X. Inheritance of the Sex-Ratio in the Thoroughbred Racehorse.

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Mr Cobb in his note in the *Miscellanea* of this number has defined the sex-ratio of an individual as the value which the ratio of male to total births tends to assume when his fraternity becomes indefinitely large.

It is suggested that the variability, σ_0 , actually observed in the sex-ratio as calculated in the ordinary way, arises in part from the variability in what he calls the true sex-ratio, σ_s , and in part from the error due to considering small families, $\sqrt{\frac{pq}{n}}$, and further that the true correlation between the sex-ratios of parent and offspring can be found by multiplying that obtained from considering the actual families by a factor $(\sigma_s^2 + pq/n)/\sigma_s^2$.

It has already been shewn that σ_s is zero, within the limits of probable error, in the case of man. We can however test the reality of this factor directly by two methods. We can compare the correlation found by considering any group of families with that found after rejecting the smaller families, or we can compare the correlation found by considering large completed families with that found from sex-ratios defined by, say, the first four members of these families. In each case, the larger families ought to give higher correlation. The results of the first test have already been given in Prof. Pearson's note. To apply the second test, 1000 mares were taken at random from The General Stud Book, and the sex-ratio of their produce and that of their dam calculated. Only those cases were considered where each mare had at least eight foals, and as only second and subsequent editions of the Stud Book were used, the sex-ratio in each case was based on a completed family. From the same families sex-ratios were also calculated from the first four members. Tables I and II give the actual correlation tables

TABLE I.

Correlation of Sex-Ratios of Mother and Daughter Mares' Produce: Completed Families. Thoroughbred Horses from The General Stud Book.

	Sex-Ratios of Daughters' Produce.											Totals
	.00— .05	.05— .15	.15— .25	.25— .35	.35— .45	.45— .55	.55— .65	.65— .75	.75— .85	.85— .95	.95— 1.00	
.00— .05	—	—	—	—	—	—	—	—	—	—	—	—
.05— .15	—	—	—	—	3	4	2	1.5	.5	—	—	11
.15— .25	—	1	3	6	9	16.5	7.5	4	2	—	—	49
.25— .35	1	2	7.5	14.5	33	28.5	24.5	20	7	2	—	140
.35— .45	—	1	10.5	28.5	58	60	58	29.5	14.5	2	2	260
.45— .55	—	2	13	33	58	76.5	68.5	20.5	16.5	3	—	291
.55— .65	—	1	11	14	34	45	31	20	8	1	—	165
.65— .75	—	—	2.5	8	10	23	12.5	6.5	4.5	—	—	67
.75— .85	—	—	1	1.5	2	6	4.5	1	—	—	—	16
.85— .95	—	—	—	—	—	—	1	—	—	—	—	1
.95— 1.00	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	7	48.5	105.5	205	259.5	207.5	103	53	8	2	1000

TABLE II.

Correlation of Sex-Ratios of First Four Members of Mother and Daughter Mares' Produce. Thoroughbred Horses from The General Stud Book.

Sex-Ratio of First Four Members of Daughter's Produce.

Sex-Ratio of First Four Members of Mother's Produce.		$\frac{2}{1}$	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	Totals
	$\frac{2}{1}$	3	12	35	20	10	80
	$\frac{1}{1}$	20	65	119	72	15	291
	$\frac{1}{2}$	19	104	137	97	23	380
	$\frac{1}{3}$	8	45	84	63	13	213
	$\frac{1}{4}$	1	8	17	10	—	36
	Totals	51	234	392	262	61	1000

of sex-ratios derived from completed families of eight and upwards and from the first four members of these families respectively, while the means and standard deviations are given in Table III.

TABLE III.

Group		Mean	S. D.
Completed Families ...	{Mother's Produce	$\cdot 4620 \pm \cdot 0029$	$\cdot 1379 \pm \cdot 0021$
	{Daughter's Produce	$\cdot 5020 \pm \cdot 0033$	$\cdot 1546 \pm \cdot 0023$
First Four Members Only	{Mother's Produce	$\cdot 4585 \pm \cdot 0062$	$\cdot 2425 \pm \cdot 0037$
	{Daughter's Produce	$\cdot 5012 \pm \cdot 0052$	$\cdot 2426 \pm \cdot 0037$

In the completed family of the mother, at least one female must appear and this of course reduces the mean sex-ratio. When the offspring were not selected in this way, the mean sex-ratio was $\cdot 502$. If we now apply the condition that at least one member is to be a female, the sex-ratio ought to become $\frac{n-1}{n} (\cdot 502)$ where n is the number in the family. The average family was found to be $11\cdot 433$ and substituting this for n we get the sex-ratio to be $\cdot 458$. Actually it was found to be $\cdot 462$, an excellent agreement.

When the question of variability is considered we find as before that σ_s is zero. Taking the first four members of the family, the standard deviation is $\cdot 2426 \pm \cdot 0037$, while that which would arise from a chance distribution in families of four is $\sqrt{\frac{\cdot 502 \times \cdot 498}{4}} = \cdot 2499$. The difference is within the probable error and it is to be noted that the actual s.d. is less than that which would arise from a chance distribution. According to Mr Cobb's suggestion, it ought to be greater.

If we consider completed families (only the 1000 cases in the second generation were taken) the theoretical variability is given by

$$N\sigma^2 = S \left[n_s \left(\bar{p} - \frac{n_s}{n_s} \right)^2 + pq \frac{n_s}{S} \right]$$

in the notation used by Prof. Pearson above, and from this formula σ was found to be $\cdot1459$, where the term depending on the means contributes practically nothing. Actually the standard deviation calculated from the grouped sex-ratios was found to be $\cdot1546 \pm \cdot0085$, so that again the difference is within the limits of the probable error.

Turning now to the question of correlation, in the case of completed families of eight and upwards r was found to be $\cdot0101 \pm \cdot0213$, while in the case of the first four members only, r was found to be $-\cdot0064 \pm \cdot0213$. These coefficients are both zero within the limits of probable error so that no significant increase in correlation is obtained by raising the number of individuals on which the sex-ratio is based from 4 to an average of 11.4.

It should also be noted that Mr Cobb's corrective factor is *positive* and if the true correlation were at all comparable with its value found for other physical characters, which is of course always positive, the result of our calculation must give a positive correlation. The fact that it gives a small negative correlation is compatible with its being really zero but not with its being the reduced value of a true positive correlation found by using a positive reducing factor.