III.—CONCEIVABILITY AND THE INFINITE.

By Professor GEORGE S. FULLERTON.

IN an examination of the mathematical antinomies printed in the *Journal of Speculative Philosophy* for January, 1884, I found a solution of those much-mooted problems in the elimination of a contradictory element—the quantitative—which had been inadvertently introduced into our notion of infinity. It was shown that the antinomies arise from the attempt to know the infinite as a whole; that the word 'whole' has necessary reference to limits beyond which there is no more of any object; and that, consequently, the attempt to know all of an infinite is simply an attempt to find the limits of the limitless. It follows, of course, that the infinite is not to be known by a successive synthesis of parts, *i.e.*, by exhausting a quantity; but that, if it is to be known at all, it must be known in some other way. The mathematical antinomies of Kant and Hamilton, reasserted more lately by Mr. Herbert Spencer, when regarded from this stand-point, are disposed of very satisfactorily, and simply disappear; but the question may still be raised whether our notion of infinity does not disappear with them, that is, whether proving the contradictory nature of the quantitative infinite does not simply prove that we have no notion of the infinite whatever, and that our supposed discussions as to infinities are really concerned only with more or less disguised indefinites. This position is not a very sensible one to take, to be sure: for, when a man says of any given object that it is not infinite but indefinite, we usually expect him to know what 'infinite' means, which would intimate that he brought the conception of the infinite before his mind—in some way thought it; and the negative quality of his proposition would imply a true distinction between the conception thus thought and that of the indefinite. But the frequency with which this position is taken, and the confidence with which it is asserted that, the quantitative infinite being proved self-contradictory and the qualitative being inconceivable, we can have no knowledge of the infinite at all, make it desirable that the exact elements of the conception be carefully separated and exhibited, and that it be shown—as it can be—that the conception of an infinite is qualitative, is perfectly conceivable, and that the

act by which such a conception is formed is, in fact, one of our most common mental operations. Those who have maintained the existence of the qualitative infinite as a real conception have been neither few nor far between; but they have not been able to make their doctrine clear to others, nor obtain for it general vogue, partly because they have usually failed to keep the notion free from admixture with other elements quite foreign to it, and partly because they have not made a careful analysis of the actual psychical elements present in the conception, and shown the essential identity of thinking it with other acts which receive general recognition. The question is simply one of psychological analysis, and it is quite possible to discuss it clearly and simply, remembering that our aim is only to find out what is in mind when we habitually use a certain very common word.

The word 'infinite' means endless; and surely it will not be denied that we make a mental distinction between an object thought as having no end at all and one thought as having an end at an indefinite distance. We at least know what we mean by the word 'endless'; and if the word presents any meaning to the mind at all, that to which it is applied must present itself as really distinguished from the merely indefinite. If then, when we speak of the infinite, we mean to say that it is really endless, we cannot know it by a continued addition of parts which may serve to bring us to a limit at some indefinite distance—there is no such limit, definite or indefinite, by the very conditions of the problem; but that which is in mind when we think an infinite must be distinctly different from what is in mind when we add to our previous thought the notion of a limit, and think the indefinite.

Now, when we analyse the mental state in which we have reference to an infinite,—let us say, in this case, an infinite straight line,—we find the following elements: in the first place, there are present the usual qualities of a line, for the fact of our conceiving it as without limits need not alter any of its usual qualities, any more than the fact of our being unable to see the ends of a telegraph-wire need force us to deny that it is a wire of a certain diameter, material or colour; and, in the second place, there is present the notion that, however far we may go in thought, we shall find a continuation of the line; in other words, there is the notion of *unlimited possibility of quantity*—a notion which, be it marked, is strictly qualitative. Quantity in general, not this or that determined quantity, is as much a qualitative

notion as colour or form ; and in thus defining the second element present in our notion of an infinite line, I have used the word advisedly to bring out what is a distinctive characteristic of the conception. The word infinite does not denote a *quantity*, but it has reference to quantity ; and it cannot, in accordance with its derivation and true signification, be rightly applied to what is incapable of being quantitatively considered. My objection to the usage of the word 'infinite' by some who yet recognise that the conception for which it stands is qualitative, is that they overlook the distinctive characteristic of this conception, which marks it out from other qualitative conceptions ; that is, its necessary reference to quantity, though not itself quantitative. If, by that process of abstraction which takes place when I compare objects similar in some of their qualities, I fix my attention upon the other qualities of any finite line, disregarding its length and leaving out of view for the time being its limits, my conception is qualitative ; and yet it is not the conception of an infinite line. In this case, so far from affirming infinite length, I do not think of length at all. But in the case of an infinite line, I add to the former complex of qualities a new quality, possibility of quantity in general, not this or that quantity. When I try to bring before my mind the notion of an infinite line, what I am conscious of is this : I represent in imagination a line of indefinite length, and then run mentally along the line representing additional line-portions—which proceeding would give so far, of course, only the finite ; but what makes my conception distinctively of the infinite is, that, in this progression, I proceed to fix my attention upon the progression itself, and eliminate by abstraction the limits to which such a progression is subject. I do not, be it marked, merely fix my attention upon other qualities of a line, abstracting from the notion of limits ; but I have in mind a progression, a possibility of ever-increasing quantity, and I abstract from the limits of this progression. The two conceptions are distinctly different, although both are qualitative, and they should not be confounded with one another.

The question, therefore, whether I can conceive an infinite line is identical with the question whether I can mentally grasp the usual qualities of a line and the notion of a continuous increase in length, without including the notion of limits : and it will be seen that this question is simply one of the phases of the broader question which is concerned with the possibility of the concept or general notion. A certain complex of qualities being necessary to the existence

of an object as apprehended in nature, or to its subjective existence as represented in the imagination, is there any mental operation by which we may grasp some of these qualities to the exclusion of others, and convey to our own and other minds by the use of the word which stands for this new complex a distinct meaning?

There have been held with reference to this problem, as is well known, three opinions: the doctrine of the Realists, that general ideas have corresponding to them a counterpart reality—a doctrine which may be passed over as now abandoned, though its influence makes itself felt in many directions; the doctrine of the Conceptualists, that, although general ideas cannot exist in nature nor be represented in imagination, yet they have a true mental existence and are the result of a distinct mental operation; and the doctrine of the Nominalists, that the only generality that has a separate existence subjective or objective is the name, which may be applied indifferently to many similar objects.

The Conceptualist may hold that it is possible, unless the words include repugnant elements, to conceive an infinite line,—that is, to grasp in mind a certain complex of psychic elements which are yet incapable of being pictured in the imagination as an infinite line. To think, in the sense of to form such a concept, is to him something other than to imagine. What cannot be imagined may yet be thought. The word 'man', which we define as comprehending the elements of rationality and animality, does not, he claims, in the least include all those other qualities which must be combined with these two before we can picture in the imagination or know as existing any given man. If we select the two qualities in which all the objects of a class resemble each other, and give to these two a special name, have we not brought them into consciousness in some way in which we have not the other qualities possessed by the objects?

When we turn to the Nominalist, it would not be hard to show that, although his doctrines, if taken in strictness, would deny the possibility of the mental operation by which we arrive at the concept, and consequently of the operation by which we may grasp in thought the various elements implied in the expression 'an infinite line,' yet one may find in his teachings by implication ample justification for assuming its possible and actual existence. I will take some extracts from four well-known Nominalists, to show how palpable is the fact stated, and I will first quote from

Berkeley, Locke's great opponent on the subject of the abstract idea :

"Whether others have this wonderful faculty of abstracting their ideas, they best can tell : for myself, I find indeed I have indeed a faculty of imagining, or representing to myself, the ideas of those particular things I have perceived, and of variously compounding and dividing them. I can imagine a man with two heads, or the upper parts of a man joined to the body of a horse. I can consider the hand, the eye, the nose, each by itself abstracted or separated from the rest of the body. But then whatever hand or eye I imagine, it must have some particular shape and colour. Likewise the idea of man that I frame to myself must be either of a white, or a black, or a tawny, a straight or a crooked, a tall or a low, or a middle-sized man. I cannot by any effort of thought conceive the abstract idea above described. And it is equally impossible for me to form the abstract idea of motion distinct from the body moving, and which is neither swift nor slow, curvilinear nor rectilinear ; and the like may be said of all other abstract general ideas whatsoever. To be plain, I own myself able to abstract in one sense, as when I consider some particular parts or qualities separated from others, with which though they are united in some object, yet it is possible they may really exist without them. But I deny that I can abstract from one another, or conceive separately, those qualities which it is impossible should exist so separated ; or that I can frame a general notion, by abstracting from particulars in the manner aforesaid—which last are the two proper acceptations of *abstraction*. And there is ground to think most men will acknowledge themselves to be in my case. The generality of men which are simple and illiterate never pretend to *abstract notions*. It is said they are difficult, and not to be attained without pains and study ; we may therefore reasonably conclude that, if such there be, they are confined only to the learned." (*Principles of Human Knowledge*, Introd., § 10.)

So much for Berkeley's position with respect to the abstract notion. But mark the concessions which he is forced to make in a later section (16) :

"But here it will be demanded, how we can know any proposition to be true of all particular triangles, except we have first seen it demonstrated of the abstract idea of a triangle which equally agrees to all? For, because a property may be demonstrated to agree to some one particular triangle, it will not thence follow that it equally belongs to any other triangle, which in all respects is not the same with it. For example, having demonstrated that the three angles of an isosceles rectangular triangle are equal to two right ones, I cannot therefore conclude this affection agrees to all other triangles which have neither a right angle nor two equal sides. It seems therefore that, to be certain this proposition is universally true, we must either make a particular demonstration for every particular triangle, which is impossible, or once for all demonstrate it of the abstract idea of a triangle, in which all the particulars do indifferently partake and by which they are all equally represented. To which I answer, that, though the idea I have in view whilst I make the demonstration be, for instance, that of an isosceles rectangular triangle whose sides are of a determinate length, I may nevertheless be certain it extends to all other rectilinear triangles, of what sort or bigness soever. And that, because neither the right angle, nor the equality, nor determinate length of the sides are at all concerned in the demonstration. It is true the diagram I have in view includes all these particulars, but then there is not the least mention made of them in

the proof of the proposition. It is not said the three angles are equal to two right ones, because one of them is a right angle, or because the sides comprehending it are of the same length. Which sufficiently shows that the right angle might have been oblique, and the sides unequal, and for all that the demonstration have held good. And for this reason it is that I conclude that to be true of any obliquangular or scalenon which I had demonstrated of a particular right-angled equi-crural triangle, and not because I demonstrated the proposition of the abstract idea of a triangle. And here it must be acknowledged that a man may consider a figure merely as triangular, without attending to the particular qualities of the angles, or relations of the sides. So far he may abstract; but this will never prove that he can frame an abstract, general, inconsistent idea of a triangle. In like manner we may consider Peter so far forth as man, or so far forth as animal, without framing the fore-mentioned abstract idea, either of man or of animal, inasmuch as all that is perceived is not considered."

In the former of these two extracts Berkeley has declared himself able to abstract only so far that he can represent to himself in imagination what can exist separately in nature. He denies that he can conceive separately those qualities which it is impossible should exist separately. But when he supposes an objector to ask, How is it possible for something, proved to be true of a particular triangle, to be known to be true of all triangles? he answers that it is seen that neither the right angle, nor the equality, nor the determinate length of the sides is at all concerned in the demonstration. In other words, he admits that, so far as that demonstration goes, we have to do only with those elements in which all triangles agree. And if we can reason about certain elements to the exclusion of others; if we can see that certain objects are alike in certain elements and unlike in others; if we can give a name to objects simply to express the presence of these same elements, however the elements accompanying them may vary,—then surely the elements of the concept have been before the mind in some way in which the others have not, and have been grasped together.

Berkeley frankly admits as much in the concluding sentences of the latter extract,—sentences which were added twenty-four years after the first publication of the essay, when mature reflection, we may suppose, had brought him to see that on his previous principles, strictly held, all comparison of objects differing in any of their qualities would be impossible. If we can consider a figure merely as triangular, without attending to the particular qualities of the angles, or relations of the sides, then we can in some sort divorce the elements included under the general word 'triangle' from the accompanying elements, and consider

them separately. In those last few sentences Berkeley admits all that a reasonable Conceptualist would care to prove, and the words 'abstract idea,' as there used, are equivalent to object of the imagination—a something which is not implied in the formation of the abstract or general notion.

Every one, Nominalist or Conceptualist, must acknowledge that we can compare objects, and recognise them as like or unlike:—not merely like or unlike as wholes, but in this or that element; like in length, unlike in breadth; like in colour, unlike in shape. Now no one claims that we can call into clear consciousness the element of length alone, and picture it divorced of breadth and colour; but when we recognise two objects as like in length and unlike in breadth, the elements must in some way have been present in mind separately, so as to be recognised separately as length and breadth. If one object that what is present in consciousness must *ipso facto* be perceived, and that we cannot perceive length as a factor by itself, nor recall in memory any perception of such a factor during the act of comparison; I answer that what is in consciousness is not necessarily in a clear analytic consciousness; and that we may by a process of deductive reasoning be sure that certain elements are present as factors in a given mental state, while we are yet quite unable to call these elements into a clear analytic consciousness, separated from certain other elements bound to them by long association and habit. As an instance I refer to vision. That distance is itself unperceivable by sight we must admit. That judgments of distance are a result of reasoning from an observed constant connexion of certain visual with certain other elements may be satisfactorily established when the above proposition is admitted. But to call into clear consciousness by itself the purely visual sensation, which forms the basis of the judgment, is altogether impossible. That it is a factor, and an important factor, in the complex consciousness which we have at the time, we admit: and yet its presence, as a single and distinct element, is capable of being only deductively known. Notice a further point which is worthy of remark. If we vary the purely visual element, allowing all other elements to remain the same, that is, if we change the *colour* of the object, but do not change in any respect the form or size of the image on the retina, a difference is at once remarked, and the change of colour recognised. But the difference is not recognised as a difference between two purely visual sensations, when the result of the actual comparison comes out into clear

consciousness, but as a difference in one of their elements between two complexes or wholes. That is to say, the two purely visual sensations cannot be brought into clear consciousness and recognised as compared with each other alone, but only come out clearly as combined with certain other elements in complexes or wholes;—it is the presence of two or more such wholes, which we wish to compare, that primarily conditions the narrowing of the attention to the particular similar or dissimilar elements; and it is from the presence of two or more such wholes, of which we are conscious as compared, that we are led to infer the presence of single psychic elements in a dim unanalytic consciousness, as a necessary condition to the possibility of all ordinary comparison and classification.

When I form the concept of length, by comparing two objects in length and affirming agreement, and then recognising as a distinct element that in which they agree, I certainly do not compare the objects simply as wholes, but compare the *lengths*; and I must certainly have had these elements in mind in some way in which I had not the other elements which go to make up the object. Whether I can call into clear consciousness the psychic elements present during the operation or not, it does not much matter; I evidently have specialised, selected one element from among others, and compared length with length as element with element. The name which we give to such resemblances is the name representing a general or abstract idea.

Hume warmly applauds the position taken by Berkeley with reference to the abstract idea, calling it "one of the greatest and most valuable discoveries that have been made of late years in the republic of letters," and he undertakes to confirm it with proofs that he hopes will put it "beyond all doubt and controversy". For the same purpose for which I have quoted the two extracts from Berkeley, I will quote the last part of the section which he devotes to the establishment of this position:

"It is certain that the mind would never have dreamed of distinguishing a figure from the body figured, as being in reality neither distinguishable, nor different, nor separable, did it not observe, that even in this simplicity there might be contained many different resemblances and relations. Thus, when a globe of white marble is presented, we receive only the impression of a white colour disposed in a certain form, nor are we able to separate and distinguish the colour from the form. But, observing afterwards a globe of black marble and a cube of white and comparing them with our former object, we find two separate resemblances in what formerly seemed, and really is, perfectly inseparable. After a little more practice of this kind, we begin to distinguish the figure from the colour by a *distinction of reason*; that is, we consider the figure and colour together, since they are,

in effect, the same and undistinguishable ; but still view them in different aspects, according to the resemblances of which they are susceptible. When we would consider only the figure of the globe of white marble, we form in reality an idea both of the figure and colour, but tacitly carry our eye to its resemblance with the globe of black marble : and in the same manner, when we would consider its colour only, we turn our view to its resemblance with the cube of white marble. By this means we accompany our ideas with a kind of reflection, of which custom renders us, in a great measure, insensible. A person who desires us to consider the figure of a globe of white marble without thinking on its colour, desires an impossibility ; but his meaning is, that we should consider the colour and figure together, but still keep in our eye the resemblance to the globe of black marble, or that to any other globe of whatever colour or substance." (*Treatise of Human Nature*, i. 7.)

It is not hard to see that we cannot distinguish in a body figured "many different resemblances and relations," without bringing the resembling elements in some sense singly into thought : if the mental complex which we call an object were an indissoluble unit, we might affirm a general likeness or unlikeness between it and other objects, but we could not affirm that the resemblance lay in the figure, or colour. If, as Hume asserts, the figure and colour "are, in effect, the same and undistinguishable," why do we find the one susceptible of the one class of resemblances, and the other of another class? If we take the words literally, should not the figure, viewed in one aspect, be susceptible of resemblances of figure, and viewed in another, of colour? And similarly, if the colour is one with the figure—the same and undistinguishable,—should not the colour, viewed in one aspect, be susceptible of resemblances of colour, and viewed in another, of figure? Hume's admission that the two elements are known as giving different resemblances in itself refutes his previous assertion that they are undistinguishable. If colour be recognised as like colour, and figure like figure, the two qualities are distinguished as different, and are in reality separately grasped.

I will now take a passage from J. S. Mill's *Examination of Sir William Hamilton's Philosophy* (c. xvii.) :

"The formation, therefore, of a Concept, does not consist in separating the attributes which are said to compose it, from all other attributes of the same object, and enabling us to conceive those attributes, disjoined from any others. We neither conceive them, nor think them, nor cognise them in any way as a thing apart, but solely as forming, in combination with numerous other attributes, the idea of an individual object. But, though thinking them only as part of a larger agglomeration, we have the power of fixing our attention on them, to the neglect of the other attributes with which we think them combined. While the concentration of attention actually lasts, if it is sufficiently intense, we may be temporarily unconscious of any of the other attributes, and may really, for a brief interval, have nothing present to our mind but the attributes constituent to the

concept. In general, however, the attention is not so completely exclusive as this; it leaves room in consciousness for other elements of the concrete idea; though of these the consciousness is faint, in proportion to the energy of the concentrative effort, and the moment the attention relaxes, if the same concrete idea continues to be contemplated, its other constituents come out into consciousness. General concepts, therefore, we have, properly speaking, none; we have only complex ideas of objects in the concrete: but we are able to attend exclusively to certain parts of the concrete idea; and, by that exclusive attention, we enable those parts to determine exclusively the course of our thoughts as subsequently called up by association; and are in a condition to carry on a train of meditation or reasoning relating to those parts only, exactly as if we were able to conceive them separately from the rest."

This passage is so clearly in harmony with the views of the Conceptualist, as I have portrayed them, that it seems scarcely necessary to comment upon it. But I cannot resist the temptation to delay for a moment over an inconsistency into which Mill was forced by his attempt to recognise, though a Nominalist, a truth which the Nominalist pure and simple cannot recognise. The formation of a concept, he insists, does not consist "in separating the attributes said to compose it, from all other attributes of the same object, and enabling us to conceive those attributes, disjoined from any others": this position he emphasises by the further affirmation, that "we neither conceive them, nor think them, nor cognise them in any way as a thing apart, but solely as forming, in combination with numerous other attributes, the idea of an individual object". These sentences are certainly unequivocal;—they contain an emphatic assertion of the Nominalistic doctrine. But side by side with such statements we find it asserted that we may fix the attention upon the attributes constituent of the concept, to the neglect of the other attributes of the object, and that, while the concentration of attention actually lasts, if it is sufficiently intense, "we may be temporarily unconscious of any of the other attributes, and may really, for a brief interval, have nothing present to our mind but the attributes constituent of the concept". Surely, if the only elements before the mind are those constituent of the concept; if we may be conscious of these, even for a brief interval, and conscious of these alone;—surely in such a case we conceive, or think, or in some way cognise the attributes forming the concept as separate and apart, and *not*, for the time being, in combination with numerous other attributes. Mill goes even further in the above admission than most of us would care to follow him. In speaking as he does of the process, and not distinguishing between the imagining of an object and the knowing of one or more of its isolated qualities, he

clearly intimates, although he does not distinctly say, that the elements before the mind during the formation or use of a concept are in consciousness in the same way in which the whole complex or object may be in consciousness. But, to recur to the before-mentioned analogy of the purely visual element in vision, we know that, although we may so concentrate the attention as to distinguish the blue colour of one object from the red colour of another, and so must have compared in some rapid manner these purely visual sensations, yet when we try to call into clear consciousness the mere sensation of colour, we cannot do it without imagining the colour as on a surface, or as combined with psychic elements not purely visual. That is to say, the single and separate sensations cannot be called into a clear consciousness, and their presence when we use the concept, or have occasion to compare them singly with each other, is something quite distinct and different from the presence in consciousness of the complex which is knowable as an object. And such would seem also to be the case wherever we call before the mind single psychic elements which can yet not be represented alone in the imagination. The element must have been grasped separately, but it can be brought into a clear consciousness only in combination.

If now we recognise in each of two objects presented to us a certain quality or complex of qualities upon which we can fix the attention ; and if we discover that, so far as these qualities go, there is an undistinguishable similarity in the objects, the differences arising altogether from other qualities ; why may we not call the complex of qualities in point a *general notion* or *general idea* ? Of course, whether we should call the qualities, in the two instances, the *same*, even if undistinguishably similar, would depend on our use of the word 'same,' and our ideas of what constitutes sameness or identity : but I can see no objection to using the words 'general notion' to indicate the fact that a certain complex of qualities is to be found in many different combinations with other qualities. Should it still be insisted that, since we cannot bring separately into clear consciousness these elements of objects known, we have no reason to assume that we actually conceive them or think them separately, I will not quarrel over the use of a word, but will simply state that I find the word 'conceive' a useful one to express that concentration of the attention upon certain qualities of an object, which takes place when objects are compared, and which eliminates from consciousness, or at least subordinates, all other qualities of the objects : and I will so use the word,

applying it to an operation the existence of which Mill has in so many words admitted.

The last author whom I will quote is Prof. Bain : I will take some passages from the chapter on Abstraction in his *Mental and Moral Science* pp. 176-8, where he supports the Nominalistic doctrine :

"We are able to attend to the points of agreement of resembling things, and to neglect the points of difference ; as when we think of the light of luminous bodies, or the roundness of round bodies. This power is named Abstraction.

"It is a fact that we can direct our attention, or our thoughts, to the points of agreement of bodies that agree. We can think of the light of the heavenly bodies, and make assertions, and draw inferences respecting it. So we can think of the roundness of spherical bodies, and discard the consideration of their colour and size. In such an object as the full moon, we can concentrate our regards upon its luminous character, wherein it agrees with one class of objects ; or upon its figure, wherein it agrees with another class of objects. We can think of the taste of a strawberry, either as agreeing with other tastes, or as agreeing with pleasures generally. . . .

"Every concrete thing falls into as many classes as it has attributes ; to refer it to one of these classes, and to think of the corresponding attribute, are one mental operation.

"When a concrete thing before the view recalls others agreeing in a certain point, our attention is awake upon that point ; when the moon recalls other luminous bodies, we are thinking of its light ; when it recalls other round bodies, we are thinking of its roundness. The two operations are not different but identical.

"On this supposition, to abstract, or to think of a property in the abstract, is to classify under some one head. To abstract the property of transparency from water, is to recall, at the instance of water, window glass, crystal, air, &c. ; to abstract its liquidity, is to recall milk, vinegar, melted butter, mercury, &c. ; to abstract its weight, is to bring it into comparison with other kinds of gravitating matter.

"Hence abstraction does not properly consist in the mental separation of one property of a thing from the other properties—as in thinking of the roundness of the moon apart from its luminosity and apparent magnitude. Such a separation is impracticable ; no one can think of a circle without colour and a definite size. All the purposes of the abstract idea are served by conceiving a concrete thing in company with others resembling it in the attribute in question ; and by affirming nothing, of the one concrete, but what is true of all those others. . . .

"In abstract reasoning, therefore, we are not so much engaged with any single thing, as with a class of things. When we are discussing government, we commonly have in view a number of governments, alternately thought of ; if we notice in any one government a certain feature, we run over the rest in our mind, to see if the same feature is present in all. There is no such thing as an idea of government in the abstract ; there is only possible a comparison of governments in the concrete ; the abstraction is the likeness or community of the individuals."

It will be noticed that throughout this extract Prof. Bain does not distinguish between those elements of an act which come out into a clear consciousness, and the elements which do not so come out but are nevertheless necessary to the

possibility of the operation. When he says, for instance, that there is no such thing as the idea of government in the abstract, but that we can compare governments in the concrete and recognise the likeness of the individuals, it is perfectly true that all that we are clearly conscious of is several individual objects and a similarity between them ; but, when we come to analyse this recognition of a similarity, it will be seen that the elements which are known as similar are quite incapable, by themselves, of forming a concrete object, and yet they are distinguished by the mind from the dissimilar elements : they must, therefore, have been in some sort grasped separately, though they cannot separately be brought into a clear consciousness. Whether, during this rapid act of concentration and comparison, the other elements which go to form the object actually disappear from consciousness, or are only dimly perceived, as Mill suggests that they are in most cases, does not affect the peculiar character of the act. When I compare in height two trees, which I see side by side in the distant landscape before me, I am perhaps conscious of several objects in their immediate vicinity in a dim and indefinite way, but the two objects compared are present in consciousness in a manner very different, and are grasped, so to speak, separately. And when I fix my attention upon the height of the two trees, finding them similar or dissimilar in this one element, we have every reason to suppose that something very analogous takes place, and that this one element is present in consciousness in some way quite different from the others, and is grasped separately, for the time being. Were it not so, we could not say that the trees were alike in height, but different in contour or colour of foliage. We are justified in assuming that, when we recognise two trees as alike in height but not in colour, we have compared height with height, and colour with colour, and not merely compared the one object as an undistinguishable complex of qualities with another object as another undistinguishable complex.

As I have before said, the name which we choose to apply to this operation is of little consequence : the point to be chiefly borne in mind is, that we have here an operation differing from ordinary imagination, in that it takes cognisance of certain psychic elements which can yet not be called into clear consciousness by themselves as a mental picture. Whether the two operations completely differ in their ultimate nature is another question. When the Conceptualist asserts that, though he cannot imagine length apart from breadth or colour, yet he can conceive or think it, he

merely marks by a distinct name his recognition of an operation different from imagination, and which is implied in all comparison of objects. What may be the peculiar psychic elements present in the operation he does not necessarily know, nor express when he uses the name.¹

Arguing from the analogy of the visual element in vision, one might conclude that what is actually present in consciousness in comparing lengths, for example, is the distinctive element or sensation which is present in combination with other elements (and consequently in a modified form) in all our experience of extended objects, and which, in the act of comparing objects, may be brought into sufficient prominence to be considered, for the moment, alone, and alone compared with its kind. When we make the attempt to call it into clear consciousness, the element appears as modified by, and in combination with, others; but it is not improbable that, in the act of comparison, it obtains in its pure state sufficient recognition to make possible a comparison with a similar element also in its pure state. However, whether we can describe just what is present during the act or not, we may be sure that a mental separation of two objects into their elements is necessary in order to a recognition of them as in some points similar and in some dissimilar.

In view of the foregoing, I would, therefore, regard the fact as beyond all doubt, that there are mental operations differing distinctly from imagination, in that certain elements, of which we have usually, as single elements, no analytic consciousness but which are merged with others into an indivisible whole, are brought for the time being into such prominence as to be compared individually with similar elements and recognised as like or unlike. We cannot hold these up to inspection as single elements, but from the fact of the comparison, about which there can be no doubt, we may be very sure that the operation in question has taken place.

Now when we return to the particular conception that we have been considering,—that of an infinite line,—we find it

¹ Kant seems to have despaired of the possibility of ever making this analysis:—"Dieser Schematismus unseres Verstandes, in Ansehung der Erscheinungen und ihrer blossen Form, ist eine verborgene Kunst in den Tiefen der menschlichen Seele, deren wahre Handgriffe wir der Natur schwerlich jemals abrathen und sie unverdeckt vor Augen legen werden." (*Kritik d. reinen Vernunft*: "Von dem Schematismus der reinen Verstandesbegriffe");—yet Kant did not doubt the existence of the operation.

merely a concrete instance of this general truth which all must either explicitly or implicitly admit. As I have said, the elements constituent of this conception are the usual qualitative attributes of a line and the notion of continued progression, of unlimited possibility of quantity. These elements may be brought into mind, to the exclusion of the notion of limits, which are yet present in all imagined lines and in all intuitions of lines in nature, by employing the usual process of forming a concept. When I think of an infinite line, I first represent to myself a line of some indefinite length, and I then run mentally along this line, adding new portions; that is, I successively think several increasing lengths: I have now before my mind what Hume and Prof. Bain insist upon and make so prominent in forming the concept, several concrete objects similar in some of their qualities. Having mentally passed over several of these line-portions, I then fix my attention, not upon that in which they differ, the quantitative element, but upon that in which they agree, the usual qualitative attributes of a line, and the element of increase or progression, which is common to all. This is precisely what I do in forming the concept man or animal. The concrete objects are compared, their differences eliminated by abstraction, and their likenesses grasped together under a distinctive name. Or I may select one of the qualities in which objects agree and consider it alone, as when I compare men, of the same age and colour, only as to their height, and pronounce them equal in height. If this be possible, if, in using the word man, I can distinguish between that in which men agree and that in which they disagree, and if it be further possible for me to fix my attention upon one of the points in which they agree to the exclusion of others; then it is possible to abstract from the particular quantities or amounts of several lines present in imagination, and think only of a constant increase or progression. That both the one and the other are not only possible but actual operations, is proved beyond possibility of doubt by our constant comparison of objects, our use of general language, our frequent use of the word 'infinite,' to indicate what is clearly distinguished, readily defined, and conveys a distinct meaning to speaker and hearer.¹

¹ It might be objected that the case cited is not fairly analogous, on the ground that an end is not an actual constituent element of an object, which may exist and be contemplated apart from the other elements. But the sensation of progression in passing the eye along a line is very clearly different from the sensation of arrested motion, of an end to the intuition, and there is no reason why these two sensations should not be separated by

To the objection which may be made to my classing the notion of this or that particular infinite line with the concept or general notion, on the ground that the individual or the intuition is something quite different and distinct from the concept, I answer that the notion of any particular infinite line is not a complete intuition, in that one of the elements of the intuition is eliminated by abstraction; and that when, in the formation of any concept, we fix the attention upon certain elements of an intuition to the exclusion of others, we have in mind, so to speak, a constituent part of an intuition: the fact that we recognise its similarity or sameness with parts of other intuitions does not alter the individual character of the elements which we actually have in mind. The operation of forming a concept and the operation of conceiving an infinite line are in nature identical.

It seems impossible that any one, having reflected upon the fact of his constantly grasping in concepts elements which can yet not be separately imagined, and having, after an analysis of what is in his mind when he calls up the notion of an infinite, discerned the identity of the latter operation with the former,—it seems impossible that such an one should hold infinite space or infinite time to be inconceivable. If, however, he should still object that, even if it be true that we can grasp in thought the notion of progression, and the notion of a line in general, this will give us no knowledge of an infinite line, but will give us only the elements of an incomplete image, which cannot be called distinctly before consciousness, and, therefore, cannot be known as an object at all;—we may answer that, if he feel himself aggrieved because he cannot represent to himself, endowed with all the qualities necessary to an object of the imagination, that which he has already defined as wanting some of those qualities, he must also be unreasonable enough to think it ground for complaint that he cannot in thought make parallel lines meet, or imagine a triangle with four sides. The word 'infinite' means devoid of limits, and it necessarily follows that an infinite line cannot be known as a quantity, consequently not as a whole. Every *object* which is seen or imagined has necessarily limits, definite or indefinite: an infinite line, as infinite, cannot become an object of the imagination. But from this it by no means follows

what Hume was pleased to call a "distinction of reason," and at least the former be held in the attention abstracted from the latter. Whether the latter were wholly eliminated or only subordinated, the complex mental state would still be quite different from that in which the elements have the relative prominence usual in our ordinary experience of objects.

that I cannot call a particular line infinite, provided I have some proof for the fact other than its conceivability, and that I cannot know my conception to be in harmony with the reality. Suppose that, either from testimony, or by means of some *a priori* chain of reasoning, I have good reason to believe a given line endless: I can conceive the line as without end, and I may know my conception, although it does not represent the total content of my consciousness when at any moment I gaze upon this or that part of the line, to be a true and real conception, and in harmony with my experience as I pass over the line; and I may be certain that, however long my experience may continue, it will yet not prove incompatible with the conception I have formed. In this sense any infinite object is conceivable, and there would seem to be no other conceivable way in which we could conceive it: an infinite object which could be known as a whole is not even an object of thought, for the elements indicated by the words cannot be so put together as to express a meaning. But the conception of the infinite, rightly defined, contains in it nothing either contradictory or beyond the grasp of the human mind, and is, indeed, a very common conception, as is evidenced by use of the word in literature ancient and modern, to say nothing of its constant occurrence in the debates of those very philosophers who find the conception such a stone of stumbling. And that the conception is a real one, having a real consonance with experience, those who hold to the Christian doctrine of Immortality will not be slow to maintain.