will be placed before the Geological Section of the British Association at their next meeting, and I trust to receive many offers of valuable assistance from different parts of the country. If copies of photographs are sent me they will be carefully kept for exhibition at the meeting. Several geological friends have favoured me with suggestions in regard to size of photograph, scales of height, length, and other details, which will all be carefully considered. Photography is now so popular and easy of accomplishment that there should be no difficulty in organizing local photographic surveys for the purpose I have indicated.

It was arranged at Bath that the delegates should get their Societies to think the matter over, and that I should meanwhile endeavour to prepare a list of local geological photographs already available for study. I am communicating with the Societies with this object, but your insertion of this letter will further aid me in directing attention to the subject over a wider circle than I am able to reach.

OSMUND W. JEFFS.

12 Queen's Road, Rock Ferry, Cheshire, April 23.

Columnar Structure in Ice.

I HAVE just read Mr. James McConnel's interesting and important paper on the plasticity of ice (*NATURE*, vol. xxxix. p. 203), and as he remarks that it would be interesting to know whether the columnar structure he describes as occurring in the ice of the St. Moritz Lake has been observed in England, I venture to ask you to record the fact that I recollect seeing a precisely similar structure in the ice of the lake in Kew Gardens in February 1880. The phenomenon occurred during a thaw that preceded by a day or two the memorable snowstorm of that month, and the aspect of the ice, where it had been broken through, recalled to my mind that of the well-known fossil *Lithostrotion basaltiforme*, as it was built up of vertical columns, irregularly hexagonal in section, about a quarter of an inch in diameter and of equal length with the thickness of the ice, about 4 or 5 inches. If I remember aright, bright sunshine had been thawing the ice during the day. I made a note on the occurrence at the time, but as I came to India shortly after I do not know what has become of it.

T. D. LA TOUCHE,
Geological Survey of India.

Camp near Cherrapunji, Assam, March 4.

Brilliant Meteor.

I SEND you an account of a meteor I saw on Saturday evening last, thinking it may interest others as much as it has myself. I was lamp-signalling at the time (8.55 p.m.), and saw far the largest meteor I have ever seen. It was far brighter than any planet, or even than a good rocket. It seemed to start from the Great Bear, and fall in a north-east direction half-way to the horizon. I immediately stopped my message, and asked my companion (a mile distant) if he had seen the meteor. He replied he had not, which surprised me, though he had the town lights not far behind him, and he was looking away from the north-east. I had not finished asking him about the meteor, when I heard a loud but distant report, which I can only put down to the same source. It sounded like distant artillery, or more particularly like a six-pounder at six miles distance on a still evening. The interval of time between the sight and the sound I should estimate at a minute.

T. HERBERT CLARK.

Wingfield, Trowbridge, April 30.

A New Mountain of the Bell.

HAVE the kindness to correct two typographical errors in my communication describing the "New Mountain of the Bell," printed in your issue of April 25. On p. 607, col. 2, line 7, an unfortunate superfluous comma after the word quartz should be expunged, so as to read quartz pebbles and veins.

Near the bottom of the same column "modern gong" should read "wooden gong." As a matter of fact the *Nagous* is far from "modern." It consists of a heavy plank nearly 2 inches

thick, 14 feet long, and suspended by ropes at two points 4 feet from either end. When struck with a wooden mallet, this primitive gong emits a loud sound. At the Monastery of St. Catherine, three of these are in use, one small one to call to their noonday meal the numerous *cats* which inhabit the rambling old building.

H. CARRINGTON BOLTON.

London, May 1.

KLEIN'S "IKOSAHEDRON."¹

IT has recently been said, with great truth, that pure mathematics is at the present moment the most progressive of all the sciences. It is, we must confess with sorrow, equally true, that the means at the disposal of English pure mathematical students for making themselves familiar with the recent advances in their science are deplorably scanty. This is not the place to discuss the reasons why it has so long been the case in this island that the stars of our mathematical firmament have been

"*Étoiles qui filent, filent et disparaient!*"

and not fixed suns, with minor but still useful bodies around them, receiving their light and completing their systems. But it is obvious that this shortcoming has been closely associated with the backward state of our textbook literature in pure mathematics. There exist in the English language so few books through whose pages the reader can so much as descry the frontier land of pure mathematics that every addition is an event of importance. Such an addition is Mr. Morrice's translation of Klein's "*Ikosahedron*." Klein's book is in many respects the most charming piece of modern mathematical writing that has appeared for many a day. It is a rare combination of great originality with wide and far-reaching views, Teutonic minuteness of scholarship, and a candour in dealing with the work of others which does not always accompany the other high qualities just mentioned. If we were asked to name a single book that would beyond others give the reader a comprehensive glance over the wide field of modern pure mathematics, and give him an introduction to this study which would at once both interest and instruct him, we should without hesitation name Klein's "*Ikosahedron*." The work interweaves, in a singularly felicitous and natural way, the most remote and apparently unconnected branches of higher pure mathematics. In the course of its perusal the reader will make acquaintance with the geometry of the regular solids, the theory of substitutions, the theory of functions of a complex variable, invariants, the theory of linear differential equations, Riemann's researches on the hypergeometric series, Galois's theory of the resolution of algebraic equations, elliptic functions, Plücker's line geometry, and the special theory of quintic equations. This enumeration will sufficiently indicate the wide sweep of the work; but let not the reader be alarmed. If he is ignorant of all these subjects, so much the more will he enjoy the pleasure of Prof. Klein's introduction to them. He will find that he is led, by easy and pleasant ways, first to see the interest and importance of these subjects, then to panoramic aspects of them, and finally to just so much detail as will make him (if he be right-minded) thirst for more. Speaking from past experience, we should say that one of the greatest disadvantages of modern specialism is the repulsive force which it establishes at every point to the entrant. Let an English student sit down, for example, to Jordan's "*Théorie des Substitutions*." He is at once plunged into a sea of new terms and definitions. He is baffled by a kaleidoscopic array of subtle distinctions between con-

¹ "Lectures on the Icosahedron, and the Solution of Equations of the Fifth Degree." By Felix Klein. Translated by George Gavin Morrice, M.A., M.B. (London: Trübner and Co., 1888.)

ceptions that are unfamiliar to him. He toils through the solution of abstract problems whose formulation he imperfectly grasps, and whose interest and importance he has not been permitted to see beforehand. The very beauty and logical rigour of the work is a hindrance to him; it entangles and suffocates him at the outset. And yet M. Jordan's work could not be otherwise, could not be better for its purpose, could not be dispensed with. But let the learner first read Klein's "Ikosaedron." He will there see the substitution theory first applied in simple cases with concrete illustration, and will then be led by degrees to see its all-embracing character. Then let him return to Jordan, and he will say of the theory of substitution,

"Her loveliness I never knew,
Until she smiled on me."

Under ordinary circumstances, any detailed analysis of a mathematical work would be out of place in the pages of NATURE. The appearance of the "Ikosaedron" in English is, however, an event of such importance that it would be wrong to miss the opportunity of giving the

English mathematical world some account, however imperfect, of its contents.

The proper subject of the work may be said to be the general theory and applications of functions which are transformable into themselves (*i.e.* are unaltered) by a finite group of substitutions. By a *group* of substitutions, or indeed of any operations whatsoever, is meant a complete set of operations of such a nature that the combination of any two or more of them is equivalent to some one of the set. The number of operations in a group is called its order, and the order may in general be finite or infinite. A leading feature of Klein's work, indicated by its title, is the geometrical connection which he establishes between certain groups of substitutions, and the rotations which cause a regular solid to return into itself. It is obvious beforehand that the totality of such rotations for any given regular solid forms a group of finite order, for any two successive rotations of the kind may be replaced by a single one. The accompanying figure will make the connection plain. It represents the stereographic projection of the traces on a circumscribed

sphere made by the planes of symmetry of a regular tetrahedron, A B C D, the vertex of projection being the antipodal point to A, and the plane of projection the diametral plane of which A is the pole. The spherical surface is obviously divided by the planes of symmetry into twelve (non-shaded) congruent triangles, and twelve other (shaded) congruent triangles, each of which is the image of one of the former in a plane of symmetry. These triangles lie in sets of $v_2 = 2$ about points such as F, which correspond to mid-edges of the tetrahedron, in sets of $v_2 = 3$ about points such as E, which correspond to centroids of the faces, and in sets of $v_3 = 3$ about the points which correspond to vertices of the tetrahedron. It is farther obvious that any one of the unshaded triangles can be transformed into each of the other twelve unshaded triangles by one of the group of $N = 12$ rotations which cause the tetrahedron to return into itself. If we mark one of these by inscribing 1, and if S denote a rotation of period 2 (*i.e.* of angular magnitude $2\pi/2$) about G, T a rotation of period 2 about F, and U a rotation of period 3 about A, then the rotations by which the region 1 is transformed into the other twelve are—

1, U, U², S, SU, SU², T, TU, TU², ST, STU, STU²;

and these give the twelve rotations of the tetrahedral group expressed succinctly in terms of three of them.

The same holds for the shaded triangles. Hence, if we pair each non-shaded triangle with a shaded one, and thus form a *fundamental domain*, then we see that any point within such a domain (boundary and summit points excepted) is transformed by the N tetrahedral rotations into N other points, one of which lies in each of the N domains. (Here we count the transformation of the point into itself, viz. the rotation represented by the identical symbol 1.) We pass over the questions that arise regarding the composition of a group and the conception of the extended group which embraces reflections in the planes of symmetry as well as rotations, and merely mention that a similar theory is established for the dihedron, *i.e.*, the figure composed of a great circle of the sphere divided into n equal parts, the octahedron, and the ikosaedron, the characteristic numbers being given by the following table:—

	v_1	v_2	v_3	N
Dihedron ...	2 ...	2 ...	n ...	$2n$
Tetrahedron ...	2 ...	3 ...	3 ...	12
Octahedron ...	2 ...	3 ...	4 ...	24
Ikosaedron ...	2 ...	3 ...	5 ...	60

The next step is to connect each point on the sphere with the value of a complex variable z , or with the ratio of two complex variables $z = z_1/z_2$. This is done, after the manner of Riemann, by representing $z = x + yi$ in an Argand-diagram on the diametral plane of the sphere, and then projecting the point (x, y) stereographically upon the sphere. The point on the sphere is then spoken of as the point (z) or (z_1, z_2) .

It is next shown that every dihedral, tetrahedral, octahedral, or ikosaedral rotation is equivalent to a non-homogeneous linear substitution of the form—

$$z' = (Az + B)/(Cz + D),$$

or to one or other of two pairs of homogeneous substitutions of the form—

$$z_1' = Az_1 + Bz_2, z_2' = Cz_1 + Dz_2;$$

and the proper values of A, B, C, D are calculated for each case.

If we consider the values of z corresponding to a point, and the $N - 1$ other points into which it is transformed by the N polyhedral rotations, we see that they are the roots of an algebraical equation of the Nth degree, say $Z \equiv f(z) = 0$, the characteristic Z of which must have the property of remaining unaltered by every one of the N

non-homogeneous substitutions. And there must likewise exist homogeneous integral functions of $F(z_1, z_2)$ of the N th degree in the two variables z_1, z_2 , containing one arbitrary parameter, which remain unaltered, to a factor *près*, for each of the group of $2N$ homogeneous polyhedral substitutions. The functions of the latter kind ("ground forms") are first determined. First, a special function F_1 (of degree N/ν_1 is determined, such that $F_1 = 0$ gives one of the ν_1 summits on the sphere, and the $N/\nu_1 - 1$ other points into which it is transformed by the polyhedral rotations. Then $F_1^{\nu_1}$ is a particular function having the property sought for. By an elegant application of the theory of invariants, two other functions, F_2 and F_3 , corresponding respectively to the ν_2 and ν_3 summits, are derived from F_1 . We then have for the general invariant function required—

$$\lambda_1 F_1^{\nu_1} + \lambda_2 F_2^{\nu_2} + \lambda_3 F_3^{\nu_3},$$

which contains only one arbitrary parameter, since there is in each case an identity of the form—

$$\lambda_1^{(0)} F_1^{\nu_1} + \lambda_2^{(0)} F_2^{\nu_2} + \lambda_3^{(0)} F_3^{\nu_3} = 0,$$

connecting F_1, F_2, F_3 . In the particular case of the ikosahedron we have—

$$F_1 \equiv z_1^{30} + z_2^{30} + 522(z_1^{25}z_2^5 - z_1^5z_2^{25}) - 10005(z_1^{20}z_2^{10} + z_1^{10}z_2^{20});$$

$$F_2 \equiv -(z_1^{20} + z_2^{20}) + 228(z_1^{15}z_2^5 - z_1^5z_2^{15}) - 494z_1^{10}z_2^{10};$$

$$F_3 \equiv z_1z_2(z_1^{10} + z_1^5z_2^5 + z_2^{10});$$

$$\text{and } F_1^2 + F_2^2 - 1728F_3^2 \equiv 0.$$

The non-homogeneous function Z is next discussed. It is shown that any form of Z , whatever, is a linear rational function, $(\alpha Z' + \beta)/(\gamma Z' + \delta)$ say, of any particular form Z^1 ; so that it is sufficient to determine a special form Z subject to the conditions that Z assumes the values $1, 0, \infty$, for the ν_1, ν_2, ν_3 summits respectively. It is found that $Z = cF_2^{\nu_2}/F_3^{\nu_3}$, and thus the synthesis of the polyhedral functions is completed.

In Chapters III. and IV. of his first part, Klein discusses the inversion of the polyhedral functions. If, in the polyhedral equation $cF_2^{\nu_2}/F_3^{\nu_3} = Z$, we suppose Z given and z required, z appears as an N -valued function of Z , whose properties it becomes our business to discuss. Parallel to this problem we have a "form-problem." There are for each of the five polyhedra a set of three forms which are *absolutely* invariant for the $2N$ homogeneous polyhedral substitutions; thus, for the ikosahedron, these are the special forms F_1, F_2, F_3 , themselves. We may then suppose the values of these absolutely invariant forms given, subject to the identical relation which in all cases connects them; and require the values of Z_1 and Z_2 . There are in each case $2N$ solutions of this "form-problem," and it is shown that these can all be obtained from the N solutions of the corresponding polyhedral equation by adjoining an accessory square root. It is, of course, obvious that the N solutions of the polyhedral equation and the $2N$ solutions of the corresponding form-problem can all be derived from any one of them by the N non-homogeneous and the $2N$ homogeneous polyhedral substitutions respectively.

A brief graphical discussion is given of the functions Z, Z_1, Z_2 ; and it is shown that Z satisfies the differential equation—

$$\frac{Z'''}{Z'} - \frac{3}{2} \left(\frac{Z''}{Z'} \right)^2 = \frac{\nu_1^2 - 1}{2\nu_1^2(Z-1)^2} + \frac{\nu_2^2 - 1}{2\nu_2^2 Z^2} + \frac{1/\nu_1^2 + 1/\nu_2^2 - 1/\nu_3^2 - 1}{2(Z-1)Z} \dots (A)$$

the left-hand side of which is the differential invariant which Cayley has called the Schwarzian derivative of z

with respect to Z (see Forsyth's "Differential Equations," § 61 and chapter vi.); and that Z_1 and Z_2 each satisfy the linear differential equation—

$$y'' + \frac{y'}{Z} + \frac{y}{4(Z-1)Z} \left\{ -\frac{1}{\nu_2^2} + Z \left(\frac{1}{\nu_2^2} + \frac{1}{\nu_3^2} - \frac{1}{\nu_1^2} + 1 \right) - \frac{Z^2}{\nu_3^2} \right\} = 0.$$

Through the latter equations z_1 and z_2 are identified as particular cases of the Riemannian P -function, and thus connected with the theory of the hypergeometric series. Here Klein's work comes in contact with the well-known researches of Schwarz—"Ueber diejenigen Fälle, in welchen die Gaussische Hypergeometrische Reihe eine Algebraische Function ihres vierten Elementes darstellt" (*Crelle*, Bd. 75).

The inversion of the polyhedral functions is next considered from the standpoint of Galois's theory of the resolution of an algebraical equation. An attractive outline of this theory is given, so far as it concerns the problem on hand. The starting-point may be said to be the famous theorem of Lagrange, which, in a generalized form, runs as follows: If $R, R_1, R_2, \dots$ be rational functions of the variables $x_1, x_2, \dots, x_n$, and if R remain unchanged by all the substitutions of the x 's which leave $R_1, R_2, \dots$ simultaneously unaltered, then R can be expressed as a rational function of $R_1, R_2, \dots$ and of the elementary symmetric functions of the x 's. In particular, if we characterize all the functions which admit (*i.e.* are unaltered by) a given group of substitutions, G , as belonging to the family G , we see that all the functions of any family are rationally expressible in terms of one another.

Suppose now that we have any algebraical equations, $f(x) = 0$, whose roots are $x_1, x_2, \dots, x_n$, and we "adjoin" thereto a group of asymmetric functions, $K_1, K_2, \dots$, of its roots, whose values along with the coefficients of $f(x)$ are supposed to be "known," then there exists a group of substitutions, G , that, viz., for which $K_1, K_2, \dots$ are unaltered, such that all functions of the family G and no others are rationally expressible in terms of the "known quantities." If R be a function of $x_1, x_2, \dots, x_n$, not belonging to the family, but say to the family g where g is a sub-group in G of the order $\nu = N/n$, then we can form an equation for R , viz.,

$$(R - R_1)(R - R_2) \dots (R - R_{\nu-1}) = 0,$$

whose coefficients are rational functions of the known quantities. Such an equation is called a resolvent of $f(x) = 0$. All the resolvents constructed by means of functions R which belong to the same family g_1 are identical in the sense that they are rationally transformable into each other, and with these are also identical all resolvents arising from functions belonging to the families $g_2, g_3, \dots$, where $g_2, g_3, \dots$ are the sub-groups "associated with g_1 " in the main group G of the original equation. There are therefore as many different kinds of resolvents, as there are different sets of associated sub-groups in G . The group Γ of every resolvent is isomorphic with the original group G ; that is to say, we can order the two groups so that to every substitution S in G corresponds one S^1 in Γ , and to every combination of substitutions $STU \dots$ in G corresponds $S^1T^1U^1 \dots$ in Γ . If this correspondence be holohedric (one S for every S^1), then the resolvent and the original equation are equivalent in the sense that every root of the one is rationally expressible in terms of the roots of the other and of the known quantities; each is in fact a resolvent of the other. Pre-eminent among this species of resolvents stands the Galois resolvent, whose R is a perfectly asymmetric function as regards the substitutions G . The degree of this resolvent is the highest possible, viz. N . Since the sub-group g_1 belonging to any root R_1 of this equation reduces to the identical substitution, it follows that we can express each

of the x 's in terms of R_1 . In fact, all the roots of the Galois resolvent are rational functions of any one of them; and it has the remarkable property of being transformable into itself by a group of N rational transformations, which stands in holohedric isomorphism with the group Γ . It is also established that an irreducible equation of the N th degree which is transformable into itself by a group of N rational transformations is its own Galois resolvent; and its group is holohedrally isomorphous with the group of the N transformations. We are now enabled to perceive a very important property of the polyhedral equations, viz. each of them is its own Galois resolvent; the N rational transformations in question being simply the N linear polyhedral transformations. Every polyhedral equation therefore stands in a fundamentally simple relation to any equation of which it can be shown to be a resolvent.

If we consider the case where the isomorphism of the groups G and Γ is not holohedric—that is, where to each of the S 's corresponds a group of the S 's, we see that this necessitates the existence of a self-conjugate (*sibi-associate*) group γ in G to which belong the whole of the R 's. If to the other known quantities we now adjoin the R 's, the solution of the original equation $f(x) = 0$ will be simplified, because its group is now γ , which is smaller than G . Moreover, the R 's themselves are calculable in terms of the known quantities by means of an equation whose group Γ is also smaller than G . In this case, therefore, an essential simplification in the formal solution of the equation $f(x) = 0$ can be effected. If the group γ be either intransitive or composite, a further simplification would ensue, in the one case by the "reduction" of $f(x) = 0$, in the other by the construction of another resolvent having a smaller group than γ .

The application of the latter part of the general theory in combination with the data regarding the groups of the polyhedral substitutions obtained in the earlier chapters, leads at once to important conclusions regarding the polyhedral equations. It is found that the octahedral equation can be solved by extracting in succession a square root, a cube root, and finally two square roots; the tetrahedral equation by the same series of operations, if we omit the first, and the dihedral equation by extracting a square root and then an n th root. All these equations are therefore soluble by means of the ordinary *elementary* algebraical irrationalities.

The ikosahedral equation stands by itself because the ikosahedral substitutions form a "simple" group; its lowest resolvents correspond to the five associate tetrahedral and the six associate dihedral sub-groups of the ikosahedral main group, and are of the fifth and sixth degrees respectively. This is, from one point of view, the main part of the theory, for it leads us to see that the solution of the ikosahedral equation involves an irrationality which exists independently of the ordinary algebraical irrationalities.

Since Abel demonstrated the impossibility of solving general equations whose degree exceeds the fourth by means of elementary algebraical irrationalities, two, or perhaps we should say three, great classes of problems in the higher theory of equations have arisen: (1) to characterize and classify all those exceptional cases of equations of a degree exceeding the fourth which can be solved by elementary irrational operations; (2) to circumscribe the domain of the higher algebraic irrationalities—that is, to characterize and exhaustively classify all the essentially distinct irrational operations which are required for the solution of any algebraical equation of finite order,—this is not to be confounded with the interesting and practically important, but perfectly distinct, question regarding the solution of such equations by means of transcendental irrationalities, such

as circular and elliptic functions; (3) in connection with each distinct higher irrationality, there arises, of course, the question as to the characteristics of the various equations which can be solved by means of this irrationality and others of a lower order.

Much has been done in the working out of the first problem by Abel, Kronecker, and others; but comparatively little progress has been made with the second class of problems. In Klein's work we have an important contribution to this new branch of the theory of equations; and a sketch of a general method which seems to promise farther advance in the immediate future. The latter part of the book under review is almost entirely occupied with this subject. He there shows by two different methods that the solution of the general quintic equation can be effected by means of the ikosahedral irrationality combined with an accessory square root. A brief sketch of his first method will enable the reader to understand the general march of the investigation. If to the rational coefficients of the quintic equation we adjoin the square root of its discriminant, its Galois group becomes the 60 even permutations of its roots. Now this is isomorphous with the group of the ikosahedral equation, and therefore (since that group is simple) with the group of any of its resolvents. But it is shown that one of the ikosahedral resolvents ("the principal resolvent") is an equation of the fifth degree of the form $y^5 + 5Qy^3 + 5Ry^2 + S$, where Q, R, S are rational functions of three arbitrary parameters m, n, Z . The question then naturally arises, Can we rationally connect the roots of this resolvent with the roots of the general quintic by properly determining the parameters m, n, Z ? By means of a Tschirnhausian transformation, we can reduce the general quintic to a "principal equation" of the form $y^5 + 5ay^2 + 5by + \gamma = 0$; and it is shown that the necessary operations become rational after the adjunction of the square root of the discriminant of the quintic. We have thus two equations, each involving three arbitrary parameters; and it is shown that the determination of m, n, Z in terms of a, b, γ so as to satisfy the equations $Q = a, R = b, S = \gamma$ involve no farther irrational operations. The calculations in both methods are full of beautiful details, partly geometrical and partly analytical in character.

In the last chapter of the first part a general survey of the theory of the polyhedral functions is given, wherein their relation to a variety of other functions is pointed out. In particular it is shown that the polyhedral functions virtually embrace all functions that "admit" a *finite* group of linear transformations. The proof of this depends essentially on the fact that the diophantine equation, $\Sigma (1 - 1/\nu_i) = 2 - 2/N$, where the ν_i 's and N are all finite and positive, has only four solutions, viz. the values of ν_1, ν_2, ν_3, N (given in the above table), which characterize the polyhedral functions. In these four cases $1/\nu_1 + 1/\nu_2 + 1/\nu_3 > 1$. If in the differential equation (A) we give to ν_1, ν_2, ν_3 other integral values for which $1/\nu_1 + 1/\nu_2 + 1/\nu_3 =$ or < 1 , we get the Schwarzian functions, which are transcendents admitting infinite groups of linear substitutions. Among these, as a limiting case corresponding to $\nu_1 = 2, \nu_2 = 3, \nu_3 = \infty$, are found the elliptic modular functions. This fact naturally leads to the attempt to solve the polyhedral equations by means of transcendently irrational functions; and it is shown that, just as the binomial equation, $Z^n = A$, can be solved by means of logarithms, and the dihedral equation, $z^n + z^{-n} = -4Z + 2$, by means of circular functions, so the tetrahedral, octahedral, and ikosahedral equations can be solved by means of elliptic modular functions.

The above imperfect sketch of Klein's "Ikosahedron" will, we trust, be held sufficient to justify us in saying that Mr. Morris's hope that his translation "may contribute towards supplying the pressing need of text-books upon

the higher branches of mathematic" is not a vain hope. He could not, in our opinion, have made a better beginning. If our critical responsibility compels us to point out some defects in the execution of the work, we trust that this will be understood as indicating our desire to see the book made perfect; and not construed into depreciation of a valuable service to the cause of pure mathematics.

We strongly advise the author to have the translation read by some one who is familiar with both English and German idioms, and who possesses also some familiarity with the departments of mathematics concerned. In proof of the necessity for such a revision, we draw the author's attention to the following points, which are merely a few of those that have attracted our attention. When Klein says (Pt. I. chap. iii., § 7), "Die Lineare Differentialgleichung zweiter Ordnung, verlangt also, . . . , zu ihrer Lösung nur noch eine einzige Quadratur;" he does *not* mean, "The linear differential equation of the second order, therefore, requires, . . . , only a single square root besides in order to solve it." *Quadratur* means simply *quadrature* (i.e., direct integration), it never means *square root*. Here a knowledge of the properties of the Schwarzian derivative might have helped the translator to divine the meaning of the German technical term. On p. 96, "But for this the determination of the R's themselves is more easy to carry out," is not a good, but in fact a misleading translation of the German "Dafür aber ist die Bestimmung . . ." *Dafür* here means "in compensation for this." At the foot of the same page occurs a very common confusion between *wenn eben* and *wenn gleich*, which has the effect of exactly reversing the meaning of the note. "Hierdurch kann $f(x) = 0$ (wenn eben γ , in den x geschrieben, nicht transitive ist) möglicherweise reducibel geworden sein," means, "Hereby $f(x) = 0$ may possibly have become reducible (namely if (or precisely if) γ , when expressed in terms of the x 's, is intransitive)" for, of course, an equation is reducible if, and not unless, its group be intransitive. A still more important error occurs on the following page, where, in the definition of the Galois resolvent, "ihre einzelne Wurzel bei jeder in G enthaltenen Vertauschung der x ungeändert wird" is translated "its individual roots are *unaltered*, &c." First of all, this makes nonsense of the definition, as definite knowledge of the subject would have shown; and farther, supposing the translator to have read *un-* by mistake for *um-*, a knowledge of German idiom would have shown him that "ungeändert wird" makes nonsense of the German. The error is deliberately repeated on the following page, where "in ungeänderter Reihenfolge" is translated in "unaltered sequence," instead of "in altered sequence." These are vital errors, which should at once be corrected by means of an "errata-slip"; for they would be a serious rock of offence for a tyro in reading the passages where they occur. There are many other cases, however, where loose translation somewhat obscures the crisp and lucid exposition of Klein, which is a pity, for this quality is not all too common among Klein's countrymen. There are a considerable number of misprints, many of them copied from the original. An amusing instance of this occurs in the first footnote on p. 73, where the title of Schwarz's well-known memoir begins "Ueber *dienigen* Fälle, &c.;" this is in the original, but the transcriber should have known that *dienigen* is impossible German. Nevertheless, we declare, with all the sanction of our critical stool of infallibility, that Mr. Morris's translation is a notable piece of good work; and he did well to publish it without waiting to perfect his knowledge of German idiom or of Galois's theory. The blemishes alluded to can be easily amended when another edition is called for, which will be speedily, if our good wishes avail.

G. CH.

THE NORTHFLEET SERIES ELECTRIC TRAMWAY.

ON Monday, April 29, there was opened for regular passenger traffic an electric tramway at Northfleet, near Gravesend, which marks an era in the history of electric traction. This line has been run experimentally for the last month, but the seven years Board of Trade certificate having been received, this line now enters on the commercial stage of its existence. Four tramways on which electricity is the motive power have been in regular use for the last few years in Great Britain; it is not, therefore, because the Northfleet line is the first electric tramway in this country that it has attracted considerable attention; nor is it because it is the longest electric tramway, for two of the other four are of much greater length; but it is because this Northfleet line has been constructed on a totally different principle from that hitherto adopted on this side of the Atlantic that it is worthy of special consideration.

When a number of electric lamps or motors have to be supplied with power from a common centre, there are two well-known methods by which this can be done. They can either be joined "in parallel," as it is technically called, or they can be coupled up "in series." In the parallel system, the one generally adopted with electric lighting, and hitherto the only method that has been employed with the electric tramways in Europe, the electric current that passes through any lamp or motor does not pass through any other, and the dynamo produces a large current equal to the sum of all the currents passing through all the lamps or motors. In proceeding, therefore, from the dynamo end of the circuit to the distant end, there is a steady falling off in the current, but the electric pressure remains, or may remain, nearly constant. In the series system, on the other hand, the whole current produced by the dynamo passes through all the lamps or motors in succession, and therefore this current can be small. The initial electric pressure, on the other hand, must be large, since the energy imparted by the current to each lamp or motor is represented by a drop in the electric pressure. Since the energy furnished by the dynamo depends on the product of the current into the electric pressure it produces, while the waste of power in heating the conductor depends on the square of the current flowing through the conductor, it is clear that while any amount of energy can be supplied by either system, the use of high pressure and small current is by far the more economical as regards the power wasted in heating the conductor, this economy being the greater the greater the number of lamps or motors on the circuit.

Until a few years ago, however, it was not clear how it was possible to run motors electrically in series when the motors were themselves in bodily motion, as they must be when employed to propel trams. In 1881, Profs. Ayrton and Perry, for the purpose of diminishing the loss of power through the leakage of the current that occurs from the insulated rail of an electric railway to the earth, and which becomes serious when the line is long, proposed a plan of electrically subdividing the railway track into sections so arranged that the electric current was only supplied to that section of the track on which a train happened to be at any moment. This system was described and shown in action at a lecture given by one of the inventors at the Royal Institution in 1882, and the late Prof. Fleeming Jenkin, on reading the account of this lecture, saw that the device of employing an electrically subdivided conductor supplied the means of running electrical trains *in series*. A combination was, therefore, brought about between these three Professors to develop electric traction. This combination resulted in the formation of the Telferage Syndicate, and lastly in the Series Electrical Traction Syndicate, to whom is due the construction of the first series line in Europe, the