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XXII. *Measurements and Law in Electro-optics.* By JOHN KERR, LL.D., *Mathematical Lecturer, Free Church Training College, Glasgow**.

MY last communication to the *Philosophical Magazine* contains an account of some electro-optic measurements in connexion with carbon disulphide, cumol, carbon dichloride, and several other liquid dielectrics. That set of measurements was very imperfect, and of little interest except as a first trial. In the leisure of my last summer holidays I resumed the inquiry with better means, and carried it forward for some weeks with great care. I had intended to examine several dielectrics in succession, but found time only for the first of them, which was carbon disulphide. These new experiments form the subject of the following paper.

1. The dioptric actions of dielectrics are generally of one kind—pure double refractions with reference to line of electric force as axis; but they vary largely in intensity, and vary even in sign, from one dielectric to another. Dielectrified bodies are therefore optically equivalent to uniaxal crystals, and exhibit like variations, both from strong to weak, and from positive to negative. Our present dielectric (carbon disulphide) is of the positive class; and in regard to strength, it holds, as yet, a place among dielectrics like that of Iceland spar among crystals. For the particular facts and the proofs I must refer to my papers already published†.

I consider it probable in the highest degree that these optical

* Communicated by the Author.

† *Philosophical Magazine*, Nov. and Dec. 1875; also Aug. and Sept. 1879.

actions are subject to uniform laws in media of the one class as of the other, in dielectrics as in crystals. I venture therefore to accept the principal result of my present experiments as a general law, and to enunciate accordingly, although the actual measurements were limited to one dielectric.

LAW.—The intensity of electro-optic action of a given dielectric—that is, the quantity of optical effect (or the difference of retardations of the ordinary and extraordinary rays) per unit of thickness of the dielectric—varies directly as the square of the resultant electric force.

2. To prepare the way for an account of the experiments, I must begin with a description of the cell which contains the dielectric. The cell-block is composed of three thick slabs of plate-glass, which stand on the experimental table, with their plate-faces vertical and in close contact.

The diagram gives an end view of the block. The two dimensions exhibited in the diagram are about 10 and 6 inches; the third dimension of the block, or joint thickness of the slabs, is exactly $3\frac{5}{8}$ inches. The inner rectangle in the diagram represents a tunnel, which passes pretty uniformly through the block, leaving wide margins of polished surface on all the faces of the slabs. As in the case of the old plate cell, the tunnel is closed by two panes of thin and carefully selected plate glass, which lie in contact with the ends of the block. Outside of the panes are two thick plates of india-rubber cloth; and outside of the cloths are two strong planks of mahogany, which are a little wider horizontally than the slabs of glass. The planks are connected by a strong screw-bolt at each end. The tunnel is continued fairly through the cloths and planks.

When the cell is charged with carbon disulphide, and well closed by screw-pressure on the planks, it is almost perfectly tight, the trifling leakage being due to a flaw in the workmanship. The 4-inch plate of powerfully refractive and dispersive liquid thus obtained is perfectly transparent; and a beam of light, incident normally on either of the limiting panes, passes through the plate without sensible traces of dispersion, deviation, or depolarization. The absence of all cement in the construction of the cell is, I think, a most important condition. The liquid is in contact with nothing more than compact masses of metal and the glass walls of the cell; and accordingly, with a fair start, it may be kept sufficiently clean for any length of time.

The shaded pieces in the diagram represent the two conductors. The lower conductor is a thick plate of solid brass, which rests on the floor of the cell. The upper is also a plate of solid brass, of the same length and thickness as the lower, but much narrower: it is supported by a small rafter of thin plate glass at each end, rafter and conductor being connected by a bullet-headed screw, which passes freely through a fine hole drilled in the rafter, and works deeply into the conductor. Each of the rafters rests on the two trapezoidal pieces shown in the diagram. These are prisms of glass taken out of one thick plate, the horizontal surfaces (upper and lower) being primitive plate-faces kept intact. These lateral supports rest simply on the brass floor, and extend from front to back of the cell.

The opposed faces of the two conductors are perfectly flat and smooth, just as they left the planing-machine: they are also sensibly parallel, and generally distant from each other about one twelfth of an inch. All edges and corners on the upper conductor have been carefully rounded away, as well as the two adjacent edges on the lower conductor. The length of the conductors (at right angles to the plane of the diagram) is such as to leave the brass barely short of contact with the limiting panes.

The dotted lines in the diagram represent three borings in the central slab. The vertical boring admits a glass tube, within which passes a fine shaft of brass, resting below in a small cavity made in the upper conductor, and terminating outwards in a perforated knob and connecting-screw. The smaller boring on the right gives passage to a wire from the lower conductor; the larger boring on the left is used for charging the cell with liquid, and for emptying it, the latter operation being effected by means of a small siphon.

3. *Working of the Cell*.—The slabs, the closing-panes, and

all the pieces that lie within the cell are cleaned with the greatest care. The lower conductor is inserted in the central slab, and has its connecting-wire drawn up through the proper boring; and the cell-block is completed by the addition of the outer slabs. The lateral supports of glass are then put in position, also the upper conductor and the external knob connected with it. The conductors are adjusted stably, with their opposed faces exactly parallel. All the pieces are now cleared of dust as thoroughly as possible, and the cell is closed.

The liquid is introduced rapidly into the large boring, through filtering-paper and funnel properly placed. To prevent the formation of air-bubbles between the two conductors during the charging of the cell, it is necessary to have the plane faces of the conductors inclined a little to the horizon; and this is done by the temporary insertion of a thin slip of wood under one end of the cell-block. The charging is continued until the liquid stands well above the balls of the upper conductor; but the cell is never quite filled, as that might result in a very inconvenient explosion. When the borings are now stoppered and the cell firmly closed by screw-pressure, there is very little loss of the liquid by evaporation and leakage.

Sometimes, in spite of all precautions, the liquid is found to be not sufficiently clean, a quantity of solid particles gathering in chains between the electrified conductors, and masking the optical effect in some degree, or even discharging the upper conductor at comparatively low potentials. In such a case the charge is drawn off (as much of it as is possible) by the siphon; the cell is opened, and the faces of the conductors are swept clear of dust, which is done well by friction with narrow slips of clean writing-paper. The cell is then closed and charged anew.

These operations in preparation for experiment are sometimes very troublesome and tedious, a great deal more so than in the case of the old plate cell. Partly on this account, and partly because of the troublesome and expensive character of the workmanship, I consider the present cell very imperfect; but these and other faults are forgotten in the presence of the following phenomena.

4. *Electro-optic Experiment in Chromatic Polarization.*—Here, as in all the following experiments, the cell is charged with rather less than a pint of clean carbon disulphide. No other optical pieces are employed at present than the charged cell and a couple of Nicol's prisms.

A beam of light from a bright cloud is reflected horizontally into the room through a port in the window-shutter, passes

through the first Nicol, then perpendicularly through the plate of liquid, then through the ocular Nicol. The pieces are so levelled and directed, that the observer at the polariscope looks fairly through the deep slit that separates the two conductors in the cell. I may mention again that the dimensions of the slit are about *one twelfth* of an inch, *one* inch, and *four* inches—the first dimension lying vertically as the lines of force, and the last lying horizontally along the line of sight. The two Nicols are fixed, the first with its principal section at 45° to the vertical, and the second at extinction, which is here quite pure. Wires are led permanently from the two conductors—from the lower to earth, and from the external knob of the upper to the prime conductor. To give greater steadiness and distinctness to the progress of the optical effect, the wire from prime conductor to cell is put in permanent contact with the knob of a Leyden jar, whose outer coating is uninsulated.

When the machine is set in motion at a moderate rate, the potential of the upper conductor rises slowly, and the black space between the two conductors is illuminated, the light passing gradually through impure black, faintly bluish grey, faint white, and so forward, up to a sensibly pure and brilliant white. Thus far there is nothing new, except that the highest potential yet reached is comparatively low, while the optical effect is very large, and already far beyond neutralization by the action of any hand compensator of strained glass.

As the potential of the prime conductor still rises, the polariscope gives a fine progression of chromatic effects, which descend regularly and continuously through a certain range of Newton's scale. The luminous band between the conductors passes first from white to a bright straw-colour, which deepens gradually to a rich yellow, then passes through orange to a deep brown, then to a pure and dense red, then to purple and very deep violet, then to a rich and full blue, then to green. All the colours are beautifully dense and pure, certainly as fine as any that I have ever seen in experiments with crystals in the polariscope.

Generally about the point last named of the scale of colours, at or near the green of the second order, the process terminates in spark-discharge through the liquid. Sometimes, but not frequently in my observations, it terminates at an earlier stage, to run its regular course at the next trial. The irregularity appears to be due to an accidental precipitation of discharge by the action of solid particles, impurities in the liquid.

Through this whole range of effect, from the pale blue or impure black of the first order to the green of the second order, the plate of electrically-charged liquid acts regularly as

a uniaxial crystal, as a plate of quartz with optic axis parallel to the lines of force, the plate increasing in thickness continuously and very rapidly as the potential rises.

I may mention here, that with the old plate cell, under favourable conditions, I obtain one step of the scale of colours, that from white to yellow, but not easily nor very distinctly.

5. By simple observation in connexion with this experiment, the electro-optic field is seen to be sensibly uniform through the greater part of the space between the two conductors, and for potentials high or low.

Beginning at very low potentials, for which the optical effect lies below intense white, and applying a hand compensator of glass, I obtain, as the first effect of stress in the glass against electric force in the liquid, a couple of extinction-bands in the outer borders of the field, just as in my old experiments with the plate cell. As the stress of the glass, horizontal tension or vertical compression, is increased, the bands move inwards, till at last, as they pass through the borders of the field, and enter the space between the parallel faces of the conductors, they widen inwards and meet each other, coalescing in one wide and uniformly black band.

At higher potentials, and using only the two Nicols, I obtain equally distinct results. Thus, at one stage of the experiment the band of light between the two conductors is of a deep red, and sensibly uniform from end to end; but at each end, in the borders of the field, where the upper conductor is rounded off, the band is tipped with a narrow fringe of brilliant yellow. Similarly, another stage of the experiment gives a violet or blue band, sensibly of the same tint from end almost to end, and tipped at each end with a narrow fringe of dense red.

As an important practical effect of the characteristics of the new electro-optic field, its *intensity* and *uniformity*, I have been able to discard the old and imperfect compensator of strained glass. One such compensator that had been elaborately prepared was found quite inadequate, the action of a fine slip of glass (not an inch wide), under a compressing weight of 50 pounds, being neutralized and far exceeded by the action of the plate of CS_2 at low potentials.

6. *Rate of Disappearance or Degradation of the optical effect after the stoppage of the machine.*—Some observations were made upon this subject at different times; and the results are worthy of brief notice in passing. The arrangements were the same as in the preceding experiment.

The light is restored from extinction up to one of the tints; and the effect is kept pretty constant for several seconds.

When the machine is stopped suddenly, the tints disappear at once and the optical effect falls very rapidly to faint white. The subsequent rate of degradation, through faint white to pure extinction, varies very much with circumstances. In damp weather the light disappears in one or two seconds, or less; but on several occasions, the weather being favourable, and the external conductors very well insulated, I have seen the light sustained clearly for five and even for six minutes. This continuously sustained effect was certainly due to the present action of electric force at each instant; for it disappeared sharply and totally at the instant of spark-discharge of the prime conductor, and then, a few seconds after the spark, it was restored quite distinctly by the action of the residual charge of the jar.

The Leyden jar is now withdrawn, and the prime conductor is connected simply by a wire with the upper conductor in the cell, the procedure and circumstances of the observation being otherwise unaltered. There is nothing new observed in this case, except a more rapid rate of degradation of the optical effect. In favourable circumstances I have seen the effect still sustained clearly for a good many seconds—40 to 60.

The permanent wire from cell to prime conductor is now detached from the cell, and its extremity is fixed in a small ball of brass, which is held by an insulating handle of glass. The polariscope being at extinction, and the machine in motion, the ball connected with the prime conductor is brought up towards the knob of the upper conductor in the cell, till one small spark passes. At the instant of the spark there is a strong restoration in the polariscope. The effect degrades at first very rapidly like a flash, but afterwards more slowly, so that it is still seen distinctly, though at last very faintly, for 12 or 15 seconds. And here, as in the former cases, the sustained effect is certainly due to the present action of electric force at each instant; for it vanishes instantaneously when the knob of the upper conductor is touched by the finger.

There are several points proved or illustrated by these observations:—that the birefringent action of the liquid is due to electrostatic force, and not to electric discharge in any form; that the dissipation of the charge of electricity in damp weather is principally from the external parts of the apparatus, as was to be expected; that within the cell there is always a sensible discharge of electricity as long as the two conductors are at different potentials; and that the rate of this discharge or dissipation through the liquid is very great at high tensions. I proceed now to the measurements.

7. *Instruments of Measurement.*—To measure potential and

optical effect respectively, I use a Thomson's Long-Range Electrometer and a Jamin's Compensator. These admirable instruments are now so widely known that any description of them here would be superfluous. Besides, they are described fully by the inventors themselves in books which are generally accessible*.

8. *Arrangements and Method.*—The diagram shows all the optical pieces, in horizontal section through the observer's eye, Q. Several of the pieces are placed exactly as in the first experiment (4). These are :—the reflector, B, in the window-

shutter; the polarizing Nicol, M; the charged cell, P; and the ocular Nicol, N. The Nicols are fixed, here as formerly, with their principal sections at 90° to each other, and at 45° to the vertical. After the cell, and close to it, comes the compensator C, which is placed with great care and very stably, with the optic axes of its quartz prisms properly directed, one vertical and the other therefore horizontal, and both perpendicular to the ray. The narrow beam transmitted by the compensator, and received at Q, passes of course through the central parts of the electro-optic field. After the compensator comes a convergent lens D, which is fixed by trial in such a position as to give the observer at Q a distinct view of the black band and reference-wires in the compensator. Some changes were made afterwards in the optical arrangements; but I will describe these in the proper place.

Two wires are led permanently from the conductors within the cell—from the lower to earth, and from the upper to prime conductor. Wires are also led permanently from prime conductor to inductive or upper plate of the electrometer, and to the inner coating of a good Leyden jar whose outer coating is connected permanently with earth.

The index of the compensator being at the zero-point, the observer at Q sees a strong black band exactly midway between the two parallel reference-wires. When the machine is set in

* For the Electrometer, see Sir W. Thomson's Reprint of Papers on Electrostatics and Magnetism, § 383; and for the Compensator, see Jamin's *Cours de Physique*, tome iii. pp. 622, 639.

motion, the band is displaced slowly (here to the right), and can be driven easily out of the field of the instrument.

Each measurement requires the conjoint and repeated action of two observers—one at the electrometer, the other at the second Nicol. The lower plate of the electrometer is fixed at a chosen point of the scale; and the screw of the compensator is worked so as to displace the black band some distance to the left. When the machine is set in motion, the potential of the prime conductor rises slowly, and the observer at the electrometer gives a continued set of sharp audible signals, which are intended partly to inform the observer at the polariscope, and partly to guide the worker at the machine, whose task is to keep the index of the electrometer as steadily as possible in or near the sighted position. The present relation of *actual* to *chosen* potential, whether of defect, or equality, or excess, is indicated incessantly by the signals.

Meanwhile the observer at the polariscope has a distinct view of the black band, and sees whether it is midway between the wires, or to the right or left, at the time of the critical signal. The screw of the compensator is now worked in the proper direction, and the observation is repeated. The screw is again worked and the observation again repeated, and so forward, until at last the black band lies exactly midway between the reference-wires when the index of the electrometer is in the sighted position—that is, when the potential of the upper conductor has the value indicated on the scale of the electrometer. Two other observations are always made at the same time, and the results recorded—one giving a superior limit of the optical effect at the chosen potential, and the other an inferior limit; and the two limits are always drawn as close as possible.

9. *Preliminary Work.*—In mastering this method, as also in applying it afterwards, I was able to keep the manipulations, readings, and records all in my own hands; but for the observations I had to depend partly on my two assistants. The preliminary drill was found useful in respects not at first contemplated; for in the course of it I discovered several sources of error, and saw how to avoid them: I found also that, when proper care was taken, the observer's error with the compensator could be certainly limited within a definite and small range of the scale.

After more than a week of incessant exercise of myself and assistants in this way in the practical management of the instruments and method, I felt confident that good results could be obtained in one day's work at least—one mounting of the cell, and one setting of all the optical pieces, to be kept

unmoved from morning till night. I determined therefore to make a new start, and to devote one day to a small number of independent and very careful measurements, through a pretty long range of potential.

10. *First Appearance of the Law.*—Accordingly on one day, Tuesday, the 12th of August, I effected eight measurements of potential and corresponding optical effect. Every precaution was adopted, every check and test that I could think of as likely to be of use. All the pieces were placed and fixed with scrupulous care. No guidance towards the results was accepted in any case, except from the actual indications of the instruments. The limits of error were drawn always as close to each other as possible; and every observation made by an assistant at either of the instruments was repeated and verified by myself before the result was recorded. The work was very tedious, some of the single measurements taking more than an hour of time; but the results were understood to be proportionately sure.

When the day's work was done and the laboratory closed, I collected the results, and reduced the readings of the compensator, subtracting the scale-reading of the true zero-point, and expressing the remainders in terms of a convenient unit, which was the twentieth part of one screw-step of the instrument*. The final results were as follows. The numbers in the first line are the scale-readings of the electrometer, or the potentials of the upper conductor; and those in the second line are the corresponding quantities of optical effect, each probably true to ± 1 .

30	40	50	60	80	90	100	110
12½	22	36	50	88	115	144	170

Observe the effects corresponding to potentials 30 and 60, exactly as 1 to 4; the effects again for potentials 40 and 80; those also for potentials 50 and 100. It was from these numbers, exactly as they stand, that I obtained my first view of what I was looking for—a definite relation between potential and optical effect. It will be apparent immediately, that the exactness of equality of those three ratios to each other and to $\frac{1}{4}$ is partly accidental; but we shall see no reason to question the truth of the law that is here so very clearly indicated.

11. It appears therefore that, in the circumstances of the present experiments, the dielectric (CS_2) being given, and the electro-optic field also given in form and dimensions, the quantity of optical effect is proportional to the square of the difference of potentials of the two conductors that limit the field.

* The unit of potential, here and afterwards, is one screw-step of the electrometer, the 200th part of the range of the instrument.

12. Dividing the square of each of the preceding potentials by the corresponding optical effect, and keeping the quotients true to half a unit by excess or defect, we find the eight numbers

72 73 $69\frac{1}{2}$ 72 73 $70\frac{1}{2}$ $69\frac{1}{2}$ 71.

There is here a very fair approximation to equality, though it is hardly so close as might have been expected.

13. *Later Measurements.*—In the next three sets of measurements the work was still of the same empirical and tedious kind as formerly (8, 9, 10). The weather was exceptionally favourable both for electrical and optical work; and the range of potential in the second set was almost as large as the instruments could reach in any circumstances.

In the actual work, I considered that each of the observed values of optical effect was very probably true to less than ± 1 , and that none of them was possibly in error by so much as ± 2 .

There is no equality to be expected between the results of one set of measurements and those of another set; for the constancy of the electro-optic field was not carefully attended to at present, except in each day's work by itself.

14. On the day after the decisive experiments of (10) I obtained the following set of results (potentials are given as formerly in the first line, and optical effects in the second):—

60	70	90	120	130	150
50	67	113	197	234	310

Dividing the squares of the successive potentials by the corresponding optical effects, and keeping each quotient true to half a unit, we find the six numbers

72 73 $71\frac{1}{2}$ 73 72 $72\frac{1}{2}$.

Applying the number $72\frac{1}{2}$ as divisor to the squares of the successive potentials, we find the following set of calculated values of optical effect, each written under the corresponding potential. For comparison, the observed values of optical effect are repeated in the third line, and the differences between observed and calculated values are written in the fourth line.

60	70	90	120	130	150
49·6	67·6	111·7	198·6	233·1	310·3
50	67	113	197	234	310
·4	—·6	1·3	—1·6	·9	—·3

The differences in the third and fourth columns are rather large (13).

15. Next day I obtained the following set of results. The

first and second lines contain potentials and optical effects as formerly; the third line contains a set of numbers which are as the squares of the first five integers 1, 2, &c.—that is, as the squares of the successive potentials.

30	60	90	120	150
13	53	117	206	321
13	52	117	208	325

Dividing the squares of the potentials by the observed optical effects, and keeping the quotients true to quarter units, we obtain the five numbers

$$69\frac{1}{4} \quad 67\frac{3}{4} \quad 69\frac{1}{4} \quad 69\frac{1}{2} \quad 70.$$

Taking now the number 70 as divisor of the squares of successive potentials, we find the following table of results (they are arranged, line for line, as those given in the end of the preceding article):—

30	60	90	120	150
$12\frac{6}{7}$	$51\frac{3}{7}$	$115\frac{5}{7}$	$205\frac{5}{7}$	$321\frac{3}{7}$
13	53	117	206	321
$\frac{1}{7}$	$1\frac{4}{7}$	$1\frac{2}{7}$	$\frac{2}{7}$	$-\frac{3}{7}$

The differences in the second and third columns are still rather large (13).

16. The two following measurements were made as one set, in new circumstances, and with great care. The light employed was that of a paraffin-lamp; and the distance of the two conductors was about one eighth of an inch, which was a good deal larger than in the former experiments. The potentials were 90 and 120; and the quotients of square of potential by optical effect were as follows:—

$$\frac{90^2}{38} = 213, \quad \frac{120^2}{67} = 215.$$

These numbers are satisfactorily near equality. Otherwise, the calculated value of optical effect at potential 120

$$= \frac{4^2}{3^2} \times 38 = 67\frac{2}{3},$$

which differs from the observed value by half a unit.

17. Is there any *regular* difference, however small, between observed and calculated values of the optical effect? Such was the question that I specially attempted to solve in some of my latest experiments on this part of the subject. Some changes of method were adopted. The lens D of (8) was withdrawn, and its place was supplied by an ocular lunette; the second Nicol, deprived of its diaphragms, was moved close up to the compensator; the room also was well darkened, so

that all other light beside that from the port was excluded. These alterations added greatly to the purity and steadiness of the optical effect, and made the observations both easier and surer.

The electrometer was set at a convenient potential; and the corresponding optical effect was measured as accurately as possible. A new potential was chosen; the corresponding optical effect was calculated; and both electrometer and compensator were set *initially* as they ought, by the law of squares, to stand *finally* in this new case. The machine was then set in motion; and the observers had to determine, without touching the instruments, whether the black band lay exactly midway between the wires of the compensator, or perceptibly to the right or left, when the index of the electrometer was exactly in the sighted position. Some careful trials were made in this way, through various ranges of potential. Differences did present themselves at first in one or two cases, but they disappeared on closer examination. Upon the whole, there was no deviation from the law of squares made out clearly in any case, in either direction, or in any degree (great or small). I must therefore leave the statement already made in (11) as the net and undoubted result of all that has been done upon the subject up to this point.

18. *Second Part of the Proof.*—Let D be the distance between the opposed faces of the two conductors, let V be the difference of potentials of the conductors, and let Q be the corresponding quantity of optical effect; also let R be the resultant electric force at any point. Comparing the present system of dielectric and conductors with one of the simplest forms of electric field imagined in the Mathematical Theory of Electricity (that, namely, which is limited by two parallel conducting planes of infinite extent, and maintained at different potentials), we obtain the following principle as one that we can safely proceed upon.

For any small given value of D , for any given value of V , and for all points lying between the opposed parallel faces of the conductors, and not very near the borders of the field, the force R is constant in direction and magnitude, its direction being perpendicular to the faces of the conductors, and its intensity proportional to the quotient of V by D .

The optical action that we are concerned with has been already characterized as a case of double refraction with reference to direction of the force R as axis. The quantity Q of the birefringent action, or the difference of retardations of the ordinary and extraordinary rays, is proportional to, and may therefore be measured by, the reduced reading of the compen-

sator. The quantity V is measured, with sufficient accuracy, by the scale-reading of the electrometer.

It is already proved that, when D is given,

$$Q \propto V^2;$$

and from this it follows that

$$Q \propto R^2.$$

To anticipate the effect of a variation of D , we may simply extend this last statement, in the form

$$Q \propto R^2 \propto \frac{V^2}{D^2};$$

and what remains to be proved is that Q varies inversely as D^2 .

Generally, we may connect the experimentally measured quantities D , V , Q by the equation

$$Q = \frac{V^2}{E \cdot D^2};$$

and then, to complete the proof of the Law of Squares as already stated in (1), we must find that the quantity E , or the equivalent quantity

$$\frac{V^2}{Q \cdot D^2},$$

preserves a constant value when D varies as well as V .

19. *Method of Experiment.*—It was necessary here to obtain a set of exactly measured values of D . The following method was adopted, as well within reach and at the same time sufficiently accurate. I assume that the reader remembers what has been said about the upper conductor and its adjuncts, the ball-headed screws, rafters of glass, and lateral supports of thick plate (2). A sheet of thin plate glass, chosen for its apparently perfect uniformity, was cut up into a number of equal slips, of about the same length and breadth as the upper conductor. The joint thickness of sixteen of the slips was exactly one inch. When the slips were laid side by side on a plane table, and tested optically and otherwise, they were found to differ sensibly, though very little, in thickness. Four of them were selected that showed no such difference; and these were reserved as *standard* slips, the thickness of each of them being taken as unit value of D .

The pieces being all in final position for experiment, the cell empty and open, and the upper conductor screwed close up to the rafters, the distance between the two conductors was found to be rather greater than 1. A standard slip was laid on the floor of the cell, between the two conductors; and

the screws were worked through small distances till the upper conductor was in close and uniform contact with the slip. The final adjustment was obtained when the standard slip was barely movable through the whole space between the conductors, sliding with tight friction, but without lifting or dragging the upper conductor. The cell was now closed and charged, the electro-optic measurements were made; the cell was again emptied and opened, the standard slip was again applied, and the value of D was found to be absolutely unaltered.

All the pieces being in position as they were left by the preceding experiment, an unreserved slip was thrust in permanently between the brass floor of the cell and each of the lateral supports; two of the standard slips, held together as one, were laid between the two conductors; and the screws were worked backwards and forwards through very small distances, till the double slip could slide without lifting or dragging the upper conductor, but with tight friction as formerly. The cell was then closed and charged, and the electro-optic measurements were made for the new value 2 of D , as formerly for the value 1. It was also seen here, as in the former case, that the faces of the conductors were parallel, and that D had exactly the same value at the end of the experiment as at the beginning.

Distances 3 and 4 were managed in the same way. The necessary time and pains were taken to make all the operations as correct as possible. No further explanation of the method is required, the electrometer and compensator being worked here in the same way exactly as before. The improvements mentioned in (17) were retained here of course.

20. *Measurements.*—As my time was now running short, I thought it best to restrict the work to one very careful measurement, or to two measurements at most, for each value of D . The limits of error of observation in the measurement of Q were drawn as close to each other as possible in every case; and, as far as I could judge, the error lay always well within the range ± 1 ; no observed value of Q could possibly be in error by so much as ± 2 .

(1) $D=1$. One measurement was made; and the result was as follows:—

$$E = \frac{V^2}{Q \cdot D^2} = \frac{60^2}{63 \times 1^2} = 57.$$

This experiment was made in very favourable circumstances; and the result was verified again and again.

(2) $D=2$. Two measurements were made and verified; and

the values of E were as follows:—

$$\frac{90^2}{36 \times 2^2} = 56, \quad \frac{120^2}{64 \times 2^2} = 56.$$

The agreement with the first case is more than satisfactory; for if we suppose the true values of Q to be less than the observed values by *half* a unit each, (a difference which lies well within the range of probable error of observation,) the corrected values of E rise to 57 and 56·7.

(3) $D=3$. Two measurements were made; and the values of E were as follows:—

$$\frac{90^2}{16 \times 3^2} = 56, \quad \frac{120^2}{27 \times 3^2} = 59.$$

The agreement of these results with each other and with the first case is pretty close, and hardly unsatisfactory; for if we suppose the true value of Q at potential 120 to be greater than the observed value by 1, the corrected value of E falls to 57.

(4) $D=4$. Two measurements were made and verified, with the following results:—

$$\frac{120^2}{15 \times 4^2} = 60, \quad \frac{150^2}{24 \times 4^2} = 58\frac{1}{2}.$$

The agreement with the first case is close, and, indeed, satisfactory; for if we suppose the true values of Q to be greater each by 1 than the observed values, the corrected values of E fall to about 56.

The uniformity of the electro-optic field was not sensibly perfect in this fourth case, and the increase of D could not safely be carried further.

21. These numerical results are as trustworthy as any that have yet been obtained on the subject. Taken by themselves also, they afford a sketch of a full proof of the Law of Squares in the actual case of CS_2 . I think it worth while, therefore, to exhibit the contents of the last article in another form, as in the following Table. The calculated values of Q are obtained from the equation

$$Q = \frac{V^2}{57 \cdot D^2}.$$

D measured	1	2	2	3	3	4	4
V measured	60	90	120	90	120	120	150
Q measured	63	36	64	16	27	15	24
Q calculated	63·1	35·5	63·1	15·8	28	15·8	24·6

It will be noticed that the differences between observed and calculated values of Q do not extend, in any case, beyond the range ± 1 of probable error of observation. With this Table I close the experimental proof of the Law of Squares in the actual case of dielectric CS_2 .

In proceeding now to my last experiment, I may mention that the rubber of my machine is insulated on pillars of glass, as well as the prime conductor. The object in view is to compare contrary electric forces.

22. *Experiment.*—The arrangements are as described in (8), all the optical pieces being placed as in the diagram.

(1) As in (8), the prime conductor is connected by wires with the cell, the electrometer, and the Leyden jar, while the rubber is connected by a wire with earth. A convenient potential is chosen; the electrometer is set accordingly; and the screw of the compensator is worked as in the former experiments, till the optical action of the dielectric is exactly neutralized at the instant of the critical signal. In the final state of the instruments, the black band lies exactly midway between the reference-wires of the compensator when the index of the electrometer is exactly in the sighted position. From now till the end of the experiment, the electrometer and the compensator are left untouched.

(2) The wires connecting prime conductor with cell and electrometer and Leyden jar are detached from the prime conductor, and fixed in the rubber; and the wire connecting rubber with earth is detached from the rubber, and inserted in the prime conductor. The electro-optic observation is repeated, and the effects are found to be the same in kind and quantity as before. As the electric force increases, the black band moves gradually to the right; and when the index of the electrometer is exactly in the sighted position, the band lies exactly midway between the wires.

It appears thus, as might have been confidently expected, that the optical effect of electric force is independent of the sense of the force, being of exactly the same kind after reversal of the force as before, equal also in amount for equal intensities of the force.

23. In conclusion, I observe that the principal result of the experiments, what I have called the Law of Squares, may be correctly stated in several very different forms. The quantity of optical effect, per unit of thickness of the dielectric, varies either

(1) Directly as the square of the resultant electric force, or

(2) Directly as the energy of the electric field per unit of volume, or

Phil. Mag. S. 5. Vol. 9. No. 55. March 1880.

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(3) Directly as the mutual attraction of the two conductors that limit the field, or

(4) Directly as the *electric tension* of the dielectric, a quantity that was conceived long ago very clearly by Faraday, and introduced afterwards definitely into the Mathematical Theory of Electricity by Professor Clerk Maxwell.

Faraday's and Clerk Maxwell's views as to the action of the dielectric in the transmission of electrostatic force, and as to the state of molecular constraint that is associated with and essential to that action, are very strongly confirmed by the new facts of electro-optics. The dioptric action of an electrically-charged medium is closely related to the electric stress of the medium, the axis of double refraction coinciding in every case with the line of electric tension, and the intensity of double refraction varying, certainly in CS_2 and probably in all other dielectrics, directly and simply as the intensity of the tension.

Glasgow, January 22, 1880.

XXIII. *Note on the Measure of the Intensity of Sound.* By R. H. M. BOSANQUET, *Fellow of St. John's College, Oxford.*

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

I AM happy to say that my proposal for the establishment of an acoustic laboratory* is in a fair way to be realized, and I hope shortly to get to work. But I reserve all account of this for the present.

The object of this note is to present certain considerations as to the expression of the measure of sound-intensities, with a deduction of Fechner's law in a manner which I do not remember to have seen before. The point was suggested to me by a criticism, in which it appeared to be supposed that the measurement of differences of sensation arising from differences of intensity by the ratio of the mechanical intensities was inconsistent with Fechner's law.

The first and simplest hypothesis concerning the difference of the sensations due to two mechanical intensities is, that the difference of the numbers which express the measure of the sensation is proportional to the difference of the mechanical intensities. (The variations due to difference of pitch are throughout disregarded, as that question forms a distinct

* Phil. Mag. Oct. 1879.