MIND

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OF

PSYCHOLOGY AND PHILOSOPHY

I.—ANALYSIS OF THINKING. (II.).

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ONLY a few words must be here interpolated in introducing the topic of the logical *conjunction*. A construct in which predications are united by logical conjunctions constitutes a compound predication, and must be treated in connexion with *negation*. A predication involving conjunctions and negations will be termed a *conjunctional* function of its constituent predications. Thus the conjunctions *and*, *or*, *if* yield respectively the conjunctive, alternative, and implicative form or function. The details of the theory of conjunctional functions belong to the preliminaries of formal logic and cannot be entered upon here. It is only necessary to remark that conjunctions—like *ties*—are, properly speaking, not genuine constituents, but merely formal elements. The same holds of logical *punctuation*, by which the bracketing and separating of sub-constructs constituting a complex construct are indicated—another topic upon which space forbids us to enter.

In discussing the logical forms of construction, we will next consider the import of the substantival prefixes—a, the, some, every, etc., which I propose to call *selectives* or *applicatives*. An applicative is always prefixed to a general substantive name; and an applicative may be defined to be a non-characterising determinator of application. Applicatives are sometimes classified by grammarians under adjectives: but an adjective proper, when prefixed to a general substantive name, restricts the range of application by a *characterising* differentia, whereas an applicative determines application

otherwise. The first applicative to be considered is the indefinite article *a*. In some languages this article does not exist;—a fact which may be explained by defining the indefinite article to be the *undetermining applicative*. This applicative is directly opposed to the *uniquely-determining applicative*, expressed most generally by the definite article *the*. The most important of the remaining applicatives are the *distributives*, *every* and *some*, which may respectively be termed conjunctive and alternative, since the former condenses a conjunctive, and the latter an alternative reference. For example: "Every one of the twelve chosen disciples was a Jew" yields the *conjunctive* proposition—"Peter and John and James and . . . was a Jew"; and "Some one of the twelve chosen disciples was a traitor" yields the *alternative* proposition "Peter or John or James or . . . was a traitor". The same interpretation holds when the substantial reference cannot be exhausted enumeratively; as in "Every one of the points on the sphere S is at distance r from the centre O"; or "Some of the points on the line L are at distance r from the point P". Again, the familiar forms of proposition "Every man is fallible" "Some men are foolish" yield, respectively, conjunctive and alternative implications. The strict use of the article *a* as the undetermining applicative, must be distinguished from that use which is more unequivocally expressed by the phrase "a certain". The simplest use of the phrase "a certain" is where it serves to introduce an object to which we may return in thought, and in such return employ the article "the". For example—"A certain boy lived in a forest, and this (or the) boy planted a bean-stalk". When the phrase "a certain" is followed, in this way, by "the" or some equivalent, the former may be called the introductory indefinite, and the latter the referential definite.

We proceed, then, to the consideration of the *unique selective*, of which there are several variations, the connexions between which must be carefully inspected. Before discussing the question of a unique selective, let us consider the more general topic of a unique *name*. A unique substantive-name will be illustrated by "Peter" or "London," a unique adjective-name by "cold" or "rigid". The use of names such as these presupposes a process of unique identification. In however simple or complex a manner this process of identification may have been effected, the *use* of the name is independent of this presupposed process of identification. A name used with literally no indication of any presupposed process of constructive identification may be called a *proper*

name. Such a name must always be *introduced* with an indication of some process of constructive identification, by means of which the application of the name may be understood by those who use it to be the same as that determined by the constructive process. Thus, when we hear that "the Prime Minister of England in 1914 was Asquith," we see that the name Asquith, as distinguished from the complex name standing as *subject* of the proposition, is a name which does not indicate any process of constructive identification. In working out logical systems of symbolism, it is an invariable custom to select letters such as *a*, *b*, *x*, *y*, *s*, *p*, to denote uniquely determined objects. These symbols do not indicate any process of construction by which their application might be uniquely determined; and, in their further use, the constructions into which they enter are unaffected, however their unique application may be—or may have been—determined. They are often at first introduced in the construction of general formulæ, and later are *applied* to denote this or that defined construct. In this case the process of constructive identification *follows* the introduction of the symbols; but in other cases, specific symbols are introduced *after*, instead of before, the process of constructive identification has been indicated. Whichever of these two methods is adopted, we see that the symbols are *used* without any reference to any process of construction, and that, therefore, they have all the characteristics of the proper name. The fact that in symbolic systems many symbols are introduced *before* any indication of constructive process has been given, has led logicians to the mistaken view that the proposition that is symbolically the simplest—*viz.*, that in which the only substantive-names and adjective-names are proper names—is logically or psychologically the earliest. We hold, on the contrary, that the *proper* name must be defined in use always, ultimately, by means of a *descriptive* name, in the wide sense of the term descriptive. More primitive than either of these is a name which can only be expressed by the prefix, "a certain," which is a special selective, not generally distinguished from the indefinite article. In fact, this selective seems to blend the characteristics of the unique "the," and the alternative indefinite. Phrases which involve "a certain" do not convey to the hearer any process of identification by which the thinker has determined the reference. Therefore, although, for the thinker "a certain" stands for a unique selective, for the hearer it is merely an alternative indefinite. The exclamation "Thunder!" regarded as an assertion, is formless as regards substantival reference, and could only be

linguistically expressed somewhat as follows: "A certain manifestation has the character thunder". When the manifestation about which such an assertion has been made is referred to again, language adopts what we may call the referential article "the" or "this". Thus when we speak of *the* object which has been just experienced by me, or about which I have just been thinking or speaking, or in reference to which a certain proposition has been put forward, then, in every case, we are using a descriptive phrase (in the widest sense of the term) which indicates uniquely the reference intended. Now names constructed in this manner may be called referentially unique and be regarded as a special class of descriptively unique names. One familiar case of their occurrence is in narratives—fictitious or historical. In order to indicate unambiguously the reference intended, such phrases as "the latter," "the former," "celuici," "celuila," "hic," and "ille" may take the place of "the" as indicating referential uniqueness; and again the referential pronouns "he," "she," "it," perform the same function. These devices must be supplemented in a continued discourse by the use of proper names, adopted either for permanent application, or for merely temporary and contextual reference. What is effected in narrative by the proper name is secured in symbolic systems, mathematical or otherwise, by arbitrary symbols such as *a*, *b*, *x*, *y*, which are understood to indicate unique identification where the *same* symbol recurs in the *same* context. Whatever device may have been adopted, we may speak of the uniquely determined object as the *selected* object.

A construct involving substantives and adjectives, united by characterising and coupling ties, and by logical conjunctions, may be called a *descript*. The strictly undetermining applicative marks the *undetermined* descript. Consider the undetermined descript "A child afraid of a dog"—which is equivalent to "A dog frightening a child". Here, such a component as "a dog" or "a child" (which is of the general nature of a substantive) will be called a *descriptum*; and such a component as "afraid of a dog" or "frightening a child" (which is of the general nature of an adjective) will be called a *description*. The presence of the indefinite article indicates undeterminedness whether it be of the descriptum or of the description. But when, in place of "a dog" we substitute "a certain dog" or "some dog" or "every dog," then instead of the undetermined description "afraid of a dog" we obtain a *determined description*. Again, if we substitute for "a child" "a certain child" or "some child" or

"every child," then we obtain a *determined descriptum*. Consider for instance "Some children as being afraid of every dog," or again "Every dog as frightening some child". In the former case we have the determined descriptum "some children" and the determined description "afraid of every dog"; in the latter case we have the determined descriptum "every dog" and the determined description "frightening some children". These examples illustrate the principle that a descript becomes determined by means of a separate and independent determination of the description-component and of the descriptum-component. In other words, the description is determined *without reference to the descriptum*, and the descriptum is determined *without reference to the description*.

Having so far indicated the distinction between the undetermined and the determined descript, we must consider the distinction in mental attitude to which the transition from the one to the other construct may lead. The mental attitude adopted towards the undetermined descript is that of mere apprehension. In passing to the determined descript we have passed from the attitude of mere apprehension to that of *proposing*; and the construct so obtained is therefore called a proposition, towards which we may be explicitly preparing to adopt an attitude of interrogation. We may compare the term "proposition" with "supposition": whereas the former suggests the preparation for a *direct* interrogative attitude, the latter suggests the preparation for an *indirect* interrogative, in which we consider the consequences which would be entailed by asserting the proposition in question. Thus the same construct might be called either a proposition or a supposition according as it is taken up in the attitude of proposing or of supposing. In the same way many terms, such as presumption, assumption, presupposition, and so on, stand for the same construct—that might be called a proposition—towards which we are adopting one or other of the various attitudes of presuming, assuming, presupposing, etc. Thinking, as so far analysed, consists in the apprehension of a quality as characterising a given object; but we must recognise another aspect of thinking which goes beyond *mere apprehension*. What I have here called mere apprehension has been usually contrasted with *belief*, and it is agreed that belief necessarily *involves* apprehension. Now the object of belief is called a *proposition*, so that a proposition may be defined as an object of *possible* belief;—possible, because while the object of belief is always a proposition, the proposition may be merely apprehended without being believed; or

it may be entertained in some attitude opposed to belief, such as disbelief or doubt. To doubt a proposition implies that we neither believe nor disbelieve it, while belief and disbelief, as opposed to doubt, have in common the characteristic that may be called assurance. Thus there are three opposed attitudes towards a proposition included in the distinction between doubt and assurance; the latter of which may be either (assured) belief or (assured) disbelief, and the former of which appears to be susceptible of varying felt degrees.

We must consider in further detail what is involved in converting an undetermined descript into a proposition, beyond the fact that all the references must be determined. Returning to the principle that a descript becomes determined by means of a *separate and independent* determination of the description-component and of the descriptum-component, it is to be noted that this mutual independence effects a severance between the descriptum and the description, which are therefore united in the proposition—not only by the characterising tie but also by what we may call the *assertive tie*. The blending of the assertive tie with the characterising tie is expressed in language by the transition from the participial or relative clause to the finite or declaratory form of the verb. Thus in passing from “a child fearing a dog” to “a child fears a dog,” the characterising tie joins the same elements in both cases, but is, in the latter, blended with the assertive tie. That the assertive tie is blended with the characterising tie is further shown by the modifications “is-not,” “may be,” “must be,” by which the verb “to be” is inflected, in order to indicate variations in the assertive attitude. The copula “is” of traditional logic is now seen to be a blend of the characterising with the assertive tie.

We thus see that the proposition, as such, is a kind of construct to which we may stand in a unique kind of attitude, determinable in more or less opposed modes. These different modes may be termed *assertive*, and the several specific determinations of the (generic) assertive attitude are known as asserting, denying, supposing, doubting, and so on. Now any assertive attitude is an occurrent relation of the thinker to the proposition—regarded as a *whole*—and not a relation to its parts. Thus the term proposition can only be defined by reference to the notion of an assertive attitude. As so defined, a proposition may be conveniently termed an *assertum*; i.e., the kind of construct towards which we may adopt one or other of the variable modes of the assertive attitude. Now when G asserts or doubts or denies that “Everything [or something] that is *p* is *q*,” his assertive attitude is directed

to the proposition as a whole; and this relation to the proposition as a whole involves *the relation of apprehension* to the component adjectives *p* and *q*. We have to examine the question whether this entails a relation to the descriptum—Everything that is *p*. Compare the above statement that “G asserts that everything that is *p* is *q*” with the following: “With respect to everything that is *p*, G asserts that *it* is *q*”. In this latter case G stands in a definite relation to the descriptum—Everything that is *p*. The distinction between the two cases may be made clear by several considerations. Thus, let us construct a syllogism by introducing a minor premiss. In the latter case, we should take as minor (say) “This is *p*” and infer that “G asserts that *this* is *q*”. But, in the former case, the required minor would be “G asserts that this is *p*” in order to reach the same conclusion. In fact, premisses can only be joined to elicit a conclusion, when they are put together at the same time by the same asserter. This was illustrated explicitly in the case of G’s two assertions above; but it is implicitly involved also in the other case, where the two premisses must be understood to have been asserted by the same person (say H). Another way of emphasising the distinction between the two cases is shown by substituting for *q* the negative not-*p*. We should then have to contrast (1) “G asserts that everything that is *p* is not-*p*” with (2) “With respect to everything that is *p*, G asserts that *it* is not-*p*”. The former is a case in which G would be guilty of self-contradiction, the latter merely a case of error on G’s part. Now the above assumes that the reference intended by the word ‘this’ or by such a phrase as ‘Every thing that is *p*’ or ‘Something that is *p*’ can be taken as a component out of the proposition and objectively identified for different thinkers or even for the same thinker at different times. This in general is not possible. Hence we are led to the problem of what it is that can be universally identified as object of reference.

Having so far considered the proposition in its mental or *subjective* aspect, we have next to examine it in what may be called its *objective* aspect. Whereas a proposition is related subjectively to *assertion*, we shall find that it is related objectively to *fact*. Our conclusion, briefly expressed, is that any proposition *characterises* some fact, so that the relation of proposition to fact is the same as that of adjective to substantive. The word fact is more or less appropriately associated with propositions of different kinds. First, we have propositions characterising particular events; secondly, propositions which generalise with respect to the character of

events ; thirdly, formal propositions which enter into pure logic and pure mathematics. The word *fact* is most naturally associated with the first kind of proposition ; it is applied less appropriately to the second ; and hardly ever to the third. The first two kinds of proposition may be called existential. The universal feature of existential propositions is their implicit reference to temporal connexions ; and a class of them have the further common feature of implicit reference to spatial connexions. We have taken as the most primitive form of proposition : " a certain manifestation of thunder has occurred ". We may say that this proposition characterises (correctly or incorrectly) a certain occurrent fact. In this case there appears to be no distinction between what is meant by a certain manifestation and a certain fact ; in other words, we may indifferently say that the proposition ' it thunders ' characterises the fact, or that the adjective ' thundering ' characterises the manifestation. Now the barest form of proposition would be expressed thus : a certain manifestation has occurred. We may assume that the circumstances in which this proposition is asserted are such that—the reference (indicated by the phrase ' a certain ') being understood—the assertion is indubitably true. We shall find that the word *fact* can never be used except in association with a residual proposition which must be assumed to be *true* ; thus the association of *fact* with *true* proposition is precisely analogous to the association of what is ultimately *given-to-be-characterised* with what is *real*. Again passing from the barest form of proposition to one in which character is to some degree determined : a certain manifestation of thunder has occurred. This proposition cannot, like the previous one, be assumed to be true. It is, however, a further specification of the previous proposition which we said could be assumed to be true and thus to express a fact. Now, when we proceed to speak of the *fact* that a manifestation of thunder has occurred, and supplement this by a further predication—*e.g.*, by stating that this *fact* has certain consequences, this last statement could not be true, and might even be regarded as non-significant, unless the proposition expressing the fact of the manifestation being one of *thunder* had been true. Starting then with the residual fact expressed in a proposition, at each stage in the process of characterising the fact, we construct a proposition which must be taken to be true as a condition required in order that any further characterising of the fact may be true. It is in this way that the term *fact* comes to be inevitably associated with a *true* proposition, rather than with a proposition either true or false.

Now we have said that a proposition characterises a fact; and from this we see that a proposition could not be either true or false unless ultimately we could characterise a fact truly, as in the bare proposition: a certain manifestation has occurred.

Let us illustrate the way in which facts enter as constituents into propositions by the following:—

"That, during the Commonwealth, the dominance of Puritanism, which followed upon the execution of Charles I. in 1649, produced by reaction the evils of the Restoration period; largely accounts for the milder form of opinion and conduct, adopted by the reformers during and after the revolution of 1688, when the power of the Stuarts was finally destroyed."

This sentence, the understanding of which essentially requires attention to punctuation, is the expression of a proposition characterising one large fact, constituted out of many sub-facts connected in various ways illustrating temporal or causal nexus. We will give some analysis of this sentence for purposes of exposition. Take the proposition that "Charles I. was executed"; this, we say, characterises a certain fact. How then can we identify the fact previously to or apart from such characterisation? Let us suppose that a spectator who was present says, "Something is happening," and asks "What is happening?" His assertion, which expresses a bare fact, is indubitably true; and his interrogation is a request for a characterisation of this fact. But many intermediate steps, in which the fact is partially characterised, might be interpolated before we reach the proposition that "Charles I. was being executed". It is a condition for the truth of this proposition that each of the propositions into which it could be analysed should truly characterise the sub-facts. We may now dispense with this analysis and take the proposition as it stands to truly characterise the fact expressed in the assertion "Something is happening". It is important to note here, what is further involved in the relation and distinction between fact and proposition, that the same fact may be truly characterised in very many different propositions the combination of which would constitute a true characterisation of *fuller determinateness* than any of the partial propositions. Moreover, without adding on one proposition to another, we may render any proposition more determinate, and so characterise the same fact with different degrees of precision. Hence we cannot speak of *one* proposition corresponding to *one* fact, since there may be many true propositions corresponding to the same

fact. We proceed to consider the proposition "Charles I. was executed in 1649". This is a further specification of the fact characterised in our previous proposition. It is what we have called an external characterisation of the fact. In considering the logical punctuation, we must put the dating outside the bracket expressing the fact that Charles I. was executed. But in predicating the date 1649, we are not predicating a character of the *proposition* that "Charles I. was executed," but of the *fact* (characterised by the proposition) that Charles I. was executed. This illustrates the importance of distinguishing as to whether it is a fact or a proposition that enters as a constituent in a construct; because the same verbal expression is used for both. Thus Charles I.'s execution took place in 1649 is a predication about a fact; whereas, Charles I.'s execution is recorded in Macaulay's history, is a predication about a proposition. Here again the notion of a proposition cannot be explained, except in terms of assertion; thus, it is implied that Macaulay *asserted* that Charles I. was executed; and it may be further suggested that Macaulay's assertion is a ground for our asserting the same proposition. Similarly when we say that one proposition *implies* another, we mean that the assertion of the one would justify the assertion of the other. Again, when we predicate of a proposition that it *is* true, we mean that any person's *assertion* of the proposition *would* be true. From this we see that such an adjective as 'true' or 'false' or any relational adjective such as 'implying' or 'compatible with' is commonly predicated of a proposition (and not of an assertion), only because the truth or the falsity or the relation of implication or compatibility holds universally (if at all), independently of the person asserting or the time of his assertion. Again, we may predicate of the *proposition* that Charles I. was executed, that the logical justification for its assertion is human testimony, or that it is approximately certain—in contrast to other propositions of which we might predicate that their assertion is justified by general experience, or again that they are highly dubious. Such predications illustrate two points; namely, that a predication about a proposition is to be interpreted ultimately as a predication about an assertion; and that the adjectives (including relational adjectives) predicable of a proposition are of a different kind from those predicable of a fact. One of the essential characters predicable of a fact is its date, which involves indirectly a temporal relation to other facts, and leads to specific propositions, in which temporal relations to assigned facts are predicated. Again the predication of a

temporal relation or connexion between one occurrent fact and another may develop, in a further process of factual characterisation, into the predication of causal relation between the two occurrent facts. The generalisation of such predications is what is called a causal law. In formulating the causal law, reference is made both to the internal characterisation of the occurrent facts and to their external characterisations, *i.e.*, the temporal and spatial connexions between them. In saying that adjectives predicable of assertions are of a different kind from those predicable of facts, we must not forget that an assertion itself is an occurrent fact, and that therefore the same kind of adjectives that are predicable of occurrent facts in general are predicable of any assertion, regarded as a mere fact. In particular, an act of assertion may be dated, and hence enters into temporal relations with other facts; and, when the act of assertion is attributed to a certain person, we can predicate causal relations between it and other facts of experience that may have occurred in the life of the person. We must therefore more precisely define the distinction between what may be predicated of an assertion as a mere psychical fact, and what may be predicated of the assertum or proposition asserted. A predication about a proposition must be defined as a predication primarily about an assertion, but one which holds independently of the time of the assertion and of the person asserting. Thus, when any predication about one person's assertion at one time necessarily holds of the same or another person's assertion at the same or at another time, this must be because the content of the two assertions is the same; and hence the predication, which is primarily about the assertion, is transferred to the proposition, or that which is asserted. If, on the other hand, the predication about an assertion holds only on account of the temporal and causal circumstances under which the assertion is made, then such predication is about the assertion as a mere psychical fact. For example, if a person's seeing of lightning causes him to assert that there will be thunder, this predication of causal relation is one about the assertion as a mere psychical fact. If, on the other hand, we predicate about the assertion of thunder that it would be justified whenever an assertion of lightning were justified, then we are predicating about the assertion of thunder a relation which holds independently of the person making the assertion and of the time at which the assertion is made. With regard to the *time* of an assertion in this last reference, it is of course obvious that the time at which the assertion is made does not mean the same

as the time at which the asserted fact takes place. It is obvious that the fact of assertion is different from the asserted fact; and hence that the time at which the fact of assertion occurs may be different from the time at which the asserted fact occurs.¹ When we state, therefore, that a predication about a proposition holds independently of time, we mean independently of the time of the act of assertion, not independently of the time of the asserted fact, which latter is of course part of the content of the proposition. Turning to the historical illustration, it will be seen that the constituents of the complex proposition are *facts*, not propositions which characterise the facts; and that the adjectives (including relational adjectives) which are predicated of the facts, are in every case temporal or causal. Taking our historical example, we will resolve it so as to show briefly its constituent facts, and the points in our discussion which it illustrates. The constituent facts are as follows:—

- (a) A commonwealth was established (after 1649).
- (b) Puritanism was dominant (after 1649).
- (c) Fact (a) was simultaneous with fact (b).
- (d) Charles I. was executed (in 1649).
- (e) The Stuarts were restored (after 1649).
- (f) The restoration of the Stuarts was attended by certain evils.
- (g) Fact (b) caused fact (f).
- (h) A revolution occurred (in 1688).
- (i) The power of the Stuarts was finally destroyed.
- (j) Fact (k) occurred after fact (h).
- (m) The opinion of the reformers was milder in 1688 than in 1649.
- (n) The conduct of the reformers was milder in 1688 than in 1649.
- (r) The fact that fact (b) caused fact (f) caused facts (m) and (n).

This brief summary illustrates first, that the temporal order of assertions is not necessarily the same as that of the facts asserted. Secondly, that the logical conjunctions uniting assertions may be transferred to the facts asserted without change of import. This may be expressed by the following mode of bracketing: the *joint* assertion of (fact *m* and fact *n*) is equivalent to the assertion of the *joint* fact (*m* and *n*); and so of the other facts *a*, *b*, *d*. . . . Similarly, although our example does not illustrate any conjunction but 'and,' the *alternative* assertion of (fact *m* or fact *n*) is equivalent to the assertion of the alternative fact (*m* or *n*). In speaking of the alternative (*m* or *n*) as representing a *fact*, we are recognising

the principle that any characterisation of a fact may be more or less determinate ; thus the characterisation of a fact as *m* is less determinate than the characterisation of it as (*m* and *n*) and more determinate than the characterisation of it as (*m* or *n*). The *fact*, of course, may be said to be absolutely determinate, but *m* and *n* here—as elsewhere—stand for the *characterisation* of the fact. In the third place, the summary illustrates the principle which I previously expressed as follows : At each stage in the process of characterising a fact, we construct a proposition which must be taken to be true, as a condition required in order that any further or more determinate characterising of the fact may be true. Thus each of the propositions into which we resolved the sentence, *where a date or temporal characterisation is inserted*, is a proposition which would not be true, and might even be considered non-significant, unless the *undated* proposition were assumed to be true. Again each of those propositions in which we assert the relation of simultaneity or before or after, could not truly characterise the larger fact unless the minor facts themselves, between which the temporal relation is asserted, have been truly characterised in the *constituent* propositions.

So far we have restricted our discussion of facts to *occurrent* facts ; we have now to consider whether the term fact can be appropriately used, in relation and contrast to proposition, in a wider sense. Compare the proposition "Charles I. is being executed" with the proposition "*a* is greater than *b*". The former may be said to be based upon the less determinate proposition, "Some kind of thing is happening there and now" ; and upon the interrogation, "What kind of thing?" It may therefore be expressed : "What is happening there and now is the execution of Charles I.". In the same way, the latter proposition may be said to be based upon the less determinate proposition, "Some relation of magnitude subsists between *a* and *b*" ; and upon the interrogation, "What precise relation of magnitude?" It may therefore be expressed : "The relation of magnitude of *a* to *b* is as greater than to less than". In both these cases the question asked refers to a subject-term that is presented with the determinateness required for a determinate answer. In short, the subject term is assumed to be determined uniquely. In the spectacular proposition, the words 'there' and 'now' may be accompanied by actual pointing, so that the same reference may be understood by the speaker and hearer : this is an example of a unique description, the uniqueness of which is not secured by mere verbal phrases that could be

understood apart from context, but mainly by the gesture which accompanies verbal expression. Granting that the reference in the subject-term is thus unique, a true answer might be given in more or less different forms of proposition ; or, as we might say, the same uniquely determined fact may be truly characterised from various different aspects. Similarly the subject term—*viz.*, the relation of *a* to *b*—in the arithmetical example, is uniquely determined by a description intelligible apart from context (though again here *a* and *b* might be names of objects presented to the senses and pointed at). Hence the answer required is determinate. Here again a true answer might be given to the question, "What specific relation holds?" in more or less different forms, for example, "As 5 to 3," instead of "As greater than to less than". Thus the arithmetical proposition characterises, we may say, a fact, namely that *a* has to *b* some relation of magnitude ; and this fact is given, with a relatively indeterminate characterisation, to be more determinately characterised.

We have shown that to the same question of fact various different true answers may be given ; this leads to the problem of false propositions. We may begin by defining a true proposition as one which is in accordance with a fact. From this definition it would seem natural to define a false proposition as one that is *not* in accordance with *any* fact. This inference would follow if, in defining a true proposition, we had used the word "a fact" in the sense of "some fact," *i.e.*, as an alternative indefinite. Now in order to bring out the relation between truth and falsity, we must speak of the true proposition as being in accordance with *a certain* fact, and the (related) false proposition as being in *discordance* with *the same* fact. The truer definition, then, of a false proposition is that it is one which is in discordance *with a certain fact*. If we had defined a false proposition merely negatively, as being not in accordance with any fact, then the false proposition might be towards every fact in no one relation whatever, or in some relation—say parentage—to *some* fact, and in the relation say of cause to some other fact, and in the relation of say "greater than" to some third fact, and so on. There is indeed one relation in which a false proposition stands to *all* facts, namely the relation of non-identity ; but, inasmuch as the same holds of true propositions, this will not provide us with the required differentia. This shows that the definition of a true proposition as being in accordance with *some* fact is incorrect, because if the subject of a proposition is not uniquely understood, a false proposition might

be in accordance with some fact. For example, the proposition "This man is tall" might be false, and yet be in accordance with *some* fact. Thus taking the proposition "This man is short" to be true, the proposition that "This man is tall" would be in accordance with some fact, and could therefore only be pronounced as false when we had secured reference to the same fact in the two propositions, namely, the height of the same man. In short, it is obvious that in pronouncing the proposition "This man is tall" to be false, we do not mean that it is not in accordance with any fact, but that it is in discordance with a certain fact. Many propositions may be in accordance with the same fact, and merely because one proposition is in accordance with a certain fact, it does not follow that a different proposition relating to the same fact is false; there must be some positive relation between two propositions relating to the same fact in order that the falsity of the one should necessarily follow from the truth of the other. This positive relation is that the one is in accordance with, and the other in discordance with one and the same fact.

There are many pairs of terms which seem to present the same antithesis, such as true and false, affirmative and negative, acceptance and rejection, accordance and discordance, affirming and denying, etc. In my own view, there is, indeed, one single antithesis to which these expressions point. The consequence of this is that, in the attempt formally to define one pair of terms, we are apt to use another pair of terms, and are thus in danger of circular definition. Another difficulty in defining these terms arises from a certain ambiguity in the use of the word 'not'. For example, if we defined 'false' as meaning 'not true,' such definition would involve two mistakes; in the first place, it is only of propositions that 'not true' would coincide with false; and in the second place, amongst propositions, the relation of the true to the false is not *merely* that they constitute two sub-divisions of propositions that are (taken together) exhaustive, and (taken separately) exclusive. In fact, any attempt literally to define the antithesis between true and false seems inevitably to involve the prior understanding of the meaning of the terms, and of their antithesis. I think we can best avoid circular explanations by not attempting to define the *general* meaning of the terms true and false, but rather by taking a pair of *related* propositions such that the truth of either of them involves the falsity of the other. In order that this relation may hold between two propositions, we may say in the most general case that they refer to the same fact; we

then attempt to define the fundamental antithesis by means of the conception of accordance and discordance. We have already indicated that discordance does not merely mean non-accordance.

It will be found that the word '*not*' has a different meaning when prefixed to a substantive from that which it has when prefixed to an adjective; but it is only in the latter connexion that its proper significance can be understood. Language frequently supplies us with the negative prefix as part of the adjective word, for example, incompatible, dishonest, unusual, etc., but where ordinary language does not supply an adjectival prefix, we can always place '*non*' before the positive adjective, as in non-identical, non-red. We ask, what is the relation of non-red to red? It might be answered that '*non-red*' means '*non-identical with red*'; but inasmuch as '*hard*' also is non-identical with red, the assertion that a thing is '*non-red*' would be compatible with the assertion that it is '*red*', if non-identity were the only relation which held between red and non-red. Again, if this were so, '*non-identical*' would mean '*non-identical with identical*'; but non-identical is also '*non-identical*' with incompatible, or any other relational adjective; so that the same question arises upon the meaning of non-identical as upon the meaning of non-red. The relation of '*non-identical*' to '*identical*' or of '*non-red*' to '*red*', etc., may be said to be that of *incompatibility*. This would seem to raise a question as to the relation of incompatibility to compatibility; since the prefix *in* here has the same significance as the more general prefix *non*. We can only say that '*incompatible*' means '*incompatible with compatible*'. This is equivalent to saying that incompatible cannot be defined; or, to put it otherwise, incompatible is just as ultimate a positive relation as compatible. In order, then, to assert that a thing is non-red, we must be able to assert that it has some quality—not merely non-identical with '*red*'—but incompatible with red. The same applies to the relation '*non-identical*' itself; in order to assert that *a* is non-identical with *b*, we must be able to assert that it has some relation to *b* incompatible with identity. Now the only relation incompatible with identity is *otherness*. We ought, then, to have amended our original definitions, and substituted: the adjective red is *other than* the adjective hard; the adjective blue—which is an example of non-red—is *incompatible* with red; the relation '*other than*' is *incompatible* with the relation '*identical with*'. These three examples bring out the additional points that whereas '*other than*' is the only relation incompatible with '*identical with*,

blue is not the only adjective incompatible with red; and that in stating that red is merely other than hard, we suggest that red is compatible with hard. Such adjectives as 'non-identical with,' 'incompatible with,' in which the negative is prefixed to the adjective-word, are legitimately used as positives whenever there is a *strictly dual* incompatibility. It is doubtful whether logicians have not been sometimes mistaken in supposing that all cases of incompatibility can be regarded as *dual* incompatibility, whereas this is the exception rather than the rule. Incompatibility in general has been technically termed contrariety; and dual incompatibility, contradiction. Now although the negative predication non-red can be treated, in purely formal processes, under the same rules as the positives red or blue; nevertheless, when we are examining philosophically the conditions necessary in order that such predications as non-red may have any import at all, we must conclude that such import is derived from incompatibility in general. The difference of function between the terms red and non-red is that, since red is comparatively determinate, non-red is comparatively indeterminate; the general rule being that of two contradictory terms, one positive and the other negative, one is indeterminate, and the other determinate in comparison. This rule follows from the fact that the total number of predications which are all mutually incompatible is generally considerable; and therefore if one of the terms includes a small number of alternatives, the other will include a large number.

We have now reduced the problem of the negative to the question of the nature of incompatibility. We had previously spoken of a pair of related propositions, one of which was in accordance and the other in discordance with a certain fact. That there are such pairs of propositions is the condition for conceiving and applying the relation of incompatibility. Thus, in examining the content of two propositions, we may discern that if one of them is in accordance with a certain fact, the other must be in discordance with that fact, without our knowing in the special case which of the two is in accordance and which in discordance; we express this relation by saying that the two propositions are incompatible with one another. The proposition which is in accordance we call true; that which is in discordance we call false. We affirm or accept the former, and we deny or reject the latter. We have said that affirming and denying are in the same antithesis as true and false; this, of course, does not mean that a proposition that has been denied is the same as a proposition

that is false. What it means is that the denial of a proposition is equivalent to the assertion that the proposition is false; and that the affirmation of a proposition is equivalent to the assertion that the proposition is true. In other words, the attitude of denying or affirming a primary proposition is equivalent to the attitude of asserting the derived secondary proposition in which falsity or truth is predicated of the primary. Again, we have reduced the antithesis of affirmative and negative to that of accordance and discordance; but this does not mean that the affirmative proposition is that which is in accordance, and the negative that which is in discordance with fact. For instance, supposing a thing to be yellow, the negative predication 'not red' would be in accordance with the fact, and the positive predication 'blue' in discordance with the fact. That is to say, 'not red' is here in indeterminate accordance with the fact, 'yellow' in relatively determinate accordance, 'not yellow' in indeterminate discordance, and 'blue' in relatively determinate discordance. In extending this principle, it will be enough to take the two forms of proposition from traditional logic, "Every P is Q" and "Some P is non-Q". The fact with which one of these is in accordance is the same as the fact with which the other is in discordance. For example, the actual fact might be truly characterised by the proposition, "Every P is QU"; in this case our universal would be true, and our particular false. On the other hand, the fact might be truly characterised by the proposition, "Every PV is non-Q". In this case our particular would be true, and our universal false. In this example I have chosen for the proposition which actually characterises the fact, a proposition *more determinate* than the original proposition (universal or particular). This was done for the purpose of illustrating the general principle that the *fact* is always actually characterisable by a proposition that is (indefinitely) more determinate than any proposition which a mere human being could discover or even conceive. Thus, if "Some P is non-Q" is true, the actual fact will be more determinately characterised by a proposition that indicates, first, *which* amongst all the objects that are P are Q, and which are non-Q; and secondly, amongst those which are non-Q, which are characterised by some one quality incompatible with Q, and which by some other. Induction, indeed, is the process in which we try to approximate to such more completely determinate knowledge, by replacing the merely indefinite particular proposition of formal logic by a set of sub-universals, to each of which the proper determinate predicate is attached. Thus, "Every PV₁ is Q₁," "Every PV₂ is Q₂," and so on.

I have said that some logicians have in effect taken the relation of contradiction to be fundamental in place of contrariety. But many of the opposite school of logicians have apparently fallen into the reverse mistake, namely of attributing priority to the contrary in place of the contradictory; *i.e.*, they appear to have argued that we must always base our assertion of the contradictory upon a knowledge of the contrary. Now it is true that in many very important cases this is so; but in an equal number of equally important cases we are able to assert a mere contradictory or a relatively indeterminate contrary, before we can assert the more determinate contrary; and in any case we can *never* know the contrary in its complete determinateness. We must adjust the balance properly between these two opposite errors. On the one side, we maintain that our *conception* of the contradictory is based upon our conception of the contrary; on the other side, that our *knowledge* of the contradictory very frequently comes before our knowledge of the contrary. What I have maintained is that when we know that the contradictory holds, we know that *some* contrary must hold, but we do not necessarily know *what* contrary.

We must conclude by returning once more to the conception of a fact as that which the proposition characterises. We have seen that, in general, we cannot find any substantial or existential component of the proposition as an object that could be identified for all persons and for all times. But the *fact* to which any given proposition refers can be so identified. We may speak of one assertion and another assertion as characterising the *same* fact; and again of two assertions as characterising *different* facts. The fact, as determinandum for thought, can be objectively identified as the ultimate object of reference; while the proposition is the character more or less determinately apprehended as characterising the fact. The formula which finally expresses the blending of the assertive with the characterising tie is thus: The fact F asserted by the thinker T to be characterised by the proposition P.