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Rev M. O'Brien

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XLIII. On the Laws of Reflexion and Refraction at the Surfaces of Substances of High Refracting and Absorbing Powers, such as Metals. By the Rev. M. O'BRIEN, late Fellow of Caius College, Cambridge, and Professor of Natural Philosophy and Astronomy in King's College, London.

[The subject resumed from p. 123.]

To the Editors of the *Philosophical Magazine and Journal*.

GENTLEMEN,

1. **I**N a former communication I mentioned that I had investigated the laws of reflexion and refraction at the surface of a transparent substance, the particles of which are supposed to *resist* the vibrations of the æthereal fluid; I shall now briefly state and prove these laws, and show how they may be tested by experiment. In a memoir printed in the Cambridge Transactions, vol. viii. p. 7, I have obtained certain results*, which, if true, show the great importance of taking properly into account the *normal* as well as the *transversal* waves, in investigating the laws of reflexion and refraction. These results, as well as the notation and method employed in that memoir, I shall now make use of without further explanation. The waves are supposed to be plane waves, and the refracting surface is also supposed to be plane.

2. The following are the *laws of reflexion and refraction* for vibrations *perpendicular* to the plane of incidence.

Let the general expression for the *incident* disturbance be

$$a \cos \{nt - k(px + sz)\},$$

the refracting surface being taken as the plane of xy , and the plane of incidence as the plane of xz ; p and s being the sine and cosine of the angle of incidence, which angle we shall denote by the letter ϕ .

Then the general expression for the *reflected* disturbance will be (as we shall presently prove),

$$-Pa \cos \{nt - k(px - sz) - \theta\};$$

and the expression for the *refracted* disturbance will be

$$ae^{-kxz} \{ \cos \{nt - k(px + \sigma z)\} - P \cos \{nt - k(px + \sigma z) - \theta\} \}.$$

* I shall just state one of these results, to show the importance of taking into account the normal waves. If a^2 be the intensity of a ray incident on a surface of high refractive power, the intensity of the reflected ray, when the tangent of the angle of incidence is μ , will be

$$\frac{(\mu^2 - 1)^2 (\nu^2 - 1)^2}{4\mu^2 (\nu^2 + 1)^2 + (\mu^2 - 1)^2 (\nu^2 - 1)^2} \cdot a^2,$$

μ being the index of refraction for transversal waves, and ν that for normal. This remarkable result fully explains the peculiarities of reflexion at highly refractive surfaces (see the Memoir, p. 26).

The quantities σ , χ , P and θ in these expressions are to be obtained by equating the possible and impossible parts of the following equations, viz.

$$(\sigma - \chi \sqrt{-1})^2 + p^2 = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right),$$

$$s^2 \left(\frac{1 + P e^{-\theta \sqrt{-1}}}{1 - P e^{-\theta \sqrt{-1}}} \right)^2 + p^2 = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right),$$

where B , B' and C' are certain constants which occur in the equations of motion of the æther. B is the same for all colours, but B' and C' are different for different colours, being given by the equations

$$B' = \frac{B}{1 + C_2 - C_4 n^2 + C_6 n^4 \dots},$$

$$C' = \frac{C_1 - C_3 n^2 + C_5 n^4 \dots}{1 + C_2 - C_4 n^2 + C_6 n^4 \dots},$$

where B , C_1 , C_2 , C_3 , &c. are certain constants depending simply upon the constitution of the æther and of the transparent body.

3. The following are the *laws of reflexion and refraction* for vibrations *parallel* to the plane of incidence.

Let the general expression for the *incident* disturbance be

$$b \cos \{ n t - k (p x + s z) \},$$

then the expression for the *reflected* disturbance will be

$$P Q b \cos \{ n t - k (p x + s z) - \theta - i \};$$

and the expression for the *refracted* disturbance will be

$$K b e^{-k x z} \left\{ \cos \{ n t - k (p x + \sigma z) + \kappa \} \right. \\ \left. + P Q \cos \{ n t - k (p x + \sigma z) + \kappa - \theta - i \} \right\},$$

where P , θ , σ and χ are the same quantities as before, and Q , i , K and κ are to be obtained by equating the possible and impossible parts of the following equations, viz.

$$\frac{1}{K^2 e^{2\kappa \sqrt{-1}}} = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right),$$

$$U^2 + 2p U \sqrt{-1} = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right),$$

where for brevity we have put U^2 to represent the expression

$$\frac{A' B}{A B'} \frac{Q e^{-i \sqrt{-1}} - e^{2\phi \sqrt{-1}}}{1 - Q e^{-i \sqrt{-1}} \cdot e^{2\phi \sqrt{-1}}},$$

A and A' being certain constants which occur in the equations of motion for *normal* waves, occupying the same places re-

spectively that B and B' do in the equations of motion for transversal waves.

4. The equations here given for finding P, Q, θ and ι are apparently complicated, but in point of fact they are not so, as will presently appear. The equations for determining Q and ι may be easily applied to determine the values of the unknown constants involved in them by comparison with experimental results, and appear to be well-adapted, not only for testing the truth of the theory here advanced, but also (supposing that theory to turn out true) for deciding several important questions in the undulatory theory of light. We shall now show how the equations for determining Q and ι may be compared with experiment.

5. These equations apply to the case of reflexion at metallic surfaces and surfaces of high refractive power. The formulæ for the incident light,

$a \cos \{nt - k(px + sz)\}$ and $b \cos \{nt - k(px + sz)\}$, represent together a *plane* polarized ray incident at an angle ϕ ($p = \sin \phi$, $s = \cos \phi$), the angle which the plane of polarization makes with the plane of incidence being

$$\tan^{-1} \frac{b}{a}.$$

The corresponding formulæ for the reflected light are

$$-P a \cos \{nt - k(px - sz) - \theta\},$$

and $P Q b \cos \{nt - k(px - sz) - \theta - \iota\}$,

which, on account of the difference of phase (ι), represent *in general*, not a *plane*, but an *elliptically* polarized ray*. If ι be any multiple of 180° , the polarization becomes plane.

If we suppose the ray to be reflected m times successively at two parallel surfaces of the same metal or substance, the formulæ for the light after the last reflexion will be

$$(-P)^m a \cos \{nt - k(px - sz) - m\theta\},$$

and $(PQ)^m b \cos \{nt - k(px - sz) - m\theta - m\iota\}$.

Let us suppose that $m\iota = 180^\circ$, then these two expressions represent together a *plane* polarized ray, the angle which the plane of polarization makes with the plane of incidence being

$$\tan^{-1} \left(Q^m \frac{b}{a} \right).$$

Now Sir D. Brewster has determined the angles at which a plane polarized ray must be incident upon the surfaces of

* Sir D. Brewster calls the polarization *elliptical* when $\iota = 90^\circ$, and *partially elliptical* when ι is not equal to 90° . It is easy to see that the polarization is always completely elliptical, only the plane of incidence does not contain either axis of the ellipse except when $\iota = 90^\circ$.

various metals, so that the light may be *plane* polarized after two reflexions, or three reflexions, or four reflexions, &c. In other words, he has determined a set of angles of incidence which respectively make $2i = 180^\circ$, $3i = 180^\circ$, $4i = 180^\circ$, &c. He has also determined the inclination of the plane of polarization of the twice-reflected ray to the plane of incidence, but I do not know whether he has done the same in the case of the rays reflected three times, four times, &c.; there would be of course no difficulty in observing the respective inclinations of the planes of polarization of the rays which have been plane polarized by three or more reflexions. Supposing this done, it is clear that we obtain a series of values of ϕ which respectively make $i = 90^\circ$, $i = 60^\circ$, $i = 45^\circ$; and at the same time we determine the corresponding values of Q , for (if we suppose for simplicity that $\tan^{-1} \frac{b}{a} = 45^\circ$, or $b = a$) the corresponding angles of inclination of the planes of polarization will be $\tan^{-1}(Q^2)$, $\tan^{-1}(Q^3)$, $\tan^{-1}(Q^4)$, &c. Sir D. Brewster has found that there is only one value of ϕ that makes $i = 90^\circ$, but that there are two values which make $i = 60^\circ$, two which make $i = 45^\circ$, &c. Hence we can determine by experiment several sets of corresponding values of ϕ , i and Q .

6. Suppose then that we know the values of Q , i and ϕ ; substituting these values in the expression for U^2 , we may put U^2 in the form

$$U^2 = c^2(\alpha + \beta \sqrt{-1})^2,$$

where α and β are known numbers, and $c^2 = \frac{A'B}{AB}$, whence the last equation in art. 3 becomes

$$c^2(\alpha + \beta \sqrt{-1})^2 + 2pc(\alpha \sqrt{-1} - \beta) = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1}\right), \quad (1.)$$

p being also a known number.

Let α_p , β_p , p , be another set of known values of α , β , p ; then we have, similarly,

$$c^2(\alpha_p + \beta_p \sqrt{-1})^2 + 2p_c c(\alpha_p \sqrt{-1} - \beta_p) = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1}\right). \quad (2.)$$

Equating the possible and impossible parts of the first members of these two equations, we find

$$c^2(\alpha^2 - \beta^2) - 2p\beta c = c^2(\alpha_p^2 - \beta_p^2) - 2p_p\beta_p c,$$

$$c^2\alpha\beta + p\alpha c = c^2\alpha_p\beta_p + p_p\alpha_p c,$$

which give

$$c = 2 \frac{p\beta - p_p\beta_p}{\alpha^2 - \beta^2 - \alpha_p^2 + \beta_p^2} = \frac{p_p\alpha_p - p\alpha}{\alpha\beta - \alpha_p\beta_p}.$$

Thus c becomes known, and consequently, by equating the

possible and impossible parts of either (1.) or (2.), we determine $\frac{B}{B'}$ and $\frac{C'}{n}$ also. Thus by two sets of observations we may determine all the unknown constants which enter into our equations. We have also an equation of condition, namely,

$$\frac{p\beta - p_1\beta_1}{\alpha^2 - \beta^2 - \alpha_1^2 + \beta_1^2} = \frac{p_1\alpha_1 - p\alpha}{\alpha\beta - \alpha_1\beta_1},$$

which must turn out true if our theory be correct.

The simplest way, perhaps, to test the truth of our theory would be this: to find from observation several corresponding values of the two quantities

$$\frac{p\beta - p_1\beta_1}{\alpha^2 - \beta^2 - \alpha_1^2 + \beta_1^2} \text{ and } \frac{p_1\alpha_1 - p\alpha}{\alpha\beta - \alpha_1\beta_1},$$

and to try whether they vary with the angle of incidence or not; and whether they are always equal to each other or not. If they turn out to be invariable and always equal to each other, it is clear we have a very decisive proof of the correctness of the above results, and *vice versa*; especially, because there are only three distinct unknown constants involved in the above equations.

7. The method here explained of determining Q and ι by repeated metallic reflexions is not so well-adapted to test the truth of the above formulæ as another method which gives the following result,

$$Q e^{-\iota\sqrt{-1}} = \frac{\tan \beta + \sqrt{-1} \tan \gamma}{\sqrt{-1} \tan \beta \tan \gamma - 1},$$

where β and γ are two angles which may be immediately observed for any angle of incidence we please. The consideration of this method I shall reserve for my next communication, and now briefly prove the laws of reflexion and refraction here stated.

8. If ξ , η , ζ be the displacements of any element of the æthereal fluid from its position of rest, it appears, as in art. 19, that the material particles will bring into play upon that element a resisting force, the components of which (parallel to the three axes) are

$$\frac{P d\xi}{dt} + Q \frac{d^2\xi}{dt^2}, \quad \frac{P d\eta}{dt} + Q \frac{d^2\eta}{dt^2}, \quad \frac{P d\zeta}{dt} + Q \frac{d^2\zeta}{dt^2};$$

where $P = C_1 - C_3 n^2 + C_5 n^4 \dots$, $Q = C_2 - C_4 n^2 + C_6 n^4 \dots$, C_1 , C_2 , C_3 , &c. being constants depending upon the constitution of the æther and of the transparent substance, and $\frac{2\pi}{n}$ the periodic time of the æthereal vibrations.

Hence the general equations of motion, for transversal vi-

brations, will be (see Cambridge Transactions, vol. vii. p. 409)*,

$$\left. \begin{aligned} \frac{d^2 \xi}{dt^2} = B \left(\frac{d^2 \xi}{dx^2} + \frac{d^2 \xi}{dy^2} + \frac{d^2 \xi}{dz^2} \right) - P \frac{d \xi}{dt} - Q \frac{d^2 \xi}{dt^2} \end{aligned} \right\} . \quad (3.)$$

and two similar equations for η and ξ .

For normal vibrations we have three equations of exactly the same form as these, differing only in having the constant A in place of B.

{N.B. By transversal vibrations, we mean those which satisfy the condition

$$\frac{d \xi}{dx} + \frac{d \eta}{dy} + \frac{d \zeta}{dz} = 0,$$

and by normal vibrations those which satisfy the three conditions

$$\frac{d \xi}{dy} = \frac{d \eta}{dx}, \quad \frac{d \eta}{dz} = \frac{d \zeta}{dy}, \quad \frac{d \zeta}{dx} = \frac{d \xi}{dz}.$$

These definitions of transversal and normal vibrations apply equally, whether ξ , η , ζ be imaginary or not.}

If, in the equations (3.), we put

$$\frac{P}{1+Q} = C', \quad \frac{B}{1+Q} = B',$$

we have

$$\left. \begin{aligned} \frac{d^2 \xi}{dt^2} + C' \frac{d \xi}{dt} = B' \left(\frac{d^2 \xi}{dx^2} + \frac{d^2 \xi}{dy^2} + \frac{d^2 \xi}{dz^2} \right), \end{aligned} \right\} . \quad (4.)$$

and two similar equations for η and ζ .

These three equations are satisfied by the following imaginary values of ξ , η , ζ , viz.

$$\xi = a' u, \quad \eta = b' u, \quad \zeta = c' u,$$

where

$$u = e\{n t - k' (p' x + q' s + s' z)\} \sqrt{-1},$$

$$p'^2 + q'^2 + s'^2 = 1;$$

and consequently by substitution,

$$n^2 - C' n \sqrt{-1} = B' k'^2. \quad . \quad . \quad . \quad . \quad (5.)$$

Also, since the vibrations are transversal, we have, by the definition above-given,

$$p' a' + q' b' + s' c' = 0.$$

From equation (5.) it is manifest that, supposing n real, k' is imaginary. By superposing two proper sets of imaginary solutions thus obtained, we shall obtain real integrals of the equations of motion.

* To conform to the most usual notation, we have in the present papers represented the displacements by the letters ξ , η , ζ , instead of α , β , γ .

9. We shall now investigate the laws of reflexion and refraction at the surface (supposed to be plane) of the transparent substance to which the above equations apply, taking for granted the results arrived at in the memoir (Cambridge Transactions, vol. viii. p. 7) above-quoted, and assuming the notation there employed (observing that $n=k$ $v=k'v'$).

10. Referring to section viii. (p. 24), we find that the equations of *motion* and *connection** are satisfied by the following imaginary values of V , V_i and V' (the vibrations being *parallel* to the plane of incidence), viz.

$$V = \frac{b}{2} e \{ n t - k (p x + s z) \} \sqrt{-1},$$

$$V_i = \frac{\mu s - s' - \eta}{\mu s + s' + \eta} \frac{b}{2} e \{ n t - k (p x - s z) \} \sqrt{-1},$$

$$V' = \frac{1}{\mu} \left\{ \frac{\mu s - s' - \eta}{\mu s + s' + \eta} + 1 \right\} \frac{b}{2} e \{ n t - k' (p' x + s' z) \} \sqrt{-1},$$

where $\mu = \frac{v}{v'} = \frac{k'}{k}$ (since $n = k v = k' v'$),

and $p = \mu p'$, or $k' p' = k p$;

v is the velocity of propagation of transversal waves in vacuum, and k' is given by equation (5.) of the present paper; p' and q' are therefore imaginary.

Let us assume

$$k' s' = k (\sigma - \chi \sqrt{-1}),$$

then, since $k p = k' p'$ and $p'^2 + s'^2 = 1$, we find

$$\begin{aligned} k^2 p^2 + k^2 (\sigma - \chi \sqrt{-1})^2 &= k'^2 \\ &= \frac{n^2 - C' n \sqrt{-1}}{B'} \text{ by (5.)} \end{aligned}$$

and $\therefore p^2 + (\sigma - \chi \sqrt{-1})^2 = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right),$

observing that

$$k^2 = \frac{v^2}{n^2} = \frac{B}{n^2}.$$

11. If the vibrations be *perpendicular* to the plane of incidence, the following are the corresponding values of V , V_i and V' , viz.

* The equations of *connection* obtained in section ii. are evidently the same, whether the particles of matter exercise forces of resistance or not; for the forces of resistance are of the *same* order of magnitude as the forces which appear in the equations of *motion*, and therefore they do not appear in the equations of *connection*.

$$V = \frac{a}{2} e^{\{nt - k(px + sz)\} \sqrt{-1}},$$

$$V_i = \frac{s - \mu s'}{s + \mu s'} \frac{a}{2} e^{\{nt - k(px - sz)\} \sqrt{-1}},$$

$$V' = \left\{ \frac{s - \mu s'}{s + \mu s'} + 1 \right\} \frac{a}{2} e^{\{nt - k'(p'x + s'z)\} \sqrt{-1}}.$$

12. We shall now proceed to deduce real solutions from these imaginary integrals, and first in the case of vibrations *perpendicular* to the plane of incidence.

Assuming

$$\frac{s - \mu s'}{s + \mu s'} = -P e^{-\theta \sqrt{-1}},$$

we find

$$\begin{aligned} \left(\frac{1 + P e^{-\theta \sqrt{-1}}}{1 - P e^{-\theta \sqrt{-1}}} \right)^2 &= \frac{\mu^2 s'^2}{s^2} \\ &= \frac{(k' s')^2}{k^2 s^2} \\ &= \frac{k'^2 - k^2 p^2}{k^2 s^2}. \end{aligned}$$

Hence

$$\begin{aligned} s^2 \left(\frac{1 + P e^{-\theta \sqrt{-1}}}{1 - P e^{-\theta \sqrt{-1}}} \right)^2 + p^2 &= \frac{k'^2}{k^2} \\ &= \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right). \end{aligned}$$

We have, therefore, for vibrations *perpendicular* to the plane of incidence,

$$V = \frac{a}{2} e^{\{nt - k(px + sz)\} \sqrt{-1}},$$

$$V_i = -P e^{-\theta \sqrt{-1}} \frac{a}{2} e^{\{nt - k(px - sz)\} \sqrt{-1}},$$

$$V' = (1 - P e^{-\theta \sqrt{-1}}) \frac{a}{2} e^{\{nt - k(px + sz)\} \sqrt{-1} - k\chi z},$$

where P , θ , σ and χ are given by the two equations

$$(\sigma - \chi \sqrt{-1})^2 + p^2 = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right),$$

and
$$s^2 \left(\frac{1 + P e^{-\theta \sqrt{-1}}}{1 - P e^{-\theta \sqrt{-1}}} \right)^2 + p^2 = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right).$$

We may now immediately find another set of values of V , V_i and V' , by simply changing the sign of the $\sqrt{-1}$ in these formulæ, which gives us

$$V = \frac{a}{2} e^{-\{nt-k(px+sz)\} \sqrt{-1}},$$

$$V_i = -P e^{\theta \sqrt{-1}} \frac{a}{2} e^{-\{nt-k(px-sz)\} \sqrt{-1}},$$

$$V^i = (1-P e^{\theta \sqrt{-1}}) \frac{a}{2} e^{-\{nt-k(px+\sigma z)\} \sqrt{-1}-k\chi z}.$$

And superposing these two sets of values we find the following real values of V , V_i and V^i , which satisfy the equations of motion and connection, viz.

$$V = a \cos \{nt - k(px + sz)\},$$

$$V_i = -P a \cos \{nt - k(px - sz) - \theta\},$$

$$V^i = ae^{-k\chi z} \left\{ \cos \{nt - k(px + \sigma z)\} - P \cos \{nt - k(px + \sigma z) - \theta\} \right\}.$$

Thus we have proved the laws of reflexion and refraction above-stated for vibrations *perpendicular* to the plane of incidence.

13. We may proceed in exactly the same way for vibrations *parallel* to the plane of incidence.

Assuming

$$\frac{\mu s - s' - \eta}{\mu s + s' + \eta} = P Q e^{-(\theta + i) \sqrt{-1}},$$

we find

$$Q e^{-i \sqrt{-1}} = -\frac{s + \mu s' \mu s - s' - \eta}{s - \mu s' \mu s + s' + \eta},$$

which gives, by a simple reduction,

$$\frac{1 - Q e^{-i \sqrt{-1}}}{1 + Q e^{-i \sqrt{-1}}} = \frac{\frac{\mu^2 - 1}{\mu} p^2 + \mu s' \eta}{(\mu^2 - 1) s s' - s \eta}.$$

Now, by section viii. (of the Memoir above-quoted), we have

$$\eta = \frac{\nu^2 - 1}{\nu^2 + 1} \frac{\mu^2 - 1}{\mu} p \sqrt{-1},$$

where ν^2 is what μ^2 becomes when B and B' are changed into A and A' , and therefore

$$\nu^2 = \frac{A}{A'} \left(1 - \frac{C'}{n} \sqrt{-1} \right) = \frac{A}{A'} \frac{B'}{B} \mu^2.$$

Hence

$$\frac{1 - Q e^{-i \sqrt{-1}}}{1 + Q e^{-i \sqrt{-1}}} = \frac{\frac{p^2}{\mu} + \frac{\nu^2 - 1}{\nu^2 + 1} p s' \sqrt{-1}}{s s' - \frac{\nu^2 - 1}{\nu^2 + 1} \frac{p}{\mu} s \sqrt{-1}},$$

$$= \frac{p}{s} \cdot \frac{p' + \frac{v^2 - 1}{v^2 + 1} s' \sqrt{-1}}{s' - \frac{v^2 - 1}{v^2 + 1} p' \sqrt{-1}}.$$

From this we may easily show that

$$\begin{aligned} \frac{e^{2\phi\sqrt{-1}} - Qe^{-\phi\sqrt{-1}}}{Qe^{-\phi\sqrt{-1}}e^{2\phi\sqrt{-1}} - 1} &= v^2 (s' - p' \sqrt{-1})^2 \\ &= \frac{A}{A'} \frac{B'}{B} \mu^2 (s' - p' \sqrt{-1})^2 \\ &= \frac{A}{A'} \frac{B'}{B} (\mu s' - p \sqrt{-1})^2. \end{aligned}$$

Hence, assuming

$$\frac{e^{2\phi\sqrt{-1}} - Qe^{-\phi\sqrt{-1}}}{Qe^{-\phi\sqrt{-1}}e^{2\phi\sqrt{-1}} - 1} = U^2, \text{ and } c^2 = \frac{A'}{A} \frac{B}{B'},$$

we find

$$c U = \mu s' - p \sqrt{-1},$$

$$\text{and } \therefore (c U + p \sqrt{-1})^2 = \mu^2 s'^2 = \mu^2 - p^2;$$

and hence, finally,

$$c^2 U^2 + 2 c U \sqrt{-1} = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right).$$

Furthermore, if we assume

$$\frac{1}{\mu} = K e^{* \sqrt{-1}},$$

we have

$$\frac{1}{K^2 e^{2* \sqrt{-1}}} = \frac{B}{B'} \left(1 - \frac{C'}{n} \sqrt{-1} \right).$$

We have, therefore, for vibrations *parallel* to the plane of incidence,

$$V = \frac{b}{2} e^{\{nt - k(px + sz)\} \sqrt{-1}},$$

$$V_i = P Q e^{-(i+\theta) \sqrt{-1}} \frac{b}{2} e^{\{nt - k(px + sz)\} \sqrt{-1}},$$

$$V' = K e^{* \sqrt{-1}} \{ 1 + P Q e^{-(i+\theta) \sqrt{-1}} \} \frac{b}{2} e^{\{nt - k(px + sz)\} \sqrt{-1} - kxz}.$$

Hence, proceeding as before, we find the following real values :

$$V = b \cos \{nt - k(px + sz)\},$$

$$V_1 = PQ \frac{b}{2} \cos \{nt - k(px + sz) - \theta - i\},$$

$$V' = Kbe^{-kxz} \left\{ \cos \{nt - k(px + \sigma z) + \kappa\} \right. \\ \left. + PQ \cos \{nt - k(px + \sigma z) + \kappa - \theta - i\} \right\}.$$

Thus we have proved the laws above-stated for vibrations *parallel* to the plane of incidence.

[To be continued.]

XLIV. *Analysis of the Bonnington Water, near Leith, Scotland.* By EDWARD G. SCHWEITZER, Esq.*

THE soil from which this mineral spring rises, according to the kind information obtained through G. Haig, Esq. of Bonnington House, is, for 20 feet below the surface, a wet, slipping sand of a yellowish colour, which is succeeded, for 20 feet, by a black hard clay, and this is followed, by a hard sandstone rock of considerable thickness, limestone of different qualities, and two small seams of coal; these have been bored, with very little interruption, through the extent of 184 feet. The mineral water when taken from the spring is quite transparent, with an empyreumatic smell, which it likewise imparts to the palate; it has also the taste of alkaline chlorides and iron. When a bottle containing the water for examination was opened, no disengagement of carbonic acid gas was perceptible, and when poured into a glass vessel but few bubbles of gas appeared at the sides; when mixed, however, with an acid, a slight discharge of carbonic acid gas took place. When exposed to the atmospheric air it becomes turbid, and a brownish sediment is formed; a few dark brown flakes were also floating in it, which were separately examined. The natural temperature of this water is about 44° Fahr., its specific gravity at 60° F. is 1.002618, and the quantity emitted from the spring amounts to about from 4 to 5 gallons per minute.

It indicated no action upon blue litmus paper; when boiled it became turbid, oxide of iron with some organic matter precipitated, and a slight alkaline reaction upon reddened litmus paper was developed. The usual well-known application of tests for a qualitative analysis evidenced the presence of potassa, soda, lime, magnesia, oxide of iron, sulphuric acid, chlorine, and carbonic acid gas.

* Communicated by the Chemical Society; having been read November 18, 1844.