On the Flux of Light from the Electric Arc with Varying Power-Supply

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XLIX. *On the Flux of Light from the Electric Arc with Varying Power-Supply.* By G. B. DYKE, B.Sc.*

[Plate XX.]

IN 1896 a paper was communicated to the Physical Society by Dr. J. A. Fleming, F.R.S., and Mr. J. E. Petavel †, recording the results of numerous observations on the Electric Arc, dealing, amongst other matters, with the question of the relation between the flux of light, or mean spherical candle-power, and the watts expended in the arc.

The time and labour necessary to obtain the mean spherical candle-power of any source of light from observations made on the point-by-point method then employed, prevented any very extended study of this relation.

The construction of the integrating photometer, described by the author in his paper "On the Practical Determination of the Mean Spherical Candle Power of Incandescent and Arc Lamps" ‡, having, to a very large extent, removed these difficulties, it was thought that a more extended series of observations in this direction might be undertaken, and facilities for this purpose were kindly granted in the Pender Electrical Laboratory of University College, London, by Prof. J. A. Fleming.

The objects in view may be thus stated :—

- (i.) To obtain a series of curves for both continuous and alternating current arcs, showing the relation between the mean spherical candle-power and the watts taken up in the arc.
- (ii.) To compare the "efficiencies" of the continuous and alternating arcs as illuminants when supplied at different powers, the "efficiency" being expressed in mean spherical candles per watt.

As the field covered by these statements was too broad to

* Read March 24, 1905.

† "An Analytical Study of the Alternating-Current Arc," by J. A. Fleming, M.A., D.Sc., F.R.S., and G. E. Petavel, Phil. Mag. April 1896, p. 315.

‡ "On the Practical Determination of the Mean Spherical Candle-Power of Incandescent and Arc Lamps," by G. B. Dyke, B.Sc., Phil. Mag. Jan. 1905, p. 136, and Proc. Phys. Soc. vol. xix.

be covered in its entirety in the time at the author's disposal, it was decided to impose the following restrictions:—

The make and diameters of the carbons used to remain unaltered throughout the experiments.

Alternating current of one frequency only to be employed, this frequency to be 80~

In continuous-current experiments the top carbon to be invariably positive.

Arc-lengths not to exceed 0·5 inch.

Power to be supplied at rates not greater than 1500 watts.

The carbons used may be specified thus:—

Upper Carbon—

Maker	C. Conradty, Nuernberg.
Description	Cored. Marke W.C.
Diameter	12 mm.
Diameter of core	2 „
Weight per cub. centim.	1·41 gramme,
Weight per c.c. of outer carbon	1·43 „
Weight per c.c. of core	0·93 „

Lower Carbon—

Maker	C. Conradty, Nueruberg.
Description	Solid. Marke C.
Diameter	10 mm.
Weight per cub. centim.	1·46 gramme.

The mean spherical candle-power was measured directly by means of an integrating photometer.

This instrument was described in detail in the paper referred to above, and it will be sufficient to say that it consists of a system of pairs of mirrors arranged in a semi-circle, about 6 feet in diameter, in such a manner that light emitted from a source placed at the centre of the system shall be incident upon a photometer-screen, also situated at the centre, at the same angle with the vertical as that which it made on emission from the source.

The intensity of illumination of the screen is then proportional to the mean spherical candle-power. As previously arranged for testing incandescent lamps, the photometer measured directly the ratio of the mean spherical candle-power to the mean horizontal candle-power. When dealing

with arc-lamps, however, this ratio is not of much practical value, so a small alteration was made to adapt the instrument to read the absolute value of the mean spherical candle-power. This was effected by replacing the horizontal sliding pair of mirrors by a standard incandescent lamp on a sliding support.

This alteration will of course alter the constant of the instrument.

The formula given before was

$$\frac{\text{M.S.C.P.}}{\text{M.H.C.P.}} = \frac{5.09}{d^2},$$

where d is the distance in feet from source to screen by way of the sliding pair of mirrors.

Now the sliding pair of mirrors reflected 70.5 per cent. of the light incident on them; hence it is obvious that if these mirrors are replaced by a lamp of known candle-power C.P. we have

$$\text{M.S.C.P.} = \frac{5.09}{.705} \times \frac{\text{C.P.}}{d^2}.$$

Hence the formula becomes

$$\text{M.S.C.P.} = \frac{7.22}{d^2} \times \text{C.P. of balancing-lamp},$$

where d is the distance of balancing-lamp from photometer-screen in feet.

As this result is obtained somewhat indirectly the arc was replaced by an incandescent lamp, and the polar diagram constructed by opening the slides in front of the mirrors one at a time and making a photometric measurement for each.

The Rousseau diagram was then drawn, and the mean spherical candle-power deduced and found to be 22.49. All the mirror-slides were then opened and a balance obtained at 15.95 inches, the candle-power of the balancing-lamp being 5.51.

Hence, applying the formula given above,

$$\begin{aligned} \text{constant of instrument} &= \frac{22.49}{5.51} \times \left(\frac{15.95}{12} \right)^2 \\ &= 7.21. \end{aligned}$$

This check-experiment being considered satisfactory, the value 7.22 was taken as the constant of the instrument.

When dealing with arcs of some 2000 mean spherical

candle-power, it was necessary to bring the balancing-lamp very close to the photometer.

In this case it was necessary to apply a correction to the

Fig. 1.

scale-reading of the distance d , due to the angle subtended by the filament of the balancing-lamp becoming appreciable.

If l is the length of the filament, and i the brightness per unit-length.

Then, considering a flat-loop lamp at a distance d from the photometer (see fig. 1), we have :—

Intensity of illumination due to 2 legs of filament

$$\begin{aligned}
 &= 2 \left\{ 2i \int_0^{\frac{l}{2}} (\delta s \cos \theta) \cos \theta / \left(\frac{d}{\cos \theta} \right)^2 \right\} \\
 &= 4i \int_0^{\tan^{-1} \frac{l}{2d}} \left(\frac{d}{\cos \theta} \times \delta \theta \right) \cos \theta \times \left(\frac{\cos \theta}{d} \right)^2 \\
 &= \frac{4i}{d} \int_0^{\tan^{-1} \frac{l}{2d}} \cos^2 \theta \delta \theta \\
 &= \frac{2i}{d} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\tan^{-1} \frac{l}{2d}} \\
 &= \frac{2i}{d} \left[\tan^{-1} \frac{l}{2d} + \frac{\frac{1}{2} l d}{d^2 + \left(\frac{1}{2} l \right)^2} \right]
 \end{aligned}$$

or approximately

$$\begin{aligned}
 &= \frac{2i}{d} \left[\frac{\frac{1}{2}d}{\sqrt{d^2 + (\frac{1}{2}l)^2}} + \frac{\frac{1}{2}ld}{d^2 + (\frac{1}{2}l^2)^2} \right] \\
 &= \frac{il}{d} \left[\frac{d + \sqrt{d^2 + (\frac{1}{2}l)^2}}{d^2 + (\frac{1}{2}l)^2} \right] \\
 &= \frac{2il}{d^2} \left(\frac{d^2 + d \sqrt{d^2 + (\frac{1}{2}l)^2}}{2(d^2 + (\frac{1}{2}l)^2)} \right)
 \end{aligned}$$

Hence

d^2 must be multiplied by the factor

$$\frac{2\{d^2 + (\frac{1}{2}l)^2\}}{d^2 + d \sqrt{d^2 + (\frac{1}{2}l)^2}}$$

The author is indebted to Mr. W. C. Clinton, B.Sc., for bringing this correction to his notice.

A hand-regulated arc was used in the experiments, the length of arc being kept constant by observing the image of the arc thrown on to a screen by means of a lens.

During each series of experiments the arc was maintained at a constant length, whilst the power supplied was varied by means of an adjustable resistance.

In order to obtain a mean result ten photometric readings were taken at each setting.

Altogether eight lengths of arc were employed, viz.

$\frac{1}{16}$ in., $\frac{3}{32}$ in., $\frac{1}{8}$ in., and hence by 16ths to $\frac{7}{16}$ in.

The table which follows gives the mean results of these experiments, involving nearly four thousand observations.

If now these results are plotted, taking watts as abscissæ and mean spherical candle-power as ordinates, we obtain the series of curves shown in Pl. XX., the full-line curves referring to continuous-current arcs and the dotted lines to alternating-current arcs.

It will be seen that, within the limits of experimental error, the relation between mean spherical candle-power and watts follows a straight-line law. This straight line, however, does not pass through the origin, but for zero candle-power there is still an outstanding amount of power amounting to some 200 to 400 watts.

The amount of this outstanding effect increases, in general,

with the arc-length, and is probably due to the energy dissipated as heat by radiation, and as chemical energy in evaporation of the carbon at the crater.

The author hopes, at some future date, to investigate this matter experimentally.

It is found from these curves that for each arc-length the point in which the efficiency curve cuts the axis of watts is

Arc length. Inches.	Continuous Current Arc.				Alternating Current Arc.			
	Volts.	Amperes.	Watts.	M.S.C.P.	Volts.	Amperes.	Watts.	M.S.C.P.
$\frac{1}{16}$	50.0	6.4	318	207	39.8	5.6	221	155
	46.3	7.7	355	267	37.8	7.1	268	236
	44.7	9.3	417	370	37.4	9.2	344	359
	45.1	12.1	544	542	(violent hissing if current increased)			
	45.4	16.3	741	777				
$\frac{3}{32}$	44.2	20.1	890	1078				
	50.5	6.7	336	257	47.5	6.7	317	156
	48.9	7.9	384	354	40.5	9.3	378	293
	48.0	9.6	459	490	39.7	11.2	442	445
	47.0	12.4	583	738	39.8	13.4	533	624
$\frac{1}{8}$	46.4	16.9	786	1119	40.1	16.0	643	855
	45.3	20.9	947	1476	40.7	19.8	807	1104
	53.5	6.0	323	290	43.8	6.8	299	124
	51.0	7.5	382	390	42.1	8.6	362	241
	52.2	9.2	481	609	41.6	10.3	429	297
$\frac{3}{16}$	49.5	12.3	609	901	41.7	12.5	520	477
	47.5	16.8	796	1295	42.1	14.6	616	579
	47.0	26.4	961	1761	42.2	18.8	793	846
					40.3	20.1	811	977
	58.0	5.7	333	237	51.0	4.9	250	80
$\frac{1}{4}$	54.6	7.1	386	353	48.2	6.3	304	132
	52.1	8.9	462	524	45.4	8.5	384	244
	51.2	11.7	599	767	43.5	12.6	547	543
	48.8	16.3	796	1219	42.1	15.8	655	712
	48.8	20.2	985	1669	42.2	19.9	837	990
$\frac{5}{16}$	64.6	6.8	441	410	56.1	4.6	259	87
	60.9	8.7	527	683	53.9	5.4	291	154
	59.4	12.2	727	1161	50.6	7.1	360	196
	55.9	15.4	863	1532	46.0	10.7	490	432
	57.0	19.4	1104	2210	45.6	15.9	727	863
$\frac{3}{8}$	63.0	8.7	548	549	57.9	8.3	478	270
	60.8	10.4	632	727	56.3	10.5	617	382
	59.0	12.5	728	921	53.6	13.7	735	596
	58.5	14.9	871	1284	49.2	17.5	861	797
	61.1	18.8	1150	1900	48.9	21.7	1062	1024
$\frac{7}{16}$	67.2	6.8	459	350	57.7	8.1	470	238
	62.5	9.0	565	557	53.2	11.3	602	446
	62.1	12.2	760	1130	54.1	14.1	763	720
	65.8	10.0	655	621	56.8	10.5	593	407
	65.9	11.6	762	810	56.6	13.8	783	660
$\frac{1}{2}$	64.9	14.3	927	1202	55.6	18.1	1003	1070
	62.4	18.9	1182	1703				

the same whether the arc is supplied with continuous or alternating currents, pointing to the fact that the power expended in forms of energy other than light, *e. g.* volatilization of carbon, radiation of heat, sound, &c., is exactly the same for similar alternating and continuous arcs.

For lengths of arc greater than $\frac{1}{8}$ in. the ratio of the efficiencies of the continuous and alternating arcs is about 3 to 2, agreeing very well with that obtained by Dr. Fleming and Mr. Petavel.

If, however, the arc be made shorter than $\frac{1}{8}$ in. the efficiency line for the alternating-current arc will be found to gradually approach the line for the continuous-current arc until for an arc-length of $\frac{3}{32}$ in. they are practically coincident (Pl. XX.).

The arc is quite stable at this point, and even in the alternating-current case may be supplied at more than 600 watts without causing hissing.

Any further decrease in the arc-length will result in the alternating-current efficiency line rising above the continuous-current line until at an arc-length of $\frac{1}{16}$ in. they have the position shown in the first curve in Pl. XX.

With so small an arc-length, however, the alternating-current arc begins to hiss badly when supplied at a rate greater than 350 watts, although the continuous-current arc is still silent up to 900 watts.

As far as the author is aware, this is the first case in which an alternating current arc of efficiency greater than the corresponding continuous-current arc has been realized in practice, although Dr. Fleming and Mr. Petavel recognized the possibility of such a phenomenon in their paper referred to above.

Why, then, does the alternating-current arc give the same efficiency as the continuous-current arc at this arc-length?

This can perhaps be most clearly seen by making some simple approximations with regard to the shape of the carbons and calculating the length of arc necessary to give the same candle-power with the same power supply.

Let the carbons be supposed to assume the shape indicated in fig. 2, truncated cones whose heights are equal to their bases; this being approximately the shape found by experiment.

Also let all the light in the case of the continuous-current arc be supposed to come from the positive crater.

Fig. 2.

Now, the expression for the mean spherical candle-power of a source symmetrical about an axis is

$$\text{M.S.C.P.} = \frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} I_{\theta} \cos \theta \, d\theta,$$

where I_{θ} is the luminous intensity in a direction making an angle θ with the horizontal.

Now let I be the luminous intensity of the crater when viewed normally.

Then, as a rough approximation, assuming the crater to be a flat circular disk, small compared with the arc length,

$$I_{\theta} = I \sin \theta.$$

Hence for the arc

$$\begin{aligned} \text{M.S.C.P.} &= \frac{1}{2} I \int_{\frac{\pi}{2}}^{\pi} \sin \theta \cos \theta \, d\theta \\ &= \frac{1}{4} I \int_{\frac{\pi}{2}}^{\pi} \sin 2\theta \, d\theta \\ &= \frac{1}{8} I \left[-\cos 2\theta \right]_{\frac{\pi}{2}}^{\pi}. \end{aligned}$$

Let a be the diameter of the lower carbon $(-)$, and b the diameter of the upper carbon $(+)$.

Then, making the above assumptions, all the light, in the continuous-current case, will be concentrated into the zone between the horizontal and an angle α below the horizontal: where

$$\alpha = \tan^{-1} \frac{a+x}{\frac{1}{2}a}.$$

Hence we get

$$\begin{aligned} \text{M.S.C.P.}_{\text{C.C.}} &= \frac{1}{8} I_{\text{C.C.}} [-\cos 2\theta]_0^\alpha \\ &= \frac{1}{4} I_{\text{C.C.}} \sin^2 \alpha. \end{aligned}$$

Similarly for the alternating-current arc, considering each crater separately, we obtain

$$\text{M.S.C.P.}_{\text{A.C.}} = \frac{1}{4} I_{\text{A.C.}} (\sin^2 \alpha + \sin^2 \beta),$$

where

$$\beta = \tan^{-1} \frac{b+x}{\frac{1}{2}b}.$$

Hence

$$\frac{\text{M.S.C.P.}_{\text{C.C.}}}{\text{M.S.C.P.}_{\text{A.C.}}} = \frac{I_{\text{C.C.}}}{I_{\text{A.C.}}} \frac{\sin^2 \alpha}{\sin^2 \alpha + \sin^2 \beta}.$$

Some experiments mentioned by Dr. Fleming and Mr. Petavel give:—

for a continuous-current arc

$$I_\theta = 910 \quad \text{when} \quad \theta = 40.5^\circ.$$

Hence

$$I_{\text{C.C.}} = \frac{910}{\sin 40.5^\circ} = 1400;$$

and for a similar alternating-current arc

$$I_\theta = 300 \quad \text{when} \quad \theta = 60^\circ \quad \text{for top crater,}$$

and

$$I_\theta = 370 \quad \text{when} \quad \theta = 63^\circ \quad \text{for bottom crater.}$$

Hence as a mean

$$I_{\text{A.C.}} = 707.$$

So we get

$$\frac{I_{\text{C.C.}}}{I_{\text{A.C.}}} = \frac{1400}{707} = 1.99.$$

Therefore

$$\frac{\text{M.S.C.P.c.c.}}{\text{M.S.C.P.a.c.}} = 1.99 \frac{\sin^2 \alpha}{\sin^2 \alpha + \sin^2 \beta}.$$

Hence for equal efficiency we have

$$1.99 \sin^2 \alpha = \sin^2 \alpha + \sin^2 \beta.$$

$$\therefore .99 \sin^2 \alpha = \sin^2 \beta;$$

or putting in the values of the angles

$$\frac{.99(a+x)^2}{(a+x)^2 + .25a^2} = \frac{(b+x)^2}{(b+x)^2 + .25b^2}.$$

Now

$$a = 10 \text{ mm.}$$

$$b = 12 \text{ mm.}$$

Hence

$$\frac{.99(10+x)^2}{(10+x)^2 + 25} = \frac{(12+x)^2}{(12+x)^2 + 36}.$$

Writing out and arranging

$$x^4 + 44x^3 - 340x^2 - 6000x + 18000 = 0.$$

Solving this equation graphically we obtain as the positive finite value of x

$$x = 2.7 \text{ mm.,}$$

which agrees very closely with that found by experiment.

This consideration, however, by no means gives a complete solution of the problem of the relative efficiencies of the continuous and alternating current arc, but is only intended to account to some extent for the phenomenon of continuous and alternating current arcs of equal efficiency.

A complete analysis of the subject could only be undertaken on the basis of the evidence of a much more extended series of experiments, of which the time at the author's disposal would not allow.

In conclusion the author wishes to thank Dr. J. A. Fleming, F.R.S., for his many suggestions and constant advice, and also for his kindness in placing the resources of the Pender Laboratory at his disposal, and to express his indebtedness to Messrs. J. S. Westerdale and H. C. Bullman for the very

efficient assistance they have rendered him, at great personal inconvenience, in making some thousands of electrical measurements, without which the investigation would have been impossible.

DISCUSSION.

Dr. J. A. FLEMING congratulated the Author, and referred to the practical value of the experiments. The ordinary method of comparing lamps by their M.H.C.P. was unscientific. What was wanted was the total flux of light per watt, and this important figure was obtained at once by the integrating photometer used by the Author. Prof. Fleming instanced the case of a Nernst lamp and an ordinary carbon-filament lamp, pointing out that the ordinary tests indicated that the Nernst lamp was twice as efficient as the ordinary lamp, whereas a comparison of the total flux per watt showed an increased efficiency in the Nernst lamp of only 25 per cent.

Mr. PATERSON expressed his interest in the paper and pointed out that the variation of the light from an electric arc necessitated a great number of readings in order to get a satisfactory mean value: the photometer used by the Author integrated the light in one plane, but it would be still more satisfactory to integrate in two planes at right angles to one another when photometering lights which changed their candle-powers so rapidly. The correcting formula for the length of the filament in a standard lamp, as given by the Author, was unnecessarily long. By a more complete expansion of the first expression given, the formula obtained for the correcting factor for the candle-power of the standard was $K = 1 - \frac{1}{6} \frac{l^2}{d^2}$, which has very nearly the same numerical value but is simpler than the expression given in the paper.