The Slide Rule and Its Use in Teaching Logarithms

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It is easy to show that x^ν as so defined, satisfies the laws of indices.

$$\begin{aligned}\text{Thus } x^\mu \cdot x^\nu &\equiv \left(1 + \frac{\mu X}{I}\right)^I \left(1 + \frac{\nu X}{I}\right)^I \equiv \left(1 + \frac{\mu + \nu \cdot X}{I}\right)^I \left[1 + \frac{\mu\nu X^2}{I^2 + \mu + \nu \cdot X}\right]^I \\ &= x^{\mu+\nu} \left[1 + \frac{a}{I^2 + bI}\right]^I \text{ say ;}\end{aligned}$$

$$\text{and, by § III., } \left(1 + \frac{a}{I^2 + bI}\right)^I > 1 + \frac{a}{I+b} \text{ and } < \frac{1}{1 - \frac{a}{I+b}};$$

and $\therefore (I \text{ being indefinitely great}) = 1$.

$$\text{Again, } \log x^\mu = \text{Lt } I \left[(x^\mu)^{\frac{1}{I}} - 1 \right] = \text{Lt } I \left[\left(1 + \frac{\mu X}{I}\right) - 1 \right] = \text{Lt } \mu X = \mu \log x;$$

$$\therefore (x^\mu)^\nu \equiv \left(1 + \frac{\nu \log x^\mu}{I}\right)^I \equiv \left(1 + \frac{\mu\nu \log x}{I}\right)^I \equiv x^{\mu\nu} \text{ by definition.}$$

Thus in the Exponential Expansion the number e can now be introduced : and we can prove $x = e^{\log x}$.

X.

On the Same Theorems for a Vector Variable.

The work in § VII. is applicable to vector values of x . The logarithm being defined exactly as before, takes the form $(\log a + \sqrt{-1} \cdot a)$, where a , a are the length and inclination of x . In the expression for the error, X and x are replaced by their moduli, and the error vanishes ultimately. The Binomial Theorem follows by § VIII. (provided that x is within two right angles of the line denoted by 1), the words 'lies between' at the end assuming a suitable meaning for vectors. The logarithmic expansion can be deduced from the exponential by expanding n^i in the finite algebraic form $\sum_{j=1}^{j=i} (n)_j \cdot i_j$, whence

$$(x-1)^j \equiv \sum_{i=j}^{i=\kappa} \left| \frac{X^i}{i!} \cdot i_j \right| \text{ with an error vanishing ultimately as } \kappa \text{ is increased.}$$

Certain simple properties of the coefficient i_j establish the logarithmic expansion and other less simple expansions for higher powers of $\log x$.

HARROW, *July*, 1903.

W. N. ROSEVEARE.

THE SLIDE RULE AND ITS USE IN TEACHING LOGARITHMS.

1. I should like at the outset to disclaim any intention of laying down the law dogmatically. The opinions expressed are stated definitely, but they are submitted with deference for your consideration.

My object is to argue two points :

- (1) That the construction of a simple form of slide rule furnishes beginners with a good mode of approaching the subject of logarithms.
- (2) That the use of the slide rule at any earlier stage than has been customary deserves every encouragement.

2. It is a commonplace remark that a principle may be presented to a pupil in such an abstract form that his mind fails to assimilate it. This

maxim is, however, not always respected. Galileo, after years of experimenting, reached clear notions about acceleration. There were great men before Galileo who never attained clearness on this point. Yet our textbooks polish off acceleration in a few paragraphs.

Even the Heuristic method, though it is in substance a protest against this form of error, is not, at any rate as presented by some of its advocates, entirely free from it. For example, a boy is invited, after contemplating a lump of coal for five minutes, to formulate a statement. The suggestion is that he will be moved to declare that "matter cannot set itself in motion." That is, the ordinary boy is to arrive in five minutes at a statement, the precise formulation of which is an item in the list of Newton's titles to fame.

These authorities—with sanguine ideas as to the ability of the average pupil—not unnaturally adopt as their cardinal rule "never teach anyone anything," an odd principle for a teacher. The better maxim is never teach a pupil that which he can find out for himself with a reasonable expenditure of time, patience and pains.

This is the true rule, because the object of education is to enable people to find out things for themselves. The application of this guiding rule is not easy; it is apt to be misunderstood and resented by pupils who have been brought up on the plan of being shewn everything; it does not pay in examinations, it results in less ground being covered; and it requires an amount of time, effort, and individual attention from the teacher which it is often, under existing conditions, simply impossible for him to give, but we all now see that it should be our ideal.

But if the plan of making the pupil cut some of the steps for himself is to be followed, the ascent, for the great majority of pupils, must be a very easy one and must consequently appear somewhat long. In the end, however, these slow experimental methods will prove to surpass purely didactic teaching in educating as distinguished from information-inserting.

3. I suggest that it is possible to approach the thorny subject of logarithms in an experimental fashion with advantage. I call the subject of logarithms a thorny one because it is within my personal knowledge that to many successful Woolwich candidates calculation by logarithms is an occult process, savouring of the black art.

How many times I have been asked how it is that

$$\log(a+b) \text{ is not equal to } \log a + \log b,$$

and wondered whether a zealous attempt to explain has had any effect when the many previous zealous and skilful explanations which candidates received at school have obviously had none. This fact suggests that the subject might perhaps be approached with advantage in a different manner.

For instance, let us begin by trying to construct a machine to do addition and subtraction. After a discussion, in which the teacher will have to be the predominant partner, we arrive at something like this: each boy using a pair of paper slips made by himself, in which a step of, say, an inch along the uniformly divided scale brings us to a number which is 2 more than the one we started with.

			0	1	2	3	4	5	6	7	8	9	10
0	1	2	3	4	5	6	7	8	9	10			
A			P					B					

If the strips * are shown overlapping as above we see that they mechanically

* A couple of wooden slips with figures stencilled on them were used for this and subsequent illustrations. Better than stencils are the indiarubber figures and letters sold for printing shop window tickets. I am indebted to Mr. J. H. Naylor, R.M. Academy, for this 'tip,' most useful when diagrams are often wanted.

add 3 to 5; for instance, for $AP + PB = AB$. The same setting mechanically subtracts 5 from 8.

Stated in a slightly different way. . . . For any 'setting' or relative position of the strips the DIFFERENCE between any corresponding pair of numbers on the scales is constant.

4. We next suggest the problem of making a machine for multiplication, and arrive at the idea that, as in the addition machine, a step of say an inch to the right *adds* 2 we will try whether we cannot make a machine in which a step of an inch to the right multiplies by 2.

This leads to a uniformly divided scale graduated as shewn.

We are now able to work out, using a pair of such scales, certain arithmetical problems—in fact we have a rudimentary slide rule.

Set the strips overlapping—thus, for instance: [I need hardly say that of course each pupil should make his own model with two strips of paper].

Notice that any number on the upper strip is 8 times that just below it, and that hence we can, from this particular 'setting' or relative position of the strips, read off the answers to certain sums in arithmetic thus:

MULTIPLICATION.—The fact that 8 times 4 is 32 is shown thus: $\begin{array}{r} 8 \ 32 \\ 1 \ 4 \end{array}$.

DIVISION.—The fact that 32 divided by 4 is 8 is shown in the same way.

PROPORTION.—The fact that 32 is to 4 as 128 is to 16 is shown thus:

$\begin{array}{r} 32 \ 128 \\ 4 \ 16 \end{array}$ The scales may, of course, be extended to the left if desired.

Other sums may now be tried in a similar way, but with the strips of paper set to a different overlap. We observe that in this case—For any 'setting' or relative position of the strips the RATIO of any corresponding numbers* on the scales is constant.

It is evident that in its present form the instrument is not a practical one, for only a few numbers can be dealt with. We could clearly construct another instrument to deal with the numbers 3, 9, 27, 81, etc., but this does not overcome the difficulty. We made our scales on the supposition that a step of an inch to the right effected multiplication by 2. It is clear that a step of 2 inches to the right multiplies by 4, of 3 inches to the right multiplies by 8, and so on, and we infer that any step of definite amount corresponds to multiplication by some definite number. The problem then is reduced to finding the proper position for any required number on the scales.

We have now got to the pinch of the case. A little time given to the class is certain to produce some good and some poor suggestions. For example, you are certain to be told that 3 should come half way between 2 and 4. The line to follow will depend on the suggestions received—so the following is only one of many possible modes of advance. Let us consider diagram (i) supposed, to begin with, to contain the marks for 10, 100, 1000, 10000 and so on. These marks will, as we have seen, be all equidistant, say a decimetre apart. Now, looking at the diagram which is at present a

* Note that if we invert one of the adding scales we get the SUM of corresponding numbers constant, and if we invert one of the multiplying scales we get the PRODUCT of corresponding numbers constant.

multiplying machine with 10, 100, and so on marked on it, we want to know where to put the mark for 2. Suppose the distance from 1 to 2 ought to be x decimetres, we have to find x . Evidently the distances 2-4, 4-8, 8-16, . . . should each be equal to x . Now if we take any value for x and mark in the series of numbers as shewn in the diagram, it is clear that all the numbers shewn, both the multiples of 10 and the multiples of 2 should always come in order. A larger number should never be reached before a smaller one. Hence we must reach 1000, the mark for which is 3 decimetres from 4, before we reach 1024, the mark for which should be $10x$ decimetres from 4.

Hence
$$x > \frac{3}{10} > \cdot 3.$$

Again we must reach the point which should be $33x$ decimetres from 4 before we reach the point which is 10 decimetres from 4.

Hence
$$x < \frac{10}{33} < \cdot 303.$$

Inviting attempts to find x still more closely, we soon find that

$$\begin{aligned} 103x &> 31, & x &> \cdot 30097, \\ 93x &< 28, & x &< \cdot 30107; \end{aligned}$$

or, if it is considered worth while* to go further,

$$\begin{aligned} 196x &> 59, & x &> \cdot 3010204, \\ 485x &< 146, & x &< \cdot 3010309. \end{aligned}$$

Of course all this is nothing but $2^{103} > 10^{31}$, and so on, which, however, is in too abstract a form for beginners.

$$\begin{array}{c} 1 \quad \quad \quad n \\ \hline \leftarrow \text{---} \log. n \text{---} \rightarrow \end{array}$$

$$\begin{array}{c} 1 \quad \quad \quad x \\ \hline \leftarrow \text{---} \log.f(x) \text{---} \rightarrow \end{array}$$

$$\begin{array}{c} m \quad \quad \quad n \\ \hline \log. n - \log. m \\ \hline x \quad \quad \quad y \\ \hline \log.f(y) - \log.f(x) \end{array}$$

We may now give x its technical name, logarithm of 2, and the calculation of the four figure logarithms of 3, 4, 5, 6, 7, 8, 9, might be undertaken—affording a lesson in the art of saving labour by thinking, for only $\log 3$ and $\log 7$ require any fresh calculations—and even these can be found from the diagram by a little ingenuity. It is my firm conviction that a great part of the difficulty which boys find in logarithms is due to the fact that the mode of calculating logarithms is to them a mystery. If they are once shewn how any logarithm *can* be calculated, they will, I think, make no difficulty in accepting the fact that the work has been done, once for all, by

* Shanks's table of powers of 2 is fairly accessible.

others. We are now in a position to construct a primitive slide rule and to work out a number of questions on it.

5. I now come to the utility of the slide rule. A distinguished French mathematician has spoken of "that inexplicable prejudice against the slide rule, against which it is impossible to protest too strongly." This prejudice is dying out, but it exists. The Diagram (2) shews the mathematical principle of the ordinary slide rule so far as it goes beyond what we have already discussed. You will observe that if the scales 1 and 2 are placed in conjunction with their units coinciding we must have $f(x)=n$, so that two such scales furnish us with a table of 'natural' values of $f(x)$. The diagram (2) also shews that we have the means of multiplying or dividing by $f(x)$, in fact of working the equation

$$\log n - \log m = \log f(y) - \log f(x), \text{ or } \frac{m}{f(x)} = \frac{n}{f(y)}.$$

It is clear that with such an instrument we can work out any question which can be done with logarithms; and several which can not. The slide rules in the market usually contain two scales for $\log n$, one on the ruler and one on the slide and logarithmic scales for some of the following functions:

$f(x)=x^2, x^{-1}, \sin x$ (which of course serves also for $\cos x$), $\tan x$, $\log x$, and a plain scale.

The $\frac{1}{x}$ scale is not very much needed, for we obtain it at once by putting the ordinary slide upside down.

Now, I claim for the use of the slide rule many advantages.

First. In very many cases an isolated numerical example does not bring out the essence of a formula. We want to observe the effect on the result of changes in the data. With a slide rule we can work out fifty numerical examples quicker than the answers can be written down.

The K.E. of a shell weighing W lbs. moving at V f.s. is

$$\frac{WV^2}{2g \times 2240} \text{ ft. tons} = \frac{WV^2}{143000} \text{ ft. tons}.$$

The diagram (3) shows how, taking an initial velocity of 2700 f.s. and a given weight of shell, we can read off the remaining K.E. E at any remaining velocity V . People who do not think say that the slide rule in these cases prevents a boy from thinking. We must distinguish. When I am trying to teach a boy about energy I don't want him to be thinking of multiplication and division; I welcome any means of preventing him from thinking about multiplication and division. I admit that the slide rule economises useless thought.* I consider this a merit.

* Mach., *Science of Mechanics*, p. 488, urges this point very forcibly.

6. Second, the slide rule familiarises us with the values of the trigonometrical functions, and enables *really accurate* graphs to be plotted very quickly. For instance, diagrams of Fourier series* can be rapidly and accurately drawn showing each term and the result of compounding any number of terms.

I have, we all have, again and again been told that there is no angle whose tangent is 3, no angle whose sine is $\frac{1}{4}$.

Mr. Siddons spoke in January of the undue importance which has been attached to 18°, 30°, 45° as compared with other angles equally worthy of respect. But that is not the worst: a gentleman has actually printed the formula, for boys to learn,

$$\sin 20\frac{1}{4}^\circ = \frac{\sqrt{3}}{5} \text{ nearly.}$$

We have all seen a good many queer formulæ, but I confess I have never seen anything to equal this. The use of a slide rule renders such nonsense impossible.

The third advantage I claim is certainty. In real life you generally needn't do a question by the shortest way, nor very quickly; but it must be right. With a slide rule there is no excuse for not checking your work.

Cube roots, plane and spherical triangles, quadratic and cubic equations, can with a little management be worked out independently with a slide rule, but at any rate they can be very easily checked by it.

Fourthly, how many authorities have denounced the practice of students who assert that the circumference of a penny bun is

$$9\cdot734933\bar{7}6274\bar{9} \text{ inches,}$$

or that the angle of inclination of a shaky chimney-pot is 84° 17' 34·2". Yet how invariably boys make this mistake,† and how difficult it is to persuade them that it is a mistake. They call it accuracy.

If a class work out the value of $\sqrt{m \cdot n}$,

- (i) graphically on a small and large scale;
- (ii) with a slide rule;
- (iii) with 4 figure logarithms;
- (iv) with 7 figure logarithms;
- (v) by actual multiplication and extraction of the square root;

they cannot very well help seeing that each method has its own degree of accuracy, and that to give results which the method does not warrant is to tell an untruth. By supposing m and n only approximately known, we can see when the inevitable error in the result due to the imperfections of the data is appreciably added to by the approximate method of calculation adopted. I can testify that such work undertaken by a class arouses considerable interest; and that the necessary logarithmic work is done without much feeling that it is against the collar.

It is noteworthy that the exact error (due to the slide rule) committed in working any problem can always be found. This familiarises pupils with some of the leading results in the mathematical theory of errors; for instance, as has been already implied—that, to do justice to data, the Probable Error due to the use of approximate methods of calculation should not exceed about $\frac{1}{4}$ of the P.E. due to the imperfections of the data.

* Such diagrams are given in Byerley, "Fourier Series," p. 63; or Merriman & Woodward, *Higher Mathematics* (article by Professor Byerley), p. 199.

† Of course "It's near enough" is a conclusion pupils need no encouragement to adopt. But do not let us teach accuracy by inaccuracy. There are occasions when 7 or even 10 figure logarithms are called for; geodesic surveys furnish plenty of instances; but do not let us bring in 7 figure logarithms 'nisi dignus vindice nodus inciderit.'

Again, let us find the cube roots of the numbers 2, 3, 4, 5, 6, 7, 9...with a slide rule. Here we have an exact graduation to set the slide to; and we find, for instance, that the p.e. of the result is, say, a unit in the fourth significant figure. Now let us take the cube roots of 2·16, 3·16, 4·16... An additional source of error arises from the fact that we have to estimate, or interpolate, for the position of the 6. This causes the p.e. of the result to be perceptibly larger. Such examples, which may be multiplied almost indefinitely, give a real insight into the way in which a result is affected by several component sources of error.

There is perhaps no paramount scientific principle so often disregarded as this—always have some idea of the degree of accuracy of which your method is capable, and of the degree of accuracy of your own work.

To take a single case—our whole national system of competitive examination rests on a total disregard of this principle. This, however, is bordering upon topics of controversy.

I have one thing to say in conclusion. If anything in this paper is of value, the credit is largely due to Colonel H. C. Dunlop, R.A., with whom I have constantly discussed slide rule problems for some years.

The Logocyclic Slide Rule.—The special object of the logologarithmic slide is to enable the user to read off powers of numbers in the same way as the common slide rule enables us to read products.

Putting $f(x) = \log x$ in the principal equation, we have at once from the diagram $\log \log y - \log \log x = \log n - \log m$ or $y^m = x^n$, while, as special cases, if $m=1$, $y=x^n$, and if $x=10$, $n=\log_{10} y$. Thus powers and logarithms can be read off at a glance.

Numerous applications at once suggest themselves. As an instance of the use of the slide in getting out empirical formulæ, we may take Dr. Vincent's formula (which, perhaps, ought hardly to be termed empirical), viz.: atomic weight of the n^{th} element, when the elements are arranged in order of their atomic weights, is (in general) $(n+2)^{1.21}$.

It is a singular fact that the logolog scale has repeatedly been invented, forgotten, and reinvented. The first inventor was Dr. Roget. It was rediscovered and patented by Captain Thomson, R.A.; again, in Germany, by M. Blanc (Dyck's *Catalog*, p. 145), and again, almost simultaneously, reinvented by Professor Perry, and reintroduced by Colonel Dunlop and myself. The use of a $\log(-\log x)$ scale for numbers less than unity was one fondly thought new, but in this idea, which Professor Perry also brought forward, we were all anticipated by M. Blanc.

R. M. ACADEMY,
WOOLWICH.

C. S. JACKSON.

MATHEMATICAL NOTES.

128. [R. 6. b.] *On Gauss' Principle of Least Constraint and the Equations of Mechanics.*

The object of this note is to point out the remarkably simple principle to which Gauss' principle leads. The new principle is much simpler—not being an integral principle—than that of Hamilton,

$$\delta \int_{t_0}^{t_1} (T + U) dt = 0,$$

but, like it, is independent of coordinates, and leads at once to equations in generalised coordinates which have important properties.

1. Consider any material particle of mass m_ν , whose coordinates are (x_ν, y_ν, z_ν) at time t , and which forms part of a system. If m_ν were quite