Review

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REVIEWS.

Homogeneous Linear Substitutions. By HAROLD HILTON. Pp. 184. 12s. 6d. net. 1914. (Oxford : Clarendon Press.)

Of late years the theory of groups of linear substitutions has been especially cultivated, and, in the second edition (1911) of his *Theory of Groups of Finite Order*, Prof. W. Burnside has given a fairly long account of it, and has even said that "for further advances in the abstract theory [of groups] one must look largely to the representation of a group as a group of linear substitutions." In the book under review, Prof. Hilton has put together those properties of the homogeneous linear substitution with real or complex coefficients, of which frequent use is made in the theory of groups and in the theory of bilinear forms and invariant-factors. Only those properties of substitutions are dealt with which do not depend on any "group" of such substitutions. The main reason for this limitation is the fact that the most interesting properties of groups of substitutions may be found in the work of Prof. Burnside referred to above.

The chapters of the book under review are on elementary properties of substitutions, invariant-factors, bilinear forms, applications, substitutions permutable with a given substitution, symmetric, alternate, and Hermitian substitutions, invariants of the second degree, orthogonal substitutions, and families of bilinear forms.

This work is intended to be a text-book, but it is written with such condensed competence that it might be more suitably regarded as a monograph on the subject of homogeneous linear substitutions treated as an independent theory. The book is rather drily written, and is not so attractive to a student as, for example, Bôcher's *Introduction to Higher Algebra*, but this is probably inseparable from a treatment as an independent theory. Thus, the greatest interest to a student will probably lie in the excellent chapter on applications to the solution of simultaneous linear differential equations with constant coefficients, the solution of simultaneous linear homogeneous equations, collineation, geometrical movements, solution of a quartic equation, critical values of a real quadratic form, the classification of conicoids, the tangential equation of a conicoid, relations between two conics, small oscillations about a position of equilibrium, and Thomson's and Bertrand's theorems.

Intégrales de Lebesgue, fonctions d'ensemble, classes de Baire. By C. DE LA VALLÉE POUSSIN. Pp. viii, 154. 7 francs. 1916. (Paris : Gauthier-Villars et Cie.)

The author, who was formerly a well-known Professor at the University of Louvain, and in 1914 and 1915 lectured at Harvard University, gave these lectures at the Collège de France in 1915-16. They form a continuation of the Harvard lectures, which were partially published in the *Transactions of the American Mathematical Society* for 1915, and the book under review is included in the collection of monographs on the theory of functions edited by Emile Borel.

In 1898 Borel gave a definition of the "measure" of aggregates, which differed in some important respects from the definitions in vogue before him, and which were ultimately based on Hankel's definition of "content" and Riemann's theory of integration. To the ideas thus set in motion by Borel belong the famous researches of Lebesgue on integration and on the general notion of an "additive function of an aggregate." Further, Lebesgue made important progress (1910) in the theory of functions representable by infinite series of functions which are either continuous or point-wise discontinuous, which was inaugurated by Baire (1899).

In this book the first part is on measurable aggregates and Lebesgue's integral; the second part is on additive functions of an aggregate, and the author deals with the question more precisely than he did in the Harvard lectures, and introduces new methods. Thus, among such additive functions the most important are "absolutely continuous" functions, which are the same things as the indefinite integrals of Lebesgue; and the derivation of these functions is studied by a new method which is a great improvement

on that used in the Harvard lectures. The third part is devoted to the transitively many classes into which Baire and Lebesgue have divided "analytically representable" functions.

This book is of the greatest value for those who are interested in the important branch of the theory of functions which is principally due to recent French writers.

PHILIP E. B. JOURDAIN.

A Shilling Arithmetic. By JOHN W. ROBERTSON, M.A., B.Sc. Pp. 191 + vi. 1s. 1917. (G. Bell & Sons.)

The chief feature of this little book is its splendid collection of examples. According to the preface, they are practically all original, and the author is to be congratulated on his achievement in this respect. *Whitaker, The Statesman's Almanac*, everyday problems of war and finance, sugar and wheat imports and so on, have all in their turn supplied inspiration, while care has been taken to insert a sufficient number of "drill" examples which "are so essential for the average pupil." The captious might take offence at the old-fashioned treatment of logarithms, the position of graphs in the book, the omission of brackets in the examples on multiplication and division of fractions (e.g. what answer is required to Ex. 81 on p. 49?), and one or two other details; but the good points of the book on the whole are so obvious and make it such an excellent shillings-worth, that those who are thinking of introducing a new Arithmetic into their classes next September can be confidently recommended to inspect a copy before making their final choice. There is a slight misprint on p. 75.

H. G. M.

A First Course in Projective Geometry. By E. HOWARD SMART, M.A. Pp. xxiii, 1-273. 7s. 6d. 1916. (Macmillan.)

The object of this book is in the first instance to serve as a first course in projection, and secondly, by means of the methods of projection and cross-ratio to introduce the student to the chief properties of the conic before proceeding to the study of the more advanced works on modern pure geometry. It may be said at once that the author has succeeded in the task which he set himself to perform, and has produced a book which, though dealing with such well-worn subjects, is distinctly original in its treatment and eminently readable.

In Chapter II. we are introduced to the elementary notions of projection and the ideas of correspondence and duality, the relations between copolar and coaxial triangles being proved in a very clear and simple manner. Chapters III. and IV. deal with the metrical theorems of Ceva and Menelaus, harmonic section and the harmonic properties of the complete quadrangle and quadrilateral. In Chapters V. and VI. the author leaves the straight line and treats of inversion, centres of similitude and coaxial circles, which naturally lead to poles and polars and the harmonic properties of the inscribed quadrangle and circumscribed quadrilateral, and attention is drawn to the principles of duality and reciprocation. Up to this point the author has been making a collection (with proofs) of the various geometrical properties which he intends to employ, and in Chapter VII. he again takes up projection and performs in full various standard projections, the figures and demonstrations being given in a clear and interesting manner, although we would suggest that fig. 42 on p. 83 would be improved by being drawn in perspective.

This may be said to constitute the first part of the book. In Chapter VIII. the author proceeds to deduce various properties of the conic from the corresponding properties of the circle by means of the method of projection. At the outset he defines a conic as the projection of a circle, any vertex and plane of projection being taken, and he shows that the conic, like the circle, is a curve of the second order and second class. He then shows how the position of the vanishing line in the plane of the circle discriminates between the parabola, ellipse and hyperbola, and introduces at once the ideas of poles and polars, conjugate points and lines, and from them obtains various properties relating to the centre, axes, conjugate diameters and asymptotes. In Chapter IX., from a consideration of Carnot's theorem, which is obtained by projection from the circle, he obtains various fundamental properties, e.g. the constant values of the ratios $PN^2 : AN \cdot NA'$ and $QV^2 : PV \cdot VP'$, and asymptotic pro-