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On the Platonist Doctrine of the ἀσxs1F7Bμβλητοι ἀριθμοxs1F77

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γενέστεροι, but Prof. Tucker's χρόνῳ δὲ δαρόν is brilliant, and deserving of acceptance.

I will add only one further note: In *fr.* 801, vv. 2, 3 should be separated from

v. 1 (as suggested in Nauck), and for δίδωσιν ὅστις κ.τ.λ. we should read δμῶς ἔστιν ὅστις κ.τ.λ.

R. G. BURY.

ON THE PLATONIST DOCTRINE OF THE ἀσύμβλητοι ἀριθμοί.

§ 1. ARISTOTLE *Nic. Eth.* I. vi. and *Metaph.* M. vi.

AN interesting article in the February number of the *Classical Review* discusses what is said to be 'a well-known difficulty,' amounting to 'seeming contradiction,'¹ between the following passages:—*Nic. Eth.* I. vi. 1096^a 17 οἱ δὲ κομίσαντες τὴν δόξαν ταύτην οὐκ ἐποιοῦν ἰδέας ἐν οἷς τὸ πρότερον καὶ ὕστερον ἔλεγον, διότι οὐδὲ τῶν ἀριθμῶν ἰδέαν κατεσκεύαζον: *Metaph.* M. vi. 1080^b 11 οἱ μὲν οὖν ἀμφοτέρους φασὶν εἶναι τοὺς ἀριθμούς, τὸν μὲν ἔχοντα τὸ πρότερον καὶ ὕστερον τὰς ἰδέας, τὸν δὲ μαθηματικὸν παρὰ τὰς ἰδέας καὶ τὰ αἰσθητά, καὶ χωριστοὺς ἀμφοτέρους τῶν αἰσθητῶν. The difficulty, surely, is imaginary, and only due to one of those slips of interpretation sometimes made by distinguished critics in very plain matters. The writer of the article must pardon a fellow-student for thinking that in it the true nature of the mistake has not been made clear, and that—perhaps in consequence—the simplicity of the issue and of its solution have not been realised: further that the interpretation given of the passage from the *Ethics* is not correct, owing to a misunderstanding of the drift of a passage referred to from *Metaph.* B. iv.

The seeming contradiction arose simply from a misinterpretation of the first clause in the passage from the *Ethics*. According to the *Metaphysics*, the Ideal numbers stand in the relation of πρότερον καὶ ὕστερον. The words in the *Ethics*, on the other hand, οὐκ ἐποιοῦν ἰδέας ἐν οἷς τὸ πρότερον καὶ ὕστερον ἔλεγον were taken to mean that there were no 'Ideas' at all² in the case of things

related as πρότερον and ὕστερον. This would necessitate that the numbers referred to in the *Metaphysics* as having in them the πρότερον καὶ ὕστερον could not be Ideas, and so would contradict the statement in the *Metaphysics* that they were.

But the first clause of the *Ethics* passage means, not that the Platonists allowed no Ideas at all in the sphere of the πρότερον καὶ ὕστερον, but that in the case of a group whose members were in this relation, there was no one single Idea to correspond to the group as such. Aristotle uses the plural ἰδέας and not the singular ἰδέαν, because he is thinking of such groups in general. There is no implication that the members of a group of the kind could not be themselves Ideas. And with this the whole difficulty disappears.

The second clause means that the Platonists in consequence of the doctrine ascribed to them in the first clause did not recognise one Idea of the Numbers, as they held the Numbers stood in the relation of πρότερον καὶ ὕστερον.

Thus if we had no other passage from the *Metaphysics* except that which was supposed to contradict the *Ethics*, we should by combining the two places arrive at this perfectly coherent result:—The later Platonists held that the Ideal numbers were in the relation of πρότερον καὶ ὕστερον to one another (*Met.*). They also held that when the members of a group stood in that relation there was no one Idea corresponding to the group (*Eth.*) Consequently they held that the Ideal numbers had no one Idea of number corresponding to them as a group; i.e. there was

¹ Cp. Trendelenburg, *Platonis de ideis et numeris doctrina ex Aristotele illustrata*, p. 80: 'locos duos inter se, ut videtur, repugnantes.' Zeller, *Plat. Stud.* p. 243: 'mit welcher Stelle, die ihr widersprechende,' etc.

² Cp. Trend. l.c.: 'his inter se collatis, alterum, ideas eum definiri numerum, qui habeat prius et posterius, altero, non ideas factas eorum in quibus sit prius et posterius, prorsus repugnat.' Brandis, *Rhein. Mus.* 2 (1828), p. 563. Zeller, *Phil. d. Gr.*, 3rd ed., II. i. p. 571: 'Wie lässt sich nun aber mit

dieser Auffassung...die Angabe vereinigen dass Plato und seine Schule von demjenigen in dem das Vor und Nach stattfindet, keine Ideen angenommen haben?' Zeller himself, who at first followed Trendelenburg, but eventually realised the essentials of the interpretation of the *Ethics* passage, does not happen to say that the difficulty was merely due to this mistranslation, his attention being taken up with the questions about the meaning of τὸ πρότερον καὶ ὕστερον raised by those who misunderstood the *Ethics* text.

no 'Idea' corresponding to ἀριθμός in general (*Eth.*). There would then be no disagreement, and we need nothing further for the reconciliation of the two passages taken alone.

But there is another passage in the *Metaphysics* itself (999^a 6) which when combined with the above passage from the same treatise gives exactly the same result as the combination of that passage with the *Ethics*.

Met. B iii. 999^a 6 ἐτι ἐν οἷς τὸ πρότερον καὶ ὑστερόν ἐστιν, οὐχ οἷον τε τὸ ἐπὶ τούτων εἶναι τι παρὰ ταῦτα. οἷον εἰ πρώτη τῶν ἀριθμῶν ἡ δυνάς, οὐκ ἔσται τις ἀριθμὸς παρὰ τὰ εἶδη τῶν ἀριθμῶν. According to this if a group of things stand in the relation of πρότερον καὶ ὑστερον, their common predicate (τὸ ἐπὶ τούτων) cannot be some common element distinguishable (παρὰ) from them. What this more exactly means will be considered later, for the moment we are only concerned with the application made of it in the context, which is that since the εἶδη τῶν ἀριθμῶν stand in the relation of 'prior and posterior,' there is no ἀριθμὸς distinguishable from these εἶδη. If the Platonists accepted this sort of reasoning—and we learn from the *Ethics* that they did, then since they held the Ideal numbers stood in such relation, they would necessarily hold that there was no one Idea corresponding to this group as such, i.e. that there was no Idea of ἀριθμός. Thus the result of the two passages from the *Metaphysics* agrees entirely with the passage from the *Ethics*, and there is not the smallest discrepancy between the two treatises.

The foregoing is all that is relevant to shew the unreality of the supposed difficulty. Nothing turns upon what the exact meaning of τὸ πρότερον καὶ ὑστερον may be, and it is not necessary to enquire, for instance, if it refers to some process of γέννησις, or even to discuss it at all for the real issue between the passages, however otherwise interesting. Nor is it right to suppose the meaning of the *Ethics* to be that 'the Idealists did not view the higher (logically soluble) genera as Ideas.' 'Logically soluble' appears to stand for 'capable of differentiation into species,' so that the meaning would be 'the Idealists allow no Idea except for *infimae species*.' This doctrine is not really contained in the passages before us. All that is ascribed to the Idealists is the doctrine that when species are related to one another as 'prior and posterior' there could be for them no one Idea. Thus for instance, it is not a true inference from *Metaph.* 999^a 6, above

quoted, that 'Ἀριθμός does not constitute in itself an ἰδέα because it is divisible into a variety of εἶδη.' The principle stated in this passage is not applied to the Platonic Ideas at all; and if it were, the result would be that there was no ἰδέα of ἀριθμός, not because ἀριθμός was divisible into εἶδη, but because its εἶδη stand to one another in the relation of πρότερον καὶ ὑστερον. The εἶδη meant are the δυνάς, τριάς, τετράς, etc., etc., of which the δυνάς is prior to the τριάς and all the rest, the τριάς, to the τετράς and all the rest, and so on. The bearing of the whole passage of which 999^a 6–9, is a part will be considered later.¹

§ 2.—*Current views on the ἀσύμβλητοι ἀριθμοὶ and the doctrine of τὰ μεταξύ.*

We may turn to some questions of somewhat greater difficulty.

What is the true meaning, and what is the origin of the doctrine that the Ideal numbers were ἀσύμβλητοι, and of the doctrine associated with it that the objects of mathematics, τὰ μαθηματικά are μεταξύ—between the Ideas and the world of sense?

According to the theory of Ideas represented by Plato's writings, there were of course Ideas of number. In a later development of Platonism, of which we hear through Aristotle, all 'Ideas' were somehow identified with Numbers. The Ideas of numbers, ἰδέαι τῶν ἀριθμῶν, of the earlier theory, and the Idea-numbers of the later may both be called 'Ideal numbers,' and this general expression has been used in the foregoing for a reason which will appear hereafter.

It is perhaps the generally accepted view that ἀσύμβλητοι ἀριθμοὶ was a designation only applied to the Idea-numbers of the later theory and suited to them alone. It might seem that 'Ideas' would only be called numbers in some metaphorical way, and that the name ἀριθμοὶ ἀσύμβλητοι indicated the Idea-numbers had not the properties of true number; for to say they could not be added would be to say that no arithmetical relation was possible between them. Bonitz expressly remarks (*Comm. on Metaph.* p. 540) that the name ἀσύμβλητοι shews nothing numerical was really intended:—hos numeros, qui idearum exprimerent naturam a mathematicis ea distinxit ratione, quae revera ipsam numerorum naturam penitus tolleretur. Cp. p. 541 Nimirum quum numeris abstractis illud, ut sint συμβλητοί, inde accidat, quod nullum in iis est qualitatis discrimen sed unice quanti-

¹ § 7.

tas diversa, ad idem prorsus redit utrum numeros dixeris ἀσύνβλητους esse an qualitate inter se differre. He thinks the numbers were mere symbols of notions: cp. p. 543. Etenim Plato quod ἀσύνβλητους dixit numeros suos ideales, qualitatis diversitatem, quae in iis cerneretur, significavit et ipsam quantitativam numeri naturam ita sustulit, ut eorum vis non amplius penderet unitatum a multitudine . . . sed numeri modo signa quaedam et quasi symbola fiant notionum. Thus it would be assumed without question that the Ideal numbers of the earlier theory being Ideas of number in the literal sense were not the ἀριθμοὶ ἀσύνβλητοι, that these latter belonged to the later theory, and were, of course, but a paradox of Platonism without any value for thought in general.

It must be contended that these views are erroneous. The theory of the ἀσύνβλητοι ἀριθμοὶ contained an important truth, though it was not appreciated by Aristotle: and it is a theory which, in the nature of the case, belonged essentially to the earlier Platonism, arose out of that and not out of anything peculiar to the later theory.

It seems also to be supposed that the doctrine of the μαθηματικὸς ἀριθμὸς, as something μεταξύ τοῦ εἰδητικοῦ καὶ τοῦ αἰσθητοῦ (1090^b 35) was a peculiar adjunct of the later Idea-number theory; probably because it is thought that as the Idea-numbers did not relate to true numbers at all some place had to be found for true, or mathematical number, and for this *Metaph.* 991^b 26¹ might be appealed to. But the origin of this doctrine again, which cannot be dissociated from the view that geometrical figures were also μεταξύ, lies in conditions which belong not to the later theory but to the earlier, and has nothing whatever to do with the Idea-numbers as such.

§ 3.—*Meaning and origin of the conception of ἀσύνβλητοι ἀριθμοί.*

We may consider first the ἀσύνβλητοι ἀριθμοί.

Plato's ἰδέα is of course the Universal. We are not concerned here with what is peculiar to his view of its nature, but with a part of it, which all must accept, and upon which Plato laid great stress: namely, the assertion that the Universal is one—a unity, that is, in contrast with the manifold to which it corresponds. This means that whereas there are many circles, for instance, Circularity, the Universal, is one—there is

only one Circularity. It is not necessary to repeat here the familiar *reductio ad absurdum* of the hypothesis that there could be more than one. 'The circle,' with the definite article, is an equivalent expression for circularity, and even if we give several definitions of 'the Circle' we suppose them all definitions of one and the same thing. 'The number Two,' as we call it, is a Universal: it is 'twoness' in general, and there is only one 'twoness.' It is because there is only one that we use the definite article in the expression 'the number Two.' Now 'the number Two' thus accurately understood cannot enter into a process of summation like a particular two. 'Two and two make four' means two things (= a particular two) added to two other things of the same kind amount to four things. The proposition is a universal one because it stands for 'any two things added to any other two things, etc., etc.,' but not because it means an addition of Universals. It does not mean, that is, that twoness added to twoness is fourness. There is only one 'twoness' and thus the expression 'twoness added to twoness' has no sense. This is the same as saying that 'two and two make four' does not mean that 'the number Two' added to 'the number Two' is 'the number Four.' Nor does anyone ever express the proposition in this way. As before, we must say there is only one 'number Two,' and that 'the number Two added to the number Two' is an unmeaning expression. Similarly 'two and three are five' means 'any particular two (i.e. any two things) added to any other particular three, etc.' and does not mean twoness added to threeness is five-ness. The latter is an absurd expression, for, as is easily seen, it would involve more than one twoness, and more than one threeness. Or it may be put thus:—if twoness (= the number Two) could be added to threeness (= the number Three) so as to produce five-ness (= the number Five) twoness must be conceived as having two units added to three units contained in threeness. But such units cannot be particulars as twoness is universal: they must be Universals, and the Universal of units is oneness. Thus twoness would have to consist of oneness added to another oneness which is absurd as there is only one 'oneness.' Thus 'the number Two,' 'the number Three,' etc., that is the universal twoness, the universal threeness, etc., or, in popular language, the abstractions of twoness and threeness, etc. do not consist of units, and are not capable of numerical addition in the same sense as a two and a

¹ This passage is considered below, § 4, paragraph 8.

three, by the combination of units. And thus if we call them Numbers (we call them 'the Numbers') they are certainly numbers which are not addible.¹ This is exactly the Platonic doctrine; for if the *ιδέαι* or Universals of number are called *ἀριθμοί* they must be *ἀριθμοὶ ἀσύμβλητοι*.

Geometry of course affords an exact parallel. Just as the Universals represented by the Numbers cannot enter into arithmetical operation, in the sense explained, so also the Universals represented by the figures cannot have geometrical constructions performed upon them, or be elements in such constructions. 'The Circle' as we have said is a Universal, and e.g. just as 'the number Two' cannot be added to 'the number Two,' the Circle cannot intersect the Circle. That is circularity cannot intersect circularity, for there is only one circularity. Accordingly no mathematician would ever think of expressing the proposition 'a circle can intersect another circle in not more than two (real) points' in the form 'the circle can intersect the circle etc.' the absurdity of which would be felt at once. The construction of Eucl. I. 1 is general, and universal in the proper sense, but it is not the construction of the Universal, 'equilateral triangularity,' upon the Universal, 'rectilinearity,' nor the construction of the equilateral triangle upon the straight line or upon 'the given straight line.'

The doctrine then of the *ἀσύμβλητοι ἀριθμοί*, instead of being a mere fantastic product of later Platonism, embodies a truth which depends on nothing peculiar to Platonism. In that philosophy it arises naturally out of the principles involved in that theory of Ideas which is represented in Plato's writings and not out of anything peculiar to the later Idea-number theory.²

¹ About this principle there is no confusion within mathematics proper: indeed no occasion for it arises there. But it is otherwise with what may be called 'reflective mathematics' or 'quasi-philosophic mathematics.' For the attempt to find continuity within number itself (cf. Dedekind) is a mistake which comes from looking on the Numbers as magnitudes, and not realising the truth attained so long ago in Greek philosophy that they are Universals. A parallel mistake would be to treat 'triangularity,' 'squareness,' etc., etc., as figures, misled by the linguistic equivalence to them of 'the Triangle,' 'the Square,' etc.

² As is well known, there are two criticisms in the *Metaphysics* of the Platonic theory of Ideas, one in bk. A, and another in bk. M. The second is a kind of revised and expanded version of the first: in some places it is a mere duplicate, lengthy passages being repeated word for word. It is noteworthy that whereas the version in bk. M has an introduction

When in this latter all Ideas were identified with Ideas of number, as Ideas of numbers they would be still *ἀσύμβλητοι*, and it is exactly on their numerical side and not as mere Ideas that the epithet belongs to them and is relevant. Accordingly when Aristotle attacks the Idea-numbers, he speaks of them as *ἀσύμβλητοι ἀριθμοί*, and while he of course points out the absurdity of identifying with Ideas of number Ideas of things which cannot be mere number, such as Man and Animal, most of his criticisms have nothing to do with this aspect of the Idea-numbers, but relate to their numerical aspect as Ideas of numbers solely, and the other aspect might be altogether non-existent as far as these criticisms are concerned.

From his treatment of the conception of *ἀσύμβλητοι ἀριθμοί* it must be gathered that Aristotle did not appreciate the truth conveyed in it. Possibly the form in which he presents it is a perversion current in the Academy, but if he had recognised the valuable side of it, we should expect him to say so.

His attack turns mainly on the assumption that while the units in one Ideal number could not be added to those of another, this constituting them *ἀσύμβλητοι*, each of them consisted of units which were added together within the number itself. He gives this as if it were but a statement of the Platonists' own view. In his division of possible opinions he gives first the one that the *μονάδες* were all *ἀσύμβλητοι*, but

which may well imply the writer held it not strictly accurate to speak of the Idea-number theory as if due to Plato himself, as was done in the first version—*περὶ δὲ τῶν ιδέων πρώτων αὐτὴν τὴν κατὰ τὴν ιδέαν δόξαν ἐπισκεπτόμεν, μὴδὲν συνάπτοντας πρὸς τὴν τῶν ἀριθμῶν φύσιν, ἀλλ' ὥς ἐπέλαβον ἐξ ἀρχῆς οἱ πρῶτοι τὰς ιδέας φήσαντες εἶναι* (1078^a 9), later on in the same book the conception of *ἀσύμβλητοι ἀριθμοί* is attributed to Plato. Cf. 1083^b 33, *εἰ δ' ἐστὶ τὸ ἐν ἀρχῇ, ἀνάγκη μᾶλλον ὅσπερ Πλάτων εἰλεγεν ἔχειν τὰ περὶ τοὺς ἀριθμούς, καὶ εἶναι τινα δυνάδα πρώτην καὶ τριάδα καὶ οὐ συνβλητοὺς εἶναι τοὺς ἀριθμούς πρὸς ἀλλήλους*. This so far confirms the view which is now put forward. But the important point is the contention that the conception of the *ἀσύμβλητοι ἀριθμοί* even if formulated by those who originated the Idea-number theory depends on the principles of the earlier theory alone.

The impression that the writer of the second version wishes to dissociate the name of Plato from the Idea-number theory, is confirmed by another interesting circumstance. The second version while repeating the greater part of a long passage at the beginning of the first version nearly word for word, omits a little way down in the context that passage in the earlier version which involves the identification of Plato's Idea theory with the Idea-number theory and substitutes for it some different matter.

says expressly that it was an opinion which no one had held.

Now, as has been shewn above, the considerations which make it impossible that a Universal of number, such as 'fourness,' or 'the number Four,' should consist of Universals of number added together—'twoness to twoness' or 'the number Two' to 'the number Two,' make it impossible that any Universal of number, i.e. any one of the Numbers, should consist of units added together, for that would involve that there should be more than one universal 'oneness.'

The two thoughts are inseparable; and it is hardly conceivable that the philosopher whose penetration enabled him to appreciate one side of the truth should have failed to see the other. But it is very credible that a conception, so remote from ordinary habits of mind, should be imperfectly understood by disciples.

The alternative, *πάσας τὰς μονάδας εἶναι ἀσυμβλήτους*, which Aristotle says no one maintained, would not in any case represent the true theory: for of course, as we have seen, that no more admits of a plurality of 'oneness' than of 'twoness': that is, just as there is *the δνός* but not *δνάδες* in the plural, so there would be *the μονάς* but not *μονάδες*.

Bonitz in a footnote (p. 555) to his commentary on *Met. M. viii.* has remarked that Aristotle's criticisms are not well directed because those who held the theory of *ἀσύμβλητοι ἀριθμοί* would not consider them as each composed of added units. But the objection as Bonitz puts it is mistaken, for it is based upon his view that the *ἀσύμβλητοι ἀριθμοί* were in no sense numbers and had nothing really numerical about them. Thus he supposes that the Platonists could not represent them as composed of units because they did not mean them to be ideas of real number at all. 'Etenim is qui eos numeros, quos pro principiis ponat, *ἀσυμβλήτους* esse dicit, ipsam numerorum naturam manifesto tollit.' Owing to the same mistake he supposes Aristotle's proper course would have been to shew that the conception of *ἀριθμὸς ἀσύμβλητος* was self-contradictory.

§ 4.—Meaning and origin of the doctrine of τὰ μεταξύ.

Before discussing the meaning of the *πρότερον καὶ ὕστερον* and certain other subjects connected with the Ideal numbers, it will be best for the confirmation of the

view put forward about the *ἀσύμβλητοι ἀριθμοί* to pass to another doctrine of later Platonism, that of τὰ μεταξύ; or the doctrine that the objects of mathematical science were neither Ideas nor particulars of sense, but something between the two; for this may be shewn to be the outcome, though a mistaken one, of the same kind of thinking as that which produced the *ἀσύμβλητοι ἀριθμοί* combined with Plato's theory of the object of scientific knowledge.

The passage of Aristotle which helps most towards ascertaining the real meaning and origin of the conception of τὰ μεταξύ is the following from *Metaph. A. vi.* 987^b 14: —*ἐτι δὲ παρὰ τὰ αἰσθητὰ καὶ τὰ εἶδη τὰ μαθηματικὰ τῶν πραγμάτων εἶναι φησι μεταξύ, διαφέροντα τῶν μὲν αἰσθητῶν τῷ αἰδέα καὶ ἀκίνητα εἶναι, τῶν δὲ εἰδῶν τῷ τὰ μὲν πολλὰ ἅττα ὅμοια εἶναι, τὸ δὲ εἶδος αὐτὸ ἐν ἑκάστῳ μόνον.* cp. 1002^b 15 τὰ μὲν μαθηματικὰ τῶν δεύρο ἄλλω μὲν τινι διαφέρει, τῷ δὲ πολλὰ ἅττα ὁμοειδῆ εἶναι οὐδὲν διαφέρει.

Plato's view as it appears in his writings is that the sciences in the strict sense dealt with Universals only, or, in his language, with 'Ideas' only; with the Idea, e.g. of the Circle and not with particular circles. But there is only one 'Idea' of the circle, and so if geometry were of that alone there would be no possibility of a proposition about two or more circles. Similarly in arithmetic, if the only Two to be had was the Idea, there is only one such, and yet in some sense arithmetic requires a plurality of twos. This difficulty is not realised, much less provided for, in the writings of Plato:¹ but it seems to explain both the meaning and the origin of the conception of τὰ μεταξύ. For this can be understood if we suppose it contrived, whether by Plato or a disciple, to meet the difficulty above stated.²

It was probably held that geometry could not really be of the Idea of the Circle which was only one, and must be of some plurality of circles. But the Platonic doctrine was retained that the objects of science were eternal, unchangeable (and perfect), and not the fleeting (and imperfect) particulars of sense: so these manifold circles were considered as eternal (and perfect). In this they were like the Idea of the circle, τῷ

¹ See below, § 8, on *διάνοια* and *μαθηματικὴ* in the *Republic*.

² The writer finds that his friend Mr. J. A. Smith, of Balliol College, has independently arrived at much the same result; and also shares the opinion that the conception of *ἀσύμβλητοι ἀριθμοί* was connected with Platonism in general, and not specially with the Idea-number theory.

αἰδία καὶ ἀκίνητα εἶναι, while as a plurality they were so far like the circles of perception, τῷ πᾶσι ἅρτα ὁμοία εἶναι. And so for all the objects of mathematics in general whether numbers or figures.¹

Thus the number with which Mathematics deal was called μαθηματικὸς ἀριθμὸς in distinction from the ἀριθμοὶ ἀσύμβλητοι and said to be μεταξύ, or between ideal numbers and concrete numbers.

The doctrine may possibly have been developed by the later Platonists who originated the Idea-number theory, but in itself it is the natural product of the line of thought which produced the conception of the ἀσύμβλητοι ἀριθμοί, and has nothing to do with the reduction of all Ideas to Ideas of number.

There were different forms of the Idea-number theory, and of the first form it is said in *Metaph.* M. 1086^a 10, ὁ δὲ πρῶτος θέμενος τὰ εἶδη εἶναι καὶ ἀριθμούς τὰ εἶδη καὶ τὰ μαθηματικά² εὐλόγως ἐχώρισεν. This means as the context shews that whereas a later

¹ τὰ μεταξύ undoubtedly included geometrical figures as well as number. Cp. *Met.* B. ii. 997^b 2 τὰ μεταξύ, περὶ δὲ τὰς μαθηματικὰς εἶναι φασιν ἐπιστήμας, where μαθηματικὰς ἐπιστήμας must include geometry. This seems also the clear implication of what is said of geometry in 997^b 26 sqq. Compare also *Metaph.* A. ix. 991^b 27, quoted below,—περὶ δὲ ἡ ἀριθμητική, καὶ πάντα τὰ μεταξύ λεγόμενα ὑπὸ τινων, which shews that τὰ μεταξύ comprises more than ἀριθμητική.

It would not be necessary to mention this were it not that *Met.* B. ii. 992^b 13 sqq. might cause a difficulty: οὐδένα δ' ἔχει λόγον οὐδὲ τὰ μετὰ τοὺς ἀριθμούς, μήκῃ καὶ ἐπίπεδα καὶ στερεά, οὔτε ὅπως ἔστιν ἡ ἔσται οὔτε τίνα ἔχει δύναμιν ταῦτα γὰρ οὔτε εἶδη οἷον τε εἶναι, οὐ γὰρ εἰσιν ἀριθμοί, οὔτε τὰ μεταξύ, μαθηματικά γὰρ ἐκεῖνα, οὔτε τὰ φθαρτά, ἀλλὰ πάλιν τέταρτον ἄλλο φαίνεται τοῦτο τί γένος. Here it is implied that geometrical objects, in some sense, are not included in τὰ μαθηματικά, and so are not among τὰ μεταξύ. But Aristotle is not referring to the ordinary form of the doctrine of τὰ μεταξύ. He is attacking certain Platonists who, while identifying all Ideas with numbers, and retaining the view of the intermediacy of the objects of mathematical science, put, it may be supposed, the Universals of geometrical figures not among the Ideas (which were numbers), but a degree below them (μετὰ), thus introducing a fourth kind of object between the Ideas and τὰ μαθηματικά. The geometrical figures which were objects of geometry would still be found in the μαθηματικά. For these Platonists see 1080^b 23, where Aristotle, after noticing certain differences among the Platonists about Ideal and Mathematical Number, says there were differences also about geometrical objects: ὁμοίως δὲ καὶ περὶ τὰ μήκη καὶ περὶ τὰ ἐπίπεδα καὶ περὶ τὰ στερεά. οἱ μὲν γὰρ ἕτερα τὰ μαθηματικά καὶ τὰ μετὰ τὰς ιδέας, which shews that they had two kinds of geometrical objects, one the mathematical and the other μετὰ τὰς ιδέας. Cp. also 1028^b 25, where these philosophers are distinguished from Plato and Speusippus:—ἐνιοὶ δὲ τὰ μὲν εἶδη καὶ τοὺς ἀριθμούς τὴν αὐτὴν ἔχειν φασὶ φύσιν, τὰ δ' ἄλλα ἐχόμενα, γραμμάς καὶ ἐπίπεδα κ.τ.λ.

² Omitting with Christ the εἶναι after μαθηματικά.

form of the Idea-number theory abandoned the distinction between Ideal number and mathematical number (1086^a 6, sqq.), the earliest form (cf. ὁ πρῶτος) maintained it. This does not necessarily imply that the distinction itself originated with the Platonists in question.

In *Metaph.* A. ix. 991^b 27, Aristotle criticising the Idea-number theory objects to it that it would necessitate the invention of another kind of number for the purpose of ἀριθμητική, and indeed the whole sphere of τὰ μεταξύ, a conception which he holds fraught with difficulty:—ἔτι δ' ἀναγκαῖον ἕτερόν τι γένος ἀριθμοῦ κατασκευάζειν, περὶ δὲ ἡ ἀριθμητική, καὶ πάντα τὰ μεταξύ λεγόμενα ὑπὸ τινων.³

This objection does not the least necessitate that the μεταξύ theory should have been a special consequence of the Idea-number theory as such. Aristotle is attacking the latter generally,—finding all the objections he can to it, and this objection would be relevant because the Idea-numbers in this later theory were of course ἀσύμβλητοι ἀριθμοί⁴ and as such seemed to provide no object (as has already been explained) for mathematical operations. Any wrong inference from this passage is corrected by Aristotle's own statements, above quoted from *Metaph.* 987^b 14 and 1002^b 15 which indicate that the μεταξύ theory simply originated in the necessity of finding a plurality for mathematics as against the absolute unity of the mere Idea of a figure or a number. It is perhaps significant (though no great stress need be laid on it) that in all the passages where τὰ μεταξύ in general, and not merely μαθηματικὸς ἀριθμὸς, are opposed to the Ideal world, the latter is designated by εἶδη or ιδέαι not ἀριθμοί (except in one which in a way combines both), and there are a considerable number of them—987^b 14, 995^b 16, 997^b 2, 998^a 7, 1002^b 12, 1028^b 20, 1059^b 6, 1069^a 35. Even in the exceptional passage, 992^b 16, the division is still into εἶδη, τὰ μεταξύ, and τὰ φθαρτά, the text implying in this case that the εἶδη are ἀριθμοί. So that it seems as if in any case the usual formula for the division had εἶδη, not ἀριθμοί. See the discussion below of the phrase εἰδητικός ἀριθμὸς.

In *Metaph.* N. iii. 1090^b 32, ὁ μαθηματικὸς ἀριθμὸς, which is one part of τὰ μεταξύ, is said to be μεταξύ τοῦ εἰδητικοῦ (sc. ἀριθμοῦ) καὶ

³ The editors have a comma after κατασκευάζειν and none after ἀριθμητική which gives what seems a wrong sense.

⁴ See above, § 3, paragraph 3.

τοῦ αἰσθητοῦ. The contrast here not being with mathematical Ideas in general but only with Ideal number, the substitution for εἰδῶν of an expression such as εἰδητικὸς ἀριθμὸς would be natural, even if Aristotle had not, as he may have in this place, the Idea-number theory before him. The expression in itself need not mean anything more than Ideal Number, as will be seen later on.¹ It may indeed refer to the Idea-numbers, but there is nothing in this passage any more than in the one above discussed, *Metaph.* 991^b 27, to prove that the conception of τὰ μετὰ ζυ is only a consequence of the Idea-number theory.

§ 5.—On τὸ πρότερον καὶ ὕστερον in the ἀσύμβλητοι ἀριθμοί.

We may next consider the meaning of the πρότερον καὶ ὕστερον assigned to Ideal Number. It has been said that the text of *Metaph.* M. vi. 1080^b 11 sqq. (quoted above, § 1, init.) is vindicated against Trendelenburg's insertion of μὴ before ἔχοντα τὸ πρότερον καὶ ὕστερον [due to his mistake about the interpretation of *N. Eth.* I. vi] and that the statement in it that Ideal Number contains 'the Before and After' is explicable, because 'Bonitz and others have clearly proved that τὸ πρότερον καὶ ὕστερον was a technical term in the ideal arithmetic for the relation of the factor (τὸ γεννῶν) to the product (τὸ γεννώμενον).' It is difficult to understand how this opinion in this precise form could be attributed to Bonitz. Bonitz in consequence of his mistaken notion that there was nothing really numerical about the Idea-Numbers thought their order must be one of quality (de ordine quodam qualitatis agi) and not a numerical one at all. This seems inconsistent with supposing that order to be one of factor and product, nor does there seem to be evidence of such a view in Bonitz' commentary. Zeller, it is true, makes the relation of πρότερον καὶ ὕστερον that of factor and product in some sense; and this passes with him in a confused way into a purely qualitative distinction,² with which latter Bonitz would agree. But Zeller's defence of the text turns solely on the circumstance that there is abundant evidence in the *Metaphysics* that some kind of πρότερον καὶ ὕστερον was attributed to the Ideal Numbers and not at all upon his theory of what kind it was.

However there is in any case a misunderstanding. A simple and sufficient explanation of the πρότερον καὶ ὕστερον is indicated by the Aristotelian text itself (in which it may be remarked the technical distinction of γεννῶν and γεννώμενον as factor and product does not even occur): whereas the one proposed is entirely inapplicable to the ἀσύμβλητοι ἀριθμοί rightly understood, is not really applicable even to the later Idea-number theory, as will be seen, and at any rate is not necessary for either. The Ideas of numbers, as being the Universals of number and therefore ἀσύμβλητοι, are as ἀσύμβλητοι entirely outside one another, in the sense that none is a part of another. Thus they form a series of different terms, which have a definite order (ἔχουσι τὸ πρότερον καὶ ὕστερον). They are nothing but what mathematicians call 'the series of natural Numbers,' where the definite article is right, because there is only one such series, consisting as it does of Universals each of which is unique. The order of this series is obviously enough to explain the passages in the *Metaphysics*, and enough for the requirements of the argument attributed to the Platonists in the *Ethics*.

This view is fully confirmed by *Metaph.* M. vi, where the order of the Idea-numbers, which, as has been said, are treated on their numerical side as Ideas of number, is closely associated with the fact that they are ἀσύμβλητοι, and not with any sort of γένεσις of them. Thus according to 1080^a 17, if there is a kind of number such that τὸ μὲν πρῶτον τι αὐτοῦ τὸ δ' ἐχόμενον, the numerical elements in such order must be ἀσύμβλητοι. Two cases are possible. Either the elements are μονάδες, and then, if these have an order of πρότερον καὶ ἐχόμενον, they must be all ἀσύμβλητοι. Or the elements are the Numbers; and here the Numbers as forming a series with πρῶτον and ἐχόμενον must be ἀσύμβλητοι with one another, that is, the units in any Number must not be συμβληταί with those of another, though its own units are συμβληταί within itself. In this case we have μετὰ τὸ ἐν πρώτῃ³ ἢ διάς, ἔπειτα ἢ τριάς, καὶ οὕτω δὴ ὁ ἄλλος ἀριθμὸς—i.e. 'the series of natural Numbers.' The mutual exclusiveness, caused by their being ἀσύμβλητοι, which enables them to form a series is expressly put

³ Christ would emend either to ἡ πρώτη διάς or ἡ διὰς ἡ πρώτη. But the text is doubtless sound, for πρώτη corresponds to ἔπειτα. That πρώτη should be assigned to διάς in another sense in the same context ought to cause no difficulty. Such carelessness is common in Aristotle, and there are far harsher instances to be found than this.

¹ See § 8,—On the expression εἰδητικὸς ἀριθμὸς.

² So also Schwegler, note on *Metaph.* B iii. 16. See below note to paragraph 5.

thus :—οὗτος δὲ (sc. ὁ ἀριθμός) μετὰ τὸ ἐν δύο ἕτερα ἀνευ τοῦ ἐνὸς τοῦ πρώτου, καὶ ἡ τριάς ἀνευ τῆς δυνάδος, ὁμοίως δὲ καὶ ὁ ἄλλος ἀριθμός. Thus if there is a serial order of the Numbers they must be ἀσύμβλητοι. And the converse of this is put. In contrast to the number which exhibits an order, the number which does not exhibit an order is such that all units in it are συμβληταί (1080^a 20), and this is the μαθηματικὸς ἀριθμός (* 21 cp. 1081^a 5). The inclusiveness of one number in another in the μαθηματικὸς ἀριθμός, which prevents it from having the πρότερον καὶ ὕστερον,¹ is put thus (1080^a 30) διὸ καὶ ὁ μαθηματικὸς ἀριθμὸς ἀριθμεῖται μετὰ τὸ ἐν δύο, πρὸς τῷ ἐμπροσθεν ἐνὶ ἄλλο ἐν, καὶ τὰ τρία πρὸς τοῖς δυοὶ τούτοις ἄλλο ἐν, καὶ ὁ λοιπὸς δὲ ὡσαύτως. Thus clearly the property of being ἀσύμβλητος is sufficient and necessary to constitute a kind of ἀριθμός a series with πρότερον καὶ ὕστερον. A little below in the same context the production of numbers out of certain elements, according to some philosophers is alluded to :—οἱ λέγοντες τὸ ἐν ἀρχὴν εἶναι καὶ οὐσίαν καὶ στοιχεῖα πάντων, καὶ ἐκ τούτου καὶ ἄλλου τινὸς εἶναι τὸν ἀριθμόν : but no attempt is made to connect the πρότερον καὶ ὕστερον with this.

We may now consider what is found on the γένεσις of number in other places, so far as it concerns the question of the priority of factor to product. The derivation of numbers from τὸ ἐν and ἡ ἀόριστος δυνάς as elements, whether original or adapted from some Pythagorean source, may perhaps belong to the fantastic Platonism which identified all Ideas with Numbers, and these elements may be accordingly regarded as a sort of arithmetical translation of πέρας and ἀπειρον, owing to the Pythagorean association of the former with τὸ ἐν and the latter with the dyad. The description however of the process did not remain a mere metaphor, but got a kind of arithmetical expression though but a vague one. The ἀόριστος δυνάς appears as an operating factor, the effect of which is to double what it operates upon, and so it is δυοποιός (1082^a 10). Thus in distinction from the ὁρισμένη δυνάς, which is the number Two and the abstraction of 'Twoness,' it really comes to be the abstraction of 'Twiceness.' Accordingly by combination of τὸ ἐν with the ἀόριστος δυνάς is produced the number Two, the δυνάς, and by the operation of the ἀόριστος δυνάς on this

again, the number Four. By successive operations of this kind on successive results there arises the series of even numbers which are powers of Two. This would not yield the odd numbers, or the even ones which have an odd factor. Aristotle expressly notes this as a difficulty in *Metaph. M. iii.* 1091^a 10 (cf. ^a22), and we may infer that the theory was originally vague enough to be open to his objection.

Another passage seems to indicate that an attempt to remedy the defect had been made by adding other principles. In *Metaph. M. viii.* 1084^a 3 Aristotle says, in an objection, ἡ δὲ γένεσις τῶν ἀριθμῶν ἢ περιττοῦ ἀριθμοῦ ἢ ἀρτίου αἰεὶ ἐστίν, ὥδι μὲν τοῦ ἐνὸς εἰς τὸν ἀρτίον πίπτοντος περιττός, ὥδι δὲ τῆς μὲν δυνάδος (sc. τῆς ἀορίστου) ἐμπιπτούσης ὁ ἀφ' ἐνὸς διπλασιαζόμενος, ὥδι δὲ τῶν περιττῶν ὁ ἄλλος ἀρτίος. The odd numbers are produced by the operation of τὸ ἐν on an even number, in the way of addition : the other even numbers (ὁ ἄλλος ἀρτίος) which do not arise from the operation of the ἀόριστος δυνάς, above explained, have their γένεσις described in ὥδι δὲ τῶν περιττῶν κ.τ.λ. This we might expect to be the operation of the ἀόριστος δυνάς on odd numbers, and so Bonitz understands it : but the Greek is not suited to this, and rather suggests τῶν περιττῶν ἐμπιπτόντων εἰς τὸν ἀρτίον, which, understood of multiplication, would give the desired result. However the text is elliptical, for after ἐμπιπτούσης is to be understood εἰς τὸ ἐν as well as εἰς τὸν ἀρτίον. This account involves addition as well as multiplication, and if therefore the πρότερον καὶ ὕστερον had to be explained by it, that formula could not be confined to the relation between 'factor (γεννῶν) and product (γεννώμενον).' Thus the proposed explanation of τὸ πρότερον καὶ ὕστερον, the one which seems to be a part² of Zeller's, fails even in the case most favourable to it.

In the case of what may be supposed to be the original meaning of the ἀσύμβλητοι ἀριθμοί such a genesis whether by multiplication or addition is not to be thought of. It may be argued, as before, that the philosopher who grasped the truth expressed in the phrase ἀσύμβλητοι ἀριθμοί, could not have regarded them as generated by multiplication or addition : that would be too crude a misunderstanding of the principle to which his own insight had led him.

¹ Hence if *Metaph.* 999^a 6 referred to the Platonist theory, the ἀριθμός there spoken of could only be μαθηματικὸς ἀριθμός. But there is really no reference to Platonism in the passage : see below, § 7.

² Zeller's utterances however (*Phil. d. Gr.* 3rd Edn. ii. 1. p. 570) are very confused. It is not necessary to discuss the nature of the confusion, as he has misunderstood the meaning of the ἀσύμβλητοι ἀριθμοί and of the πρότερον καὶ ὕστερον.

It is true that the *γένεσις* theory of some other Platonists involves a *πρότερον* καὶ *ὑστερον* of factor and product and a *πρότερον* καὶ *ὑστερον* of part and whole in the way of addition, and Aristotle makes use of the former kind in criticisms, as in 1082^a 26, and elsewhere. But though he often refers to the *γένεσις* from τὸ ἐν and ἡ ἀόριστος δυνάς he never says that the *πρότερον* καὶ *ὑστερον* characteristic of the ἀσύμβλητοι ἀριθμοί in the Platonist theory was either relative to multiplication or relative to addition, or relative to any sort of *γένεσις*; but, as we have seen he connects the order of these numbers solely with the fact that they are ἀσύμβλητοι. To the passages already quoted for this, another *Met. M.* vii. 1081^a 17 *sqq.* may be added, partly misunderstood both by ancient and modern commentators,¹ the key to which is that if all μονάδες are ἀσύμβλητοι they necessarily form a series in which one is before another in order, according to the principle already enunciated, as we have seen, in *Met. M.* vi. 1080^a 18. Here of course there could be no question of the production of μονάδες by multiplication.

The Idea-numbers then, as Ideas of number are ἀσύμβλητοι, and as Ideas of number have the *πρότερον* and *ὑστερον* of the serial order of 'the natural Numbers.'

§ 6.—On the meaning of ἀριθμῶν in *Nic. Eth.* I. vi.

The question now naturally presents itself, Are the ἀριθμοί in *Nic. Eth.* I. vi, the Ideas of number as in the Ideal theory represented by Plato's writings, or are they the Idea-numbers of later Platonism? It is evident that what is said in this chapter of the *Ethics* is true of both stages of the theory, and it is hardly possible to decide which stage Aristotle had in mind. There is no trace either here or anywhere else in the *Ethics* of the Idea-number theory. In the corresponding chapter of the *Eudemian Ethics*, there is a reference to it, but not in the part which corresponds to the passage in the *Nicomachean Ethics* on the *πρότερον* καὶ

ὑστερον in the ἀριθμοί. It is in an argument added by the author of the *Eudemian Ethics* on his own account, and stands in contrast to the *Nicomachean Ethics* because it bases a criticism of the Idea theory upon its later number form, a thing never done in the *Nicomachean Ethics*. But even if the reference had been in a passage which had a counterpart in the *Nicomachean Ethics*, it would not in the least settle the bearing of the Aristotelian text, such additions being quite in the Eudemian manner. But the really important matter is that even if Aristotle had the Idea-numbers in mind, what he says of them, as is evident from the foregoing investigation, applies to their numerical aspect as Ideas of numbers, and not at all to them as Idea-numbers. Besides if he were regarding them as Ideas without any reference to a numerical character and as standing in an order of *πρότερον* καὶ *ὑστερον* according to some principle not numerical² he would surely have written ἰδεῶν and not ἀριθμῶν, the latter word being in such a context merely irrelevant and confusing.

§ 7.—On the Interpretation of *Metaph.* B iii. 999^a 6.

Some have held that τὰ εἶδη τῶν ἀριθμῶν here refers to the μαθηματικὸς ἀριθμὸς, others that it refers to the εἰδητικὸς ἀριθμὸς. If the choice were really between these alternatives, it is obvious that the only possible one would be that εἰδητικὸς ἀριθμὸς was intended, since, as has been seen (§ 5), μαθηματικὸς ἀριθμὸς does not admit of the *πρότερον* καὶ *ὑστερον*. But the truth is both of these contentions are irrelevant, for in this place Aristotle has not even got the Platonist distinction of μαθηματικὸς ἀριθμὸς from the ἀσύμβλητος ἀριθμὸς before him: he is not criticising anything specially Platonic, and he is speaking in the language of his own philosophy.

The passage before us belongs to a series of questions about ἀρχαί raised at the beginning of bk. B of the *Metaphysics*. In the preceding chapter (ch. 2) Aristotle has considered the Platonic theory. In the present chapter (ch. 3) he passes from that to a general question περὶ τῶν ἀρχῶν which has no special reference to Platonism as such—

¹ ἅμα γὰρ αἱ ἐν τῇ δυνάδι τῇ πρώτῃ μονάδες γεννῶνται. The meaning of an objection brought in these words is that in the process of *γένεσις* referred to the μονάδες would have to be coordinate, whereas (ex hyp.) as ἀσύμβλητοι one must be necessarily prior to the other. Christ notices that Alexander thinks ἡ γὰρ ἅμα preferable, and himself conjectures ἅμα γὰρ αἱ ἐν τῇ δυνάδι πρώτῃ καὶ αἱ ἐν τῇ τριᾷ μονάδες, which can hardly be due to anything else than a misunderstanding. Bonitz appears to have taken the place rightly, but perhaps has not brought the point out clearly enough to prevent Christ's misconception.

² The current expression 'logical order,' which we often use for some order not a time order, is vague. It seems properly to mean an order determined by some principle or conception, and thus should include every kind of numerical order, to which nevertheless it is sometimes opposed.

πότερον δὲ τὰ γένη στοιχεῖα καὶ ἀρχὰς ὑπολαμβάνειν ἢ μᾶλλον ἐξ ὧν ἐνυπαρχόντων ἐστὶν ἕκαστον πρῶτον. After an aporematic discussion of this (998^a20—^b13) he proposes in the present context (998^b 14 *sqq.*) the question whether, supposing the γένη really were ἀρχαί these would be the highest γένη or the 'infimae species,'—πρὸς δὲ τούτοις εἰ καὶ μάλιστα ἀρχαί τὰ γένη εἰσὶ, πότερον δὲ νομίζειν τὰ πρῶτα τῶν γενῶν ἀρχὰς ἢ τὰ ἔσχατα κατηγορούμενα ἐπὶ τῶν ἀτόμων; On this again there is a set of aporematic arguments, the first part of them being directed against the claims of the highest γένη. To this part belongs the passage on the πρότερον καὶ ὕστερον: and it is immediately preceded by an argument the analysis of which will shew how remote is the consideration of anything Platonic. τὸ ὄν and τὸ ἐν having been given as examples of γένη which might seem suited beyond all others to be ἀρχαί, a subtle attempt is made to reduce the view that τὸ ἐν could be an ἀρχή to a contradiction (999^a 1—6). If, it is said, τὸ ἀδιαίρετον is ἐν, then the infima species as being ἀδιαίρετον (*sc. κατ' εἶδος*) has a better title to be considered a ἐν (and so an ἀρχή) than τὸ ἐν itself, because this latter as a γένος is διαίρετον εἰς εἶδη. Then follows the passage in question:—ἐν ἐν οἷς τὸ πρότερον καὶ ὕστερον ἐστίν, οὐχ οἷον τε τὸ ἐπὶ τούτων εἶναι τι παρὰ ταῦτα, οἷον εἰ πρώτη τῶν ἀριθμῶν ἢ δυνάς, οὐκ ἔσται τις ἀριθμὸς παρὰ τὰ εἶδη τῶν ἀριθμῶν ὁμοίως δὲ οὐδὲ σχῆμα παρὰ τὰ εἶδη τῶν σχημάτων. Here τὸ ἐπὶ τούτων means the common predicate of certain species. εἶδη does not mean 'Ideas' but simply species as contrasted with the γένος. Aristotle is maintaining that the common predicate of εἶδη which stand in the relation of πρότερον καὶ ὕστερον cannot be something separable (παρά) from each species and abstractable from it, in the sense that each of them contains not only what this predicate means, but also something more. Thus figure, σχῆμα, is predicated of the various species of figure, τὰ εἶδη τῶν σχημάτων, the triangle, the quadrilateral, etc., but 'triangle' is not figure together with something else other than figure. 'Triangle' contains nothing but what is comprised in 'figure,' so that if we abstract 'figure' from 'triangle,' no differentia of it is left. Or, if we leave any such, what is left is a determination of 'figure,' and so 'figure' is left and has not been abstracted. Similarly for the εἶδη τῶν ἀριθμῶν, the various species of number, that is the Numbers Two, Three, Four, etc, the common predicate number is not τι παρὰ ταῦτα; if we abstract 'number' or numerical-

ness from Two, no differentia of it is left.

This has no connexion whatever with the Platonist distinction of ἀσύμβλητος ἀριθμὸς and μαθηματικὸς ἀριθμὸς, nor is there any reference to Platonism as such.

Though the principle ἐν ἐν οἷς τὸ πρότερον καὶ ὕστερον κ.τ.λ. was, as we learn from the *Ethics*, one recognised by the Platonists, it was also recognised by Aristotle and is here put as his own view. In a familiar passage also from the *Politics* (1275^a 34 *sqq.*) quoted by Zeller, Aristotle puts the view as his own and argues from it. Similarly εἰ πρώτη τῶν ἀριθμῶν ἢ δυνάς has nothing to do with Platonism, but simply means that the number Two comes first in the series of Numbers. Nor has the passage anything to do with the notion of a γένεσις not completed till an ἀτομον εἶδος is reached.

The inseparableness of the generic predicate from species which stand in a certain relative order is an example of a more general principle. If a generic notion is such as to include within itself the differentiae of certain given species, then none of the species contain anything outside (παρά) and distinguishable from what is contained in the given notion: so that to abstract the determination represented by that notion from the species is to take the differentiae as well. The species, it is true, may be said to agree with one another in the genus, but what they differ in is not something outside the genus but it also belongs to the genus. What red and blue agree in is colour, but what they differ in is also colour. The general principle is appreciated by Aristotle and is the basis of the argument—perhaps not always understood—which he urges at the end of *Nic. Ethics* I. vi. 1096^b23. The goods agree in goodness, but they do not therefore differ in something which is not goodness but just in goodness. Hence the definition of the goodness of one good is different from the definition of the goodness of another: which of course prevents there being one single criterion of all good.

The case of the πρότερον καὶ ὕστερον falls under this general principle, because the differentiae in respect of order are comprised in the generic notion itself as is obviously the case with the generic notion of number.¹

¹ Eudemus' attempt to explain this case in *Eud. Eth.* I. viii., where he reproduces *Nic. Eth.* I. vi. 1096^a 17 *sqq.*, is an entire misunderstanding.

§ 8.—On the expression εἰδητικός ἀριθμός.

It seems always assumed, perhaps without a sufficient examination of the evidence, that εἰδητικός ἀριθμός was the special designation of the Idea-numbers as such. It may have been so, but it is not clear that this was the original meaning of the phrase or even that Aristotle uses it in this special reference.

The conception of 'mathematical number,' as distinguished from 'Ideal number,' arises, as we have seen, out of the earlier Platonic theory of Ideas and not out of the later Idea-number theory. That number from which 'mathematical number' was distinguished was then simply the 'Ideas,' *idéai* or *εἶδη*, of number, in true reference to number. If a collective expression for the *εἶδη* τῶν ἀριθμῶν, parallel to the collective μαθηματικός ἀριθμός was wanted, εἰδητικός ἀριθμός might very naturally suggest itself. It would be neater and more convenient than οἱ ἐν τοῖς εἶδεσι ἀριθμοί which occurs in *Metaph.* N. vi. 1093^b 21, and that such a terminology should spring up in this way is no more than might be expected. If this were so, when later the Ideas of number became the Idea-numbers, they would in opposition to μαθηματικός ἀριθμός still be called εἰδητικός ἀριθμός in the same sense as before.

The three Aristotelian passages in which εἰδητικός ἀριθμός is named, *Metaph.* M. ix. 1086^a 2 *sqq.* and N. iii. 1090^b 32 *sqq.*, are compatible with this view: they favour it but hardly prove it.

In 1086^a 2 there comes first a statement about certain Platonists who acknowledged only one kind of number, the μαθηματικός ἀριθμός:—οἱ μὲν γὰρ τὰ μαθηματικά μόνον ποιοῦντες παρὰ τὰ αἰσθητά, ὁρῶντες τὴν περὶ τὰ εἶδη δυσχέρειαν καὶ πλάσιν, ἀπέστησαν ἀπὸ τοῦ εἰδητικοῦ ἀριθμοῦ καὶ τὸν μαθηματικὸν ἐποίησαν. The difficulty referred to is not one which arose out of the identification of all Ideas with the Ideas of numbers, but solely out of the distinction of an Ideal number from a mathematical, and there is no reason whatever why εἰδητικός ἀριθμός should mean anything but Ideal number, in the sense of Ideas of number proper. εἰδητικός just corresponds to τὰ εἶδη in the preceding clause. Next we are told that certain Platonists who identified all the Ideas with Ideas of numbers acknowledged only the εἰδητικός ἀριθμός and rejected the μαθηματικός:—οἱ δὲ τὰ εἶδη βουλόμενοι ἅμα καὶ ἀριθμοὺς ποιεῖν, οὐχ ὁρῶντες δὲ εἰ τὰς ἀρχάς τις ταύτας¹ θήσεται, πῶς ἔσται ὁ μαθηματικός

παρὰ τὸν εἰδητικόν, τὸν αὐτὸν εἰδητικὸν καὶ μαθηματικὸν ἐποίησαν ἀριθμὸν τῷ λόγῳ, ἐπεὶ ἔργῳ γ' ἀνήρηται ὁ μαθηματικός. Here again εἰδητικός is defined by its opposition to μαθηματικός. With these Platonists, of course, εἰδητικός ἀριθμός, even if properly meaning Ideal number, would include all Ideas because they reduced all Ideas to Ideal numbers; yet it was not from this point of view that they rejected μαθηματικός ἀριθμός, but from considerations of number as number: and so in this passage too there is no reason why Aristotle should be using εἰδητικός ἀριθμός in any other sense than that of Ideal number.

A second passage, 1090^b 32, is as follows:—οἱ δὲ πρῶτοι δύο τοὺς ἀριθμοὺς ποιήσαντες, τὸν τε τῶν εἰδῶν καὶ τὸν μαθηματικὸν ἄλλον, οὐδαμῶς οὐτ' εἰρήκασιν οὐτ' ἔχουσιν ἂν εἰπεῖν πῶς καὶ ἐκ τίνος ἔσται ὁ μαθηματικός. ποιῶσι γὰρ αὐτὸν μεταξὺ τοῦ εἰδητικοῦ καὶ τοῦ αἰσθητοῦ. εἰ μὲν γὰρ ἐκ τοῦ μεγάλου καὶ μικροῦ, ὁ αὐτὸς ἐκείνῳ ἔσται τῷ τῶν ιδεῶν κ.τ.λ. Here εἰδητικός ἀριθμός is the equivalent of τὸν τῶν εἰδῶν in the previous sentence, and of τῷ τῶν ιδεῶν in the following one, and corresponds to τὰ εἶδη in the parallel place 987^b 15 (where, as we have seen, the origin of the conception of *μεταξύ* is not put in connexion with anything which concerns the Idea-number theory, but with a difficulty arising from the ideas of number as such), and the meaning 'Ideal number' is the only one required in the passage and the most suitable.

The remaining passage is 1088^b 29:—ἐάν τε τὸν εἰδητικὸν ἀριθμὸν ἐξ αὐτῶν (sc. τοῦ ἐνὸς καὶ τῆς ἀορίστου δυνάδος) ποιῶσιν, ἐάν τε τὸν μαθηματικόν. Here also the only meaning of εἰδητικός ἀριθμός which is relevant is that of Ideas of number and not that of Idea-numbers.

In this matter the usage of the Greek Commentators does not carry us farther than the Aristotelian text. εἰδητικοὶ ἀριθμοί would be a convenient formula for Idea-numbers as opposed to numbers in the ordinary sense, and may therefore have come to be the equivalent of Idea-numbers with them, and it should be noticed that the plural εἰδητικοὶ ἀριθμοί which they use never seems to occur in Aristotle but only the singular collective εἰδητικός ἀριθμός.

§ 9.—Is the doctrine of τὰ μεταξύ to be found in Plato's 'Republic'?

In the familiar passage at the end of the Sixth Book of Plato's *Republic*, διάνοια which is concerned with the objects of mathematics

¹ There is a curious slip here in the Teubner edition:—ταύτας codd. edd., ταῦτας emendavi.

(though as will be seen not with these alone) is said to be between νοῦς and δόξα, the latter being concerned with the αἰσθητά; and the objects of διάνοια are somehow between the νοητά and αἰσθητά. The field of διάνοια is treated by Ueberweg as if it corresponded to the τὰ μεταξύ of the *Metaphysics*, and accordingly he puts in it τὰ μαθηματικά which are not ἰδέαι, the ἰδέαι being confined to the upper division of νοητά corresponding to νοῦς. If this were right it would be extraordinary that Aristotle should in the *Metaphysics* make no kind of reference to the *Republic* or to its terminology. But there is really no such doctrine in the *Republic*. The objects of διάνοια are ἰδέαι, and διάνοια itself is a faculty of apprehending universals in clear distinction from any perception of particulars. Moreover—though this is of less importance—τὰ μαθηματικά do not constitute the whole of the objects of διάνοια. A careful examination of the text of the *Republic* can hardly leave any doubt on these points. Perhaps some misunderstanding might arise from the fact that one of the distinctions Plato makes between the objects of διάνοια and νοῦς is in the manner in which they are studied and not in the objects themselves. The objects of διάνοια are studied with the help of sensible objects, i.e. figures, etc. perceived by the senses. But the text clearly distinguishes between the object really studied and those sensible objects which are but aids to the study. The geometrical object studied is the original of which the sensible figure is but the copy, and this language in Plato shews that the geometrical object is the ἰδέα:—τοῖς ὁρωμένοις εἶδει προσχρῶνται... οὐ περὶ τούτων διανοοῦμενοι, ἀλλ' ἐκείνων περὶ οἷς τὰτα ἔοικε, and similar expressions. This is made unmistakable by the examples given—τετράγωνος αὐτός, διάμετρος αὐτή, which can only be the ἰδέαι of the τετράγωνος and of the διάμετρος respectively. By his reiteration Plato shews his concern lest we should be misled by the use made of the sensible figures into confounding them with the true objects of mathematics, which are these Universals and these alone—ἀλλ' οὐ ταύτης (τῆς διαμέτρου) ἦν γράφουσι. And it is to these objects and just in this sharp distinction from the objects of sense, i.e. from the particular figures which are drawn to illustrate them, that διάνοια is assigned, as a faculty which is not of the sensible objects: οὐ περὶ τούτων διανοοῦμενοι ἀλλ' ἐκείνων περὶ οἷς τὰτ' ἔοικε, τοῦ τετραγώνου αὐτοῦ, etc. So again ζητοῦντές τε αὐτὰ ἐκείνα ἀδύναντα οὐκ ἂν ἄλλως ἴδοι ἢ τῇ διανοίᾳ, where

αὐτὰ ἐκείνα refers to the ἰδέαι of the τετράγωνος of the διάμετρος and such like. Cp. 529 E ἀδύναντα δὲ λόγῳ μὲν καὶ διανοίᾳ ληπτὰ, ὅψει δ' οὐ; 526 A, ὧν διανοηθῆναι μόνον ἐγγωρεῖ: 511 C, καὶ διανοία μὲν ἀναγκάζονται ἀλλὰ μὴ αἰσθήσεσιν αὐτὰ θεᾶσθαι οἱ θεώμενοι.

Notwithstanding these express statements confusion may have been caused perhaps by the reflection that on the one hand Plato makes a use of perceived particulars characteristic of the mathematical procedure, and on the other hand seems to assign διάνοια as the special faculty of the mathematician: whence it might seem διάνοια must include the reference to particulars.

But there ought to be no difficulty. The faculty of mathematical study, as a process of investigation, is not διάνοια: if any name is to be given to it μαθηματική would be the most appropriate. This, as the mathematician's procedure, involves αἰσθησις so far as it employs the sensible figure and διάνοια so far as the Universal is apprehended. διαλεκτική as a process stands in a very similar relation to the two 'faculties' of διάνοια and νοῦς: it is neither, but involves both, as will be seen presently.

If Plato had intended that there should be anything of the nature of sensuous intuition in διάνοια, such, for instance, as some plurality distinguishing its object from the unity of the Universal, he could not have failed to say so, and, in such a context, say it with great clearness: certainly he would never have employed expressions the only natural interpretation of which is the very opposite.

If then the objects of διάνοια are ἰδέαι, how does it differ from νοῦς, and how do its objects differ from νοητά, in the narrower sense of that term? To these questions Plato himself, at the end of *Rep.* vi., supplies a clear answer, and one which once more shews that the objects of διάνοια are ἰδέαι.

He holds that all the Ideas are connected with one another in a system which depends on one Idea, the Idea of the Good. This is the absolute and self-sufficient principle or ἀρχή on which they all depend. The knowledge of this system in its completeness, the ideal at which διαλεκτική aims and which a completed διαλεκτική would realise, is called νοῦς; and thus an ἰδέα is not a νοητόν, in the higher sense as object of νοῦς, unless it is known as a member of this system and therefore in connexion with the ἀρχή. If not seen in this connexion (μετ' ἀρχῆς) its full nature is not apprehended, and, its validity is not really known, because

it is this connexion which alone guarantees validity.

Now in mathematics the Ideas concerned are not entirely without connexion: for by processes of consistent reasoning groups of Ideas are shewn to depend on a limited number of Ideas. But these latter are not connected with one another and the ultimate ἀρχή, the Idea of the Good. Thus though treated as absolute starting-points (ἀρχαί) in mathematics they are not self-sufficient, therefore not absolute but only ὑποθέσεις, and their validity can only be assured when derived from the Idea of the Good. The connexions then between the Ideas in mathematics are not complete, and thus an Idea in it, though seen in some of its connexions with others, is not seen in the completeness of such connexions in the system which depends on the ultimate ἀρχή, and therefore not in the completeness of its own nature.

On this account obviously an Idea as seen in mathematics is not νοητόν; it is not object of νοῦς: and the faculty which apprehends them in this incomplete way though a faculty of Universals, a faculty of thought, not of sense (διανοία καὶ μὴ αἰσθήσεω), and as thought falling with νοῦς under the general designation νόησις, is inferior to νοῦς. This is the distinction Plato makes between the two faculties when he sums up his result at the end of the sixth book of the *Republic*; and it is the only distinction.

It follows of course that an object of διάνοια when its full nature is apprehended, when, that is, its connexion with the true ἀρχή is seen, is νοητόν in the higher sense, i.e. object of νοῦς; and this is exactly what Plato says:—καίτοι νοητῶν ὄντων μετ' ἀρχῆς. This is a confirmation of the view that the objects of διάνοια are ἰδέαι, for nothing but an ἰδέα can be object of νοῦς.

There are certain characteristics of Plato's terminology which also confirm this result. The sphere of διάνοια is a part of the whole sphere of νοητόν, in opposition to the ὁρατόν (= αἰσθητόν), and διάνοια itself is a species of νόησις in opposition to αἰσθησις. Now such a passage as that in 507, where it is said τὰς ἰδέας νοεῖσθαι ὁρᾶσθαι δ' οὐ, taken in connexion with this indicates that the objects of νόησις as such are ἰδέαι. Again in the seventh book of the *Republic* the objects of perception which stimulate the mind to the exercise of thought and the discovery of the Universal are called variously παρακλητικὰ διανοίας and ἐγερτικὰ νοήσεως (p. 524).

διαλεκτική must not be confused with νοῦς. It is a process which begins with Ideas incompletely connected as seen in διάνοια. It therefore involves the use of this faculty. Its work is to effect more and more connexions between the Ideas and finally effect their full connexion with the ἰδέα τοῦ ἀγαθοῦ. Only when this result had been consummated and only when διαλεκτική had finished this work would νοῦς appear. διαλεκτική therefore begins from διάνοια, and its ideal ending is in νοῦς. As already indicated, it is important for the proper interpretation of διάνοια to recognise that διαλεκτική and μαθηματική are processes of investigation to be distinguished from the fourfold division of 'faculties of objects'—as they may be called for want of a better term—νοῦς, διάνοια, πίστις, εἰκασία.

From the general description Plato gives of διαλεκτική taken with the allegory of the Cave it would seem that διαλεκτική in its upward progress brings to light ἰδέαι not found in the field of the sciences. These too, properly speaking, would not be νοητά till the process of διαλεκτική was complete, but Plato has not considered this point. There is another point also omitted in the sixth book, but implied in the allegory of the Cave. From the distinction he makes between διάνοια and νοῦς, it should follow that any ἰδέα whether mathematical or not, before its connexion with the ἰδέα τοῦ ἀγαθοῦ was understood would be object of διάνοια. An important class of these would be the ethical Ideas. Important as these are to him Plato, carried away by his interest in defining the position of the sciences, seems at the end of the sixth book of the *Republic* to have forgotten all about them—great thinkers sometimes do these things. But there is a passage in the Allegory of the Cave which shews that ethical notions are conceived of us having the same kind of gradations in respect of reality and truth as scientific notions:—517 E, περὶ τῶν τοῦ δικαίου σκιῶν ἢ ἀγαλμάτων ὧν αἱ σκιαί, opposed to αὐτῇ δικαιοσύνῃ.

It may be permissible to add that in a paper read before the Oxford Philological Society in 1892 the writer maintained that the plural γεωμετρίαι in these passages in the *Republic* did not mean Plane and Solid Geometry, but acts or operations of Geometry, and related to γεωμετεῖν just as θεωρίαι to θεωρεῖν: γεωμετρία by itself, and without qualification indicated by context, standing

normally for plane geometry. The plural γεωμετρίας is thus exactly parallel to λογισμοί with which it is in these passages associated. The same result, if not quite in

the same way, has been independently given lately by Dr. Adam in his edition of the *Republic*.

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ON A PASSAGE IN XENOPHON'S *MEMORABILIA*.

XENOPHON *memorabilia* A vi. 13 Ὁ Ἀντιφῶν, παρ' ἡμῖν νομίζεται τὴν ὥραν καὶ τὴν σοφίαν ὁμοίως μὲν καλόν, ὁμοίως δὲ αἰσχρόν διατίθεσθαι εἶναι τὴν τε γὰρ ὥραν ἂν μὲν τις ἀργυρίου πωλῇ τῷ βουλομένῳ, πόρον αὐτὸν ἀποκαλοῦσιν, ἂν δὲ τις, ὃν ἂν γνῶ καλόν τε ἀγαθὸν ἐραστὴν ὄντα, τοῦτον φίλον ἑαυτῷ ποιῇται, σῶφρονά νομίζομεν καὶ τὴν σοφίαν ὡσαύτως τοὺς μὲν ἀργυρίου τῷ βουλομένῳ πωλοῦντας σοφιστὰς ὥσπερ πόρους ἀποκαλοῦσιν, οὗτοι δέ, ὃν ἂν γνῶ εὐφρῶντα, διδάσκων ὃ τι ἂν ἔχη ἀγαθόν, φίλον ποιῇται, τοῦτον νομίζομεν, ὃ τῷ καλῷ ἀγαθῷ πολίτῃ προσήκει, ταῦτα ποιεῖν.

On this passage Mr. H. Richards (*Classical Review*, xvi 270) remarks: 'The point is not that it is equally creditable and discreditable to dispose of the two things, but that in both cases there is a creditable as well as a discreditable way of doing it, two possibilities with regard to each of them, set forth with correspondence of μὲν and δέ. It follows that in the early part of the sentence we want adverbs, not adjectives: ὁμοίως μὲν καλῶς ὁμοίως δὲ αἰσχρῶς διατίθεσθαι εἶναι, εἶναι meaning *it is possible*. For the double adverb cf. Thuc. 2. 60. 6 οὐκ ἂν ὁμοίως τι οἰκείως φράζοι.' It is not quite clear to me how Mr. Richards understands ὁμοίως: but I think that he would probably agree with me when I say that Socrates

distinguished between honourable teaching and dishonourable teaching on the same principle on which he distinguished between honourable and dishonourable love, and that ὁμοίως μὲν, ὁμοίως δέ, mark the identity of the principle. If so, both he and I dissent from Kühner, who translates 'apud nos existimatur et pulchrum et turpe esse pulchritudinem et sapientiam aliis venundare,' and makes ὁμοίως μὲν, ὁμοίως δέ mean no more than 'pariter ac.' I am however of opinion that the meaning which I desire, and which I suppose Mr. Richards to desire, can be got from the ordinary text, and indeed that the ordinary text expresses that meaning better than Mr. Richards' emended text would do. I would translate: 'Our notion is that the communication of beauty and the communication of wisdom are creditable under the like circumstances, and that under the like circumstances they are discreditable.' This seems to me exactly the right sense. My objections to Mr. Richards' reading are, that, however he understands ὁμοίως, first, it is not necessary to desert the tradition, and, secondly, 'εἶναι, meaning *it is possible*,' is, not only superfluous, but even, by the substitution of 'may be' for 'are,' injurious to the sense.

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15 April, 1904.

NOTES ON THE TEXT OF DIONYSIUS HALICARNASSENSIS: *THE THREE LITERARY LETTERS*.

p. 751 (=p. 88 Roberts' text, line 15). Read ἀπαντας, ὅσοι τὰς αὐτῶν ἐπινοίας εἰς τὴν κοινὴν φέρουσιν ὠφέλειαν ἐπανορθοῦντες ἡμῶν <τοὺς> βίους τε καὶ λόγους. Usener inserted τοὺς before ἡμῶν: but the rule that pronouns of this class come outside the article is observed even in p. 756 (=p. 94 Roberts line 8) ἐγὼ γὰρ τὰς ἄλλας αὐτοῦ γραφὰς παραφέρειν, and unmistakably in p. 757 (=p. 96 Roberts line 2) καὶ γὰρ τὰ δόγματα

διέβαλον αὐτοῦ τινας, and p. 783 (=p. 122 Roberts line 15) γνώη δ' ἂν τις αὐτοῦ τὸν πόρον.

p. 780 (=p. 120 Roberts line 1). Read τῆς δὲ λέξεως ἣ Οὐκυνδίδης κέχρηται τὸ μὲν σημειῶδες καὶ περιέργον πέφηνεν (sc. Philistus), τὸ δὲ στρογγύλον καὶ πυκνὸν καὶ ἐνθυμηματικὸν μεμίμηται. Krüger conjectured ἐκμέμακται: the MSS have μεμίκαται. For the turn of expression then cf. Marcellin. 51