XXVI. On the range of freedom of electrons in metals

Prof. J. Larmor F.R.S.

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leaks in each case at a particular temperature can be calculated.

In this way the following numbers were obtained for the temperature 1600° C.

Leak in air	5×10^{-7}	ampere.
Air with lime.....	5×10^{-2}	„
Hydrogen	10^{-1}	„
Hydrogen with lime ...	10^{+3}	„

Thus the leak with hydrogen, as with lime, is about 10^5 times greater than with air alone, while that with both hydrogen and lime is 10^{10} times greater. These ratios do not vary much with the temperature.

In conclusion I have to thank Prof. H. A. Wilson, at whose suggestion these experiments were undertaken, and who was ever ready with kind advice and assistance.

XXVI. On the Range of Freedom of Electrons in Metals.

By Prof. J. LARMOR, F.R.S.*

IT has been ascertained that complete metallic conduction is established in a small fraction of the period of low ultra-red radiation. This is proved by the experimental result of Hagen and Rubens that radiation of about ten or more times the period of light is reflected from all metals in

* Communicated by the Author.

This brief discussion was drawn up in forgetfulness of Prof. Schuster's estimate of the number of free electrons in metals (Phil. Mag. vii. 1904). His method was, I find, criticised and enlarged on in an interesting way later in the same year by Drude (*Ann. der Phys.* xiv. p. 936), with a view to greater precision; but the widely uncertain assumption that the optical frictional term is represented for visible radiation by the full ohmic resistance seems to enter fundamentally. The values there obtained are about 10 times larger than the admittedly very rough estimate in the text. Later in the same paper (p. 956) Drude estimates the length of free path of the electrons, obtaining results ranging in order of magnitude from 10^{-6} cm. for the nobler metals to 10^{-8} cm. for bismuth; these were intended to replace estimates (of almost the same order for the nobler metals) deduced by J. J. Thomson and J. Patterson from magnetic influence on the resistance of thin sheets. The lower estimate, about 10^{-8} cm. for most metals, deduced in the text directly from the time necessary for establishment of complete conduction, is the main point of this paper; the result might possibly be stretched toward 10^{-7} , but hardly further, if the velocities of the free electrons are really determined by the gas-law. If velocities so small as these are retained, the negative conclusion that hardly any portions of the paths by which the electrons travel are free, and that therefore estimates of number made in this way are uncertain, seems not unnatural.

proportions determined by their ohmic conductivities alone. For that type of radiation, the square of the *quasi*-index of refraction is therefore a pure imaginary quantity.

The opposite extreme case, that of radiation of period rapid compared with the times of undisturbed motion of the electrons, is also of interest. It will appear that, if the effective electrons are free, the square of the *quasi*-index must be a real negative quantity. The optical determinations of Drude indicate that this is not very far from being true with light-waves for some of the nobler metals; in the case of white metals such as silver, the property, moreover, persists over a considerable range of period, though at length it fails. Thus, both on this ground and by reason of the shorter period, over a rather wide range of optical period the electrons are perhaps not far removed from being free. Moreover, theories of ordinary complete metallic conduction have been developed with some promise which ascribe it to electrons entirely free.

When the electrons in the optical problem are supposed to be virtually free, then for each of them, of inertia m and charge e , under electric force (P, Q, R) ,

$$m(\ddot{x}, \ddot{y}, \ddot{z}) = e(P, Q, R);$$

thus if there are N' of them per unit volume, and (u', v', w') is the current of conduction,

$$\frac{d}{dt}(u', v', w') = N' \frac{e^2}{m}(P, Q, R),$$

the sign of the charge e not entering if m is the same for all.

Consider a plane-polarized wave-train travelling along z , with electric vector (P) along x and magnetic vector (β) along y . Its equations of propagation are, by the circuital relations of Ampère and Faraday,

$$\frac{\partial \beta}{\partial z} = -4\pi u, \quad \frac{\partial P}{\partial z} = -\frac{d\beta}{dt},$$

where

$$4\pi u = 4\pi u' + e^{-2} \frac{dP}{dt},$$

the last term representing the *quasi*-current of æther strain, which will prove to be here negligible. Thus

$$\begin{aligned} \frac{\partial^2 P}{\partial z^2} &= 4\pi \frac{du}{dt} \\ &= 4\pi N' \frac{e^2}{m} P + e^{-2} \frac{d^2 P}{dt^2}, \end{aligned}$$

which determines the mode of propagation.

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In a simple wave-train of period $2\pi/p$,

$$P = P_0 e^{i\left(\frac{p}{c}z - \varphi t\right)}$$

which yields on substitution

$$-\frac{p^2}{c'^2} = 4\pi N' \frac{e^2}{m} - \frac{p^2}{c^2},$$

thus giving for the square of the *quasi-index* μ ($= c/c'$) the value

$$\mu^2 = -4\pi N' \frac{e^2 c^2}{m p^2} + 1.$$

For radiation (D line) of wave-length 6.10^{-5} cm., $2\pi/p$ is 2.10^{-15} , so that

$$\begin{aligned} \mu^2 &= -\frac{N'}{\pi} \frac{e}{m} \cdot 10^{-20} (9.10^{20}) (4.10^{-30}) + 1 \\ &= -N' \frac{e}{m} \cdot \frac{9}{8} \cdot 10^{-29} \text{ approximately.} \end{aligned}$$

If the ions are all free electrons, e/m is 17.10^6 , thus

$$\mu^2 = -N' \cdot 2.10^{-22}.$$

For the nobler metals, in red and yellow light, the real (and five times preponderant for gold, silver, magnesium) part of μ^2 is of the order -10 ; this would give $N' = \frac{1}{5} \cdot 10^{23}$ *, so that the number of free electrons taking part would be of somewhere about the same order as that of the molecules of the metal.

A view of metallic electric conduction, which fits in with its parallelism to thermal conduction, is that it takes place by free electrons whose velocities are prescribed by collisions with the molecules, and so are taken as determined by the law of equality of mean energies in gas-theory. For hydrogen under standard conditions, the average velocity in the free path is 2.10^5 cm./sec.; as the electrons are a thousand times less massive, their average velocity would thus be about 6.10^6 . And we know that metallic conduction is fully established in a small fraction of the time 2.10^{-14} sec., which is the period of ultra-red radiation of ten times the wave-length of light. If then, as theories involving free electrons require, the establishment of conduction is intimately con-

* If undamped resonance of adjacent free molecular periods contributes sensibly to the real part of $-\mu^2$, otherwise than by shaking out more electrons into freedom, this would be an over-estimate.

cerned with the duration of the free paths*, the mean free path of the electrons must not much exceed 10^{-8} cm., so that their excursions must in fact be largely confined to the spaces between each molecule and the next. If their free paths were larger than this, complete conduction could not become established in so short a time.

If a free path longer than this were demanded, then the optical incipient conduction would have to be ascribed mainly to the gradual deflexion of the path by the electric field without introducing limitation due to interruption of the path by collision with the molecules; and the theory of propagation which has been developed above would be the one applicable to ultra-red radiation. The results of Hagen and Rubens seem emphatically to preclude that type of hypothesis.

If then metallic conduction is due to free electrons, their freedom is spacially very much restricted, almost in fact within molecular limits.

While if it is inferred from the approximation of μ^2 to a real negative value, that the *incipient* conduction, which is the main agent in the optical phenomena of the nobler metals, must be largely due in the above manner to deflexion of paths of electrons, effectively free for times exceeding the period of the vibration, it would appear that the number of them that are concerned is roughly of about the same order as the number of the metallic molecules.

The value of the mean velocity of the free electrons, above employed, about 6.10^9 , is involved in Drude's form of the theory (*Ann. der Phys.* i. 1900, p. 577), in order to get the right (universal) ratio of electric to thermal conduction, on the assumption that both of them are effected through the agency of the free electrons. If, however, their velocity were of the order 3.10^9 , like the electrons from radio-active substances, then the free path might be as much as 5.10^{-4} cm. without vitiating the conditions laid down. The work of

* Because the mean additional velocity imposed upon the electron by the electric field is proportional to the duration of the free path.

This view of conduction, as stated by Drude, seems to require tacitly that the average velocity is in most instances restored by collision at the end of each free path. Moreover the conductivity varies as the temperature for pure metals: thus $N \times \text{free path} \times \text{velocity}$ must be constant. These conditions are difficult to interpret, unless each molecule may be taken to emit electrons at a constant rate and absorb a definite proportion of those that encounter it, the same at all temperatures, thus establishing an equilibrium in which N' varies inversely as the velocity. Such increase of N' at low temperatures would make μ^2 , and also the absorbing power, for those shorter wave-lengths for which $-\mu^2$ is large, tend to vary inversely as the square root of the temperature.

O. W. Richardson, on the escape of ions from incandescent solids, which he finds to agree with the requirements of the gas-law, supports the former view, in fact seems to be its main experimental support; he concludes, moreover, (Phil. Trans. 1906) that the number of free electrons in platinum ranges somewhere about 10^{21} , which is not discordant, under the circumstances, with the general estimate $\frac{1}{2} \cdot 10^{23}$ made above.

The conclusion reached above, that in fact the free electron is deviated or entangled by each molecule which it meets or traverses, also points to the smaller velocity, as the rapid electrons are known to pass straight through thin sheets of metal. But the free path now seems hardly long or definite enough to substantiate, except very roughly, Drude's formula for metallic conduction as effected entirely by the steady deflexion of the free paths by the electric field. If the rough general estimates of order of magnitude, made above, for the number of free electrons and their velocities and free paths in the best conducting metals, are substituted in Drude's formula (*loc. cit.*), they give a conductivity of the order $12 \cdot 10^{15}$, which is about one-fiftieth of that of silver, and errs on the wrong side, though hardly so far as to cause surprise when the necessary vagueness of the data is kept in view.

Cambridge, July 15.

XXVII. Notices respecting New Books.

Exercices et Projets d'Electrotechnique. Publiés sous la direction de ERIC GERARD et OMER DE BAST. Tome Premier. Paris: Gauthier-Villars. 1907. Pp. 240.

THIS is the first instalment of a collection of problems primarily intended for the use of students attending the electrotechnical courses at the Montefiore Institute. It is devoted mainly to general magnetic and electrical theory, and deals with magnetic moments, intensity of magnetization, electrostatic theory, the laws of the electric current, electromagnetism, electromagnetic induction, and alternate current theory. This latter subject is treated very fully, and with special reference to its technical applications. The total number of problems is 104, and each problem is followed by a fully-worked solution. A book of this type should prove of great use to all students of electrotechnology, but especially so to those who are more or less thrown on their own resources, and can only occasionally take advantage of the guidance of a teacher. The solutions supplied should have an excellent effect in training students to those systematic methods of attacking problems which so many find it difficult to acquire. Altogether, the work is to be very highly commended, and should find a wide sphere of usefulness as a companion volume to Eric Gerard's well-known *Leçons sur l'Electricité*.