DISCUSSION ON "THREE-PHASE POWER-FACTOR." ATLANTIC CITY, N. J., JUNE 30, 1908

Comfort A. Adams: I think that at least a part of Mr. Burt's demonstration may be rendered less mathematical, to the advantage of one's physical conception of the problem. I refer to that part leading up to equation (34) which is the algebraic statement of the validity of the ordinary method of measuring three-phase power by means of two wattmeters.

Since in any three-wire system the algebraic sum at any instant of the three currents counted positive in the same direction along the circuit is zero, one of these currents may be legitimately looked upon as the common return for the other two, since when counted positive backwards it is equal at each instant to the algebraic sum of the other two currents counted

FIG. 1

positive outwards. Any three-wire three-phase system may then be considered as a three-wire two-phase system.

In Fig. 1, *c* is taken as the common return and the total power at any instant is, $p = e_{ac}i_a + e_{bc}i_b = p_{ac} + p_{bc}$. The small letters indicate instantaneous values, the order of voltage subscripts indicates the direction in which the voltage is counted positive, and the arrows the directions in which the currents are counted positive.

What is true of the instantaneous values of the powers is true of their average values and of the readings of the wattmeters; or using Mr. Burt's notation, $P = P_{ac} + P_{bc}$. No assumption is here made as to the shape of the electromotive force or current waves. If the power flow is from left to right in both circuits, both wattmeters will give positive deflections when connected as shown. If one gives a negative deflection when so connected, it indicates a backward flow of power in that circuit and

the reading of this instrument, when its pressure-coil connections have been reversed, must be subtracted from that of the other wattmeter to give the total outward flow of power.

Thus the point of view involved in this proof (if it may be dignified by that name) not only emphasizes the direction of power flow and its relation to the relative direction of current and electromotive force, but it also points out a positive method of connecting the two wattmeters into the circuit.

The proof given above is only a special statement of the more general proposition* that if in any three-wire system, (Fig. 1), the currents i_a and i_b and the simultaneous electromotive forces e_{ac} and e_{bc} are given for any instant, the system is for that instant completely determined in all its vital aspects, and that it is entirely immaterial as to how the three wires may be interconnected in the receiving apparatus, which may be two-phase, single-phase, three-phase star, three-phase delta, or a complicated network of many branches.

There should therefore be a power-factor so definable that it is wholly determined by E_{ac} , E_{bc} , I_a , I_b and their phase relations, and thus independent of the number of branches of the receiving circuit. Of such a power-factor Mr. Burt has very opportunely supplied the definition.

The proposition given above, as well as Mr. Burt's definition of power-factor, apply equally well to an n wire system of which $n-1$ line currents and $n-1$ line voltages are given.†

Frederick Bedell: As I understand Professor Adams, the explanation he has given can be made to form a rigorous proof if it is expressed in mathematical language by the method given by Professor Blondel in his paper on "Measurements of the Energy of Polyphase Currents" in the *Proceedings of the International Electrical Congress of 1893*.

H. L. Wallau: The method of measuring the power-factor of an unbalanced delta system appeals to me because in our system we use exclusively a three-phase delta, 2300-volt distribution. The main power feeders are separate three-phase feeders and are, generally speaking, balanced. However, our lighting circuits also have small amounts of power connected to them, a great many having single-phase loads in the form of small motors, mercury rectifiers, and arc lamps, together with single-phase branches supplying straight incandescent loads. For that reason our lines are to a greater or less extent unbalanced, and it has been of a great deal of importance to

* At the time of the oral discussion of this subject at Atlantic City the writer was convinced and stated that the proposition given above covered also the proof leading up to Mr. Burt's equation (35), but while the conviction remains, he has been unable to see the connection clearly enough from the physical point of view to express it in simple non-mathematical form.

† "Polyphase Power Measurement" by C. A. Adams, *Electrical World Jan. 19, 1907*, p 143.

us to obtain an average power-factor which would truly represent the conditions on any particular feeder.

I think that in all cases where similar systems are being used, the power-factor on the system, as a whole, can be closely approximated by obtaining power-factor on various feeders. I have used various approximations to get at the average power-factor on the sub-station, for the purpose of determining the proper size of rotary condenser to be installed, to bring the power-factor up to such a value as would give an economical transmission from the generator to the sub-station, and also under certain cases the proper size of condenser to be installed on the customers' premises, in order to bring the power-factor up, sometimes, from a value as low as fifty per cent. to what might be called a livable value, one commercially practicable.

B. A. Behrend: I think that Messrs. Adams and Bedell are correct in regard to their simple way of proving the time-honored method of measuring the power-factor and the kilowatts of a three-phase alternating-current circuit. Their method is very interesting and instructive.