

186. Indeterminate Forms

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$$\begin{aligned}\therefore f(m) &= f(-n) \\ &= \frac{1}{f(n)} = \frac{1}{(1+x)^n}, \text{ by Case I.,} \\ &= (1+x)^{-n}, \text{ by the theory of Indices,} \\ &= (1+x)^m.\end{aligned}$$

CASE III. Let m be a negative quantity numerically less than unity, $= -m'$ say.

$$\begin{aligned}\text{Then } f(m) &= 1 + mx + \frac{m(m-1)}{1 \cdot 2} x^2 + \frac{m(m-1)(m-2)}{1 \cdot 2 \cdot 3} x^3 + \dots \\ &= 1 - m'x + \frac{m'(m'+1)}{1 \cdot 2} x^2 - \frac{m'(m'+1)(m'+2)}{1 \cdot 2 \cdot 3} x^3 + \dots\end{aligned}$$

If x be negative, each term in the series is positive, and therefore $f(m)$ is positive. If x be positive, the terms are alternately positive and negative, and each term is numerically greater than the preceding term. Hence in this case also the value of $f(m)$ is positive. Thus, whether x be positive or negative, $f(m)$ is, *a priori*, seen to be a positive quantity.

Let now $m = -p/q$ where p and q are positive integers, $p < q$.

$$\begin{aligned}\text{Then } \{f(m)\}^q &= f(mq) = f(-p) \\ &= (1+x)^{-p}, \text{ by Case II. ;}\end{aligned}$$

$$\therefore f(m) = (1+x)^{-\frac{p}{q}}, \text{ that is, the positive value denoted by this expression,} \\ = (1+x)^m.$$

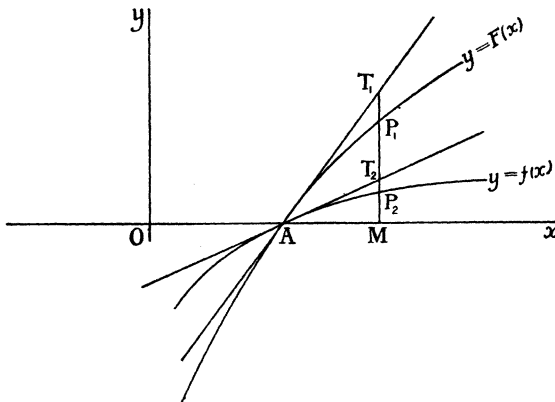
CASE IV. Lastly, let m be a quantity not falling under any of the above cases. Then we can put $m = n - m'$ where n is a positive or negative integer, and m' is positive and less than 1. Accordingly,

$$\begin{aligned}f(m) &= f(n - m') = f(n) \cdot f(-m') \\ &= (1+x)^n \cdot (1+x)^{-m'}, \text{ by Cases I. or II., and III.,} \\ &= (1+x)^{n-m'}, \text{ by the theory of Indices,} \\ &= (1+x)^m.\end{aligned}$$

V. RAMASWAMI AYYAR.

186. [C. 1. e. a.] Indeterminate Forms.

A geometrical illustration.



Let $y = F(x)$ and $y = f(x)$ be the equations of two curves, each intersecting the axis of x at the point $x = a$.

The ratio of corresponding ordinates is indeterminate when $x=a$.

But, drawing the tangents AT_1AT_2 at A , we have

$$\begin{aligned}\frac{F(x)}{f(x)} &= \frac{P_1M}{P_2M} = \frac{P_1M}{T_1M} \cdot \frac{T_1M}{T_2M} \cdot \frac{T_2M}{P_2M} \\ &= \frac{P_1M}{T_1M} \cdot \frac{AM \tan T_1AM}{AM \tan T_2AM} \cdot \frac{T_2M}{P_2M}\end{aligned}$$

and therefore when AM diminishes without limit

$$\begin{aligned}\lim_{x=a} \frac{F(x)}{f(x)} &= 1 \times \frac{\tan T_1AM}{\tan T_2AM} \times 1 \\ &= \frac{F'(a)}{f'(a)}.\end{aligned}$$

The extension to the case where $F'(a)$ and $f'(a)$ both vanish is obvious.

C. S. JACKSON.

187. [J. 5.] *The Continuum.*

Readers of the *Gazette* may be interested to know that Professor E. V. Huntington has concluded his series of articles in the *Annals*, "Mathematics on the Continuum" and the "Transfinite Numbers." They form an elementary introduction to some of the problems so actively debated at the present time in the field of Cantor's *Mengenlehre*.

W. J. G.

188. [C. 1. e.] *Proof of Taylor's Theorem.*

$$\text{Let } R = f(z) - f(z-h) - hf'(z-h) - \frac{h^2}{2}f''(z-h) - \dots - \frac{h^n}{n!}f^{(n)}(z-h),$$

$$\text{then } \frac{dR}{dh} = \frac{h^n}{n!}f^{(n+1)}(z-h).$$

$$\text{Keeping } z \text{ constant, } R = \int \frac{h^n}{n!}f^{(n+1)}(z-h)dh + \text{const.}$$

But $R=0$, if $h=0$;

$$\therefore R = \int_0^h \frac{h^n}{n!}f^{(n+1)}(z-h)dh,$$

which can be transformed or discussed in the usual way. The only thing the student has to remember is to put z for $x+h$, and therefore $z-h$ for x in the usual formula.

G. H. BRYAN.

189. [K. 13. a.] The *Remarque Minuscule* (Note 167, *Gazette*, May 1905) occurred to me also in 1887, and has been set in Aberystwyth and University of Wales' Examinations. It is, however, probably older, and contained in Bellavite's striking theorem, viz. if $ABCD\dots, A'B'C'D'\dots$ be similar polygons inversely situated, and if $AA'BB'CC'$, etc., be divided at P, Q, R , etc., each in the ratio of the linear dimensions of the polygons, then P, Q, R , etc., lie in a straight line.

The following extension of simple proof by Vector methods. If P_1, P_2 be points in AA' such that $AP_1 \cdot AP_2 : A'P_1 \cdot A'P_2 :: AB^2 : A'B'^2$, and if BB' be divided at Q_1Q_2 similarly to AP_1P_2A' , then P_1Q_1, P_2Q_2 are equally inclined to $AB, A'B'$.

R. W. GENESE.

190. [V. 1. a.] *An apparatus for teaching long multiplication.*

I devised this, in the first instance, to enable my pupils to change from the old method (left to right) to the new (right to left), without confusion.

It consists essentially of a blackboard composed of four (or more) rectangular slats ($1\frac{1}{2} \times 15$ m.s each) which slide in horizontal grooves. The local carpenter made mine for 12s. 6d.