The Flight of Projectiles, Together with the Description of an Instrument Constructed by him to Show the Trajectory of a Ball from a Given Elevation and Range, Together with its Initial Velocity and Force of Striking

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Monday, July 10th, 1865.

CAPTAIN E. GARDINER FISHBOURNE, R.N., C.B., in the Chair.

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ANNUAL.

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THE FLIGHT OF PROJECTILES, TOGETHER WITH THE DESCRIPTION OF AN INSTRUMENT CONSTRUCTED BY HIM TO SHOW THE TRAJECTORY OF A BALL FROM A GIVEN ELEVATION AND RANGE, TOGETHER WITH ITS INITIAL VELOCITY AND FORCE OF STRIKING.

A Paper prepared by Major-General ANSTRUTHER, C.B., late Madras Artillery, and read by Major-General J. T. BOILEAU, R.E., F.R.S.

PREFACE.

Extract from "Strait's Memoir of Artillery," pp. 81, 82.

"DR. GREGORY, in his lectures upon gunnery, observes on the difference between the times employed by a ball in ascending and des-

THE TRAJECTORY,

From the "Madras Artillery Records" Vol. 25, No. 1, for January, 1862.

*Solid Shot, 68 Pdr Iron Gun, Length 10 feet, Weight 95 cwt.**Windage 0.198, Charge 16 lbs. or 0.2355.*

GIVEN.

Elevation. Range.

Feet.	
0.15	660
0.30	1020
0.45	1500
1.	1920
1.30	2580
2	3180
2.30	3720
3.	4200
4.	5100
5.	5880
6.	6540
7.	7050
8.	7680
9.	8160
10.	8640
11.	9120
12.	9600
13.	10020
14.	10440
15.	10860
16.	11220
17.	11580
18.	11940

FIND.

A. B.	B. C.
660	2.8798
1020	8.9014
1500.1	19.636
1920.3	33.57
2580.9	67.67
3181.9	710.05
3723.5	762.41
4205.8	220.11
512.5	356.63
5802.5	514.44
6576.	667.38
7163.	865.63
7755.	1079.35
8362.	7292.42
8772.	1523.40
9281.	1772.75
9814.	2040.55
10284.	2313.23
10760.	2603
11243.	2910
11672.	3225
12109.	3540.4
12554.47	3879.54

P. Anstruther, Major General.
late of the Madras Artillery

“cending vertically through the same space. If a 24-pound iron ball “were projected vertically upwards, with a velocity of 2,000 feet per “second, it would ascend to the height of 6,424 feet before its upward “motion was extinguished, and it would pass over that space in less “than $9\frac{1}{2}$ seconds. (This is computed in Hutton’s Mathematics, vol. 3.) “It might, on a cursory view of the subject, be supposed that the “circumstance of the descent would be analogous to those of the “ascent, but in an inverted order; and so they would, in a non- “resisting medium, but in the air the case is widely different. After “the ball had descended 2,700 feet, the resistance of the air would be “equal to the weight of the ball, there would remain no farther cause “of acceleration, and the ball would descend uniformly with its “terminal velocity (that is, the greatest velocity which a heavy body “can acquire when falling in the air), which does not exceed 419 feet “per second. It would require, therefore, $\frac{6424-2700}{419}$ or 6 seconds, to “descend the remaining 3,724 feet, in addition to the time, about 10 “seconds, which had been occupied in descending through the first “2,700 feet; so that, in this instance the time of descent would be “about double that of ascent. In all cases where the projectile “velocity exceeds 300 or 400 feet, the time of descent will exceed “that of ascent; and their difference is greater the more the initial “velocity exceeds that limit.”

SECTION I.

The preface shows what was taught at our artillery academies of Woolwich and Addiscombe as the science of gunnery. We read that a ball, fired vertically upwards with a velocity of 2,000 feet per second, would ascend during a time “less than $9\frac{1}{2}$ seconds,” after which, all its upward motion being extinguished, it would descend during “about” 16 seconds, thus making a total time of flight of only $25\frac{1}{2}$ seconds, or less, for this most enormous velocity at the greatest possible elevation. Yet almost every table of recorded practice shows very considerably longer time of flight, obtained at elevations much less than 90° , by the use of moderate charges, therefore with no great velocities. The concluding passage tells us that “*in this instance the time of descent would be about double that of ascent. In all cases where the projectile velocity exceeds 300 or 400 feet, the time of descent will exceed that of ascent.*” These words inculcate a principle so erroneous that it is impossible to calculate the flight of any projectile until this mistake has been corrected.

SECTION II.

Whatever the elevation, whatever the velocity, it is impossible that the time of the descent of any ball fired upwards, can be either greater or less than at the time of its ascent, or can be greater or less than the whole time of flight. For, the descent is the motion which is produced by the force of gravity, and this force acts upon the ball from the very moment of its being set in motion, and continues to act upon it during

the whole time of its flight. The second law of motion, rightly understood, teaches this: it tells us that the motion which is produced by the force of gravity acting upon the ball whilst in motion upwards is the same, both in magnitude and direction, as that motion which would be produced in the same time, by the same force, if it were acting upon the same ball suffered to fall freely from a state of rest. Whatever the elevation, whatever the velocity, the ascent and descent of any ball forced upwards must be of equal duration. These two motions may differ in magnitude, and must differ in direction, but they cannot differ in duration; they are simultaneous and not successive; they are synchronous, not consecutive. Hence it follows that if we know the descent we can calculate the ascent; we can graduate the line representing the ascending motion, measuring the initial velocity, and determining the trajectory for that velocity at any elevation whatever. This constitutes the whole science of gunnery.

SECTION III.

There are three forces acting upon every ball throughout its flight:

- I. The projectile force.
- II. The force of gravity.
- III. The resistance of the atmosphere.

Of these three the first and the second produce motions which we shall take into consideration separately. The third force produces no motion, but acts by reducing the magnitude of the motions produced by the other two forces. Thus, although there are three forces acting, there are only two motions produced, and we have only to compound the motion which is produced by the force of gravity during the flight of the ball with that motion which is produced by the projectile force during the same time, to obtain the actual motion of the ball, its true trajectory, or path through the air.

SECTION IV.

By supposing the angle of elevation to be 90° we throw upon one single vertical line the two motions which we seek to compound, we thus conceal from our own view the fact of their distinct and separate simultaneous existence. We make each of them in succession cover, and so hide, the other. But, by supposing any other elevation, we shall be enabled to employ as a diagram a right-angled triangle, in which the oblique and vertical sides shall represent the two motions which we are to compound, visibly separate and distinct from each other; then, if we can determine the graduation for time of the entire magnitude of each of these two straight lines, we shall be able to draw the true trajectory for any elevation whatever to any degree of minute accuracy that may be desired.

Galileo showed how this was to be done; he proved that the flight of a ball, projected obliquely, must be in the curve of a parabola, *"unless so far as it would be diverted from that track by the resistance of*

the air." But Hutton, followed by Gregory and Straith, taught that the descent commences when the ascent ceases. If this were the case, the motions of every ball would be rectilinear, its angle of descent must always be a right angle, which would render any second graze an impossibility, and "*ricochet*" a word without meaning.

SECTION V.

Let A B G, Fig. 1, be a right-angled triangle, within which we are to inscribe the trajectory of a ball, which being fired from A towards B strikes G at the expiration of the time of flight, let us suppose 25.5 seconds, then

I. A B represents the motion produced by the projectile force in 25.5 seconds.

II. B G represents the motion produced by the force of gravity in 25.5 seconds.

III. A G represents the ground over which A B extends.

The curve described by the ball in going from A to G is usually defined by giving the abscissæ and ordinates, the abscissæ being the graduation of A G, and the ordinates, perpendiculars showing the height attained, these are the "internal," measurements of the curve. By this method we require to alter every figure of the calculation for every change of elevation, the consequence is that no one has ever ventured to draw a trajectory for any service velocity with service charge. But if artillerymen would define the curve by its "external" measurements, by the graduation of the two lines A B and B G, they would find their advantage in this, that one calculation will serve for all elevations, and that the first space in the graduation of A B denotes the initial velocity at once.

The graduation of A B can only be determined from that of B G which therefore we shall now find.

SECTION VI.

The vertical, B G, is the measure of the fall by gravity in 25.5 seconds of time, we are to find its magnitude and graduation. The fall by gravity in any time, t , is equal to $t^2 \times \frac{1}{2} g$, where the letter g expresses the velocity which would result from a fall of one second's duration, in a vacuum. The value of g varies slightly according to the latitude of the place, but we shall consider its value, in a vacuum, to be 32.2 feet. The resistance of the atmosphere very greatly alters the value of g , but we have no difficulty whatever in determining the extent of this alteration of value, for thirty seconds or more, and we only require a few experiments, easily made, to raise this to sixty or seventy seconds.

We have a very great number of experiments recorded, in which

the range, elevation, and time of flight are given; we multiply the given range by the tangent of the given elevation; the product is the measure of the fall by gravity in the given time of flight. A careful comparison of very many recorded ranges, &c., led the present writer to adopt the following scale of value for the letter g , and it was offered to the world in a lecture delivered at the Royal United Service Institution in the summer of 1861.

For 2	seconds the fall is	$\left\{ \begin{array}{l} 2^2 \times 16 \cdot 1 = 64 \cdot 4 \\ 3^2 \times 16 = 144 \cdot \\ 8^2 \times 15 \cdot 5 = 992 \cdot \\ 13^2 \times 15 = 2535 \cdot \\ 18^2 \times 14 \cdot 5 = 4698 \cdot \\ 23^2 \times 14 = 7406 \cdot \\ 28^2 \times 13 \cdot 5 = 10584 \cdot \\ 33^2 \times 13 = 14157 \cdot \\ 38^2 \times 12 \cdot 5 = 18050 \cdot \end{array} \right\}$	feet, the value of g being	$\left\{ \begin{array}{l} 32 \cdot 2 \text{ feet.} \\ 32 \text{ } \\ 31 \text{ } \\ 30 \text{ } \\ 29 \text{ } \\ 28 \text{ } \\ 27 \text{ } \\ 26 \text{ } \\ 25 \text{ } \end{array} \right\}$
3				
8				
13				
18				
23				
28				
33				
38				

The latter line being uncertain.

The Ordnance Select Committee may be said to have approved and confirmed the principle of the foregoing, as they did, in the winter of 1862, denounce as erroneous the belief which they ascribed to the present writer, that is, a belief that "*the value of g , expressing the fall of an object in the atmosphere under the attraction of gravity, is inva-riable—always 32·16 feet.*"

The latest experiment of which we have any record, the firing at *Shoeburyness*, on the 20th of July, 1864, shows that we are not far wrong, if not perfectly right. The elevation was $23^\circ \cdot 9'$, and the range 7,300 yards or 21,900 feet. Here the tangent of $23^\circ \cdot 09' \cdot 00''$, multiplied by 21900, gives 9363·74 feet, as the measure of the fall by gravity, from which we deduce the time of flight.

We are to determine the time in which a ball will fall through the space of 9363·7392 feet, and we see by a glance at the above table that it is between 23 seconds and 28 seconds; we try 26 seconds and 27 seconds, and we then try 26·1 seconds, and 26·2 seconds, which we find to give as follows, viz:—

$$\begin{aligned} 26 \cdot 1^2 \times 13 \cdot 69 &= 9325 \cdot 7649 \text{ feet, } 37 \cdot 9743 \text{ feet, short} \\ 26 \cdot 2^2 \times 13 \cdot 68 &= 9390 \cdot 4992 \text{ feet, } 26 \cdot 76 \text{ feet, over.} \end{aligned}$$

Lastly, we try $26 \cdot 16^2 \times 13 \cdot 684 = 9364 \cdot 5852$ feet, which is only 0·846 feet too long, and this we say is the time of flight for a range of 7,300 yards at an elevation of $23^\circ \cdot 9' \cdot 00''$. Now the "*Times*" tells us that the observed time was 26·2 seconds, which is 0·04 second more than we should expect, but this suffices to show that the above table is true in principle, and nearly true in fact. The fall for 25·5 seconds of time, by this rule, would be $25 \cdot 5^2 \times 13 \cdot 75^2 = 8940 \cdot 9375$ feet, and this we accept as the magnitude of BG in Fig. 1.

SECTION VII.

Knowing the magnitude of BG and its graduation for time, we can deduce the magnitude of AB for any named angle of elevation, and we shall show how to determine the graduation of AB when its magnitude is known. For convenience of comparison with the tables given in our books, we shall suppose the elevation 45° , and proceed to work out the trajectory from these two data, the elevation, 45° , and the time of flight, $25\cdot5$ seconds.

Let ABG , Fig. 2, be a right angled triangle in which the angle BAG is the given elevation, 45° , and the side BG is the fall by gravity for $25\cdot5$ seconds, $8940\cdot9375$ feet, then we find $AG = BG$, and $AB = AG \sqrt{2} = 8940\cdot9375 \sqrt{2} = 12614\cdot395$ feet. Draw AY , parallel to, and equal to, BG , and join BY , then is $AGBY$ a parallelogram, of which AB is the diagonal, and we add to the definitions given in section V, this new one, IV, AY represents the vertical height attained by AB .

We know, from the law of the composition and re-resolution of forces, that the force which produces the motion represented by the line AB , is the resultant or equivalent of two forces producing motions represented, both in magnitude and direction, by the two sides AY , AG , of the parallelogram $AGBY$ of which said AB is the diagonal. Therefore it is absolutely certain—

First. That the duration of the motion represented by AB , the time during which the projectile force is acting upon the ball, to drive it from A in a direction parallel to AB , cannot possibly be greater than, or less than, the duration of either of the two forces of which it is compounded.

Secondly. That it is impossible that the duration of either one of the two forces of which the force producing AB is compounded can be either greater than, or less than, the duration of the other of the two.

If we can, therefore, determine the graduation of the vertical ascent AY , we shall be enabled to deduce the graduation of AB , which is the only thing required to enable us to draw the trajectory of the ball.

SECTION VIII.

We have to determine the graduation of the vertical ascent AY , of which we know the magnitude and the duration. Both in magnitude and duration this motion is exactly equal to the motion represented by the line BG , but the graduation of the descent is invariable, it has

been given in table on page 376, whereas the graduation of the ascent, $A Y_t$ varies with the angle of elevation. When the angle of elevation is 90° , when the ball is fired vertically upwards, supposing such a thing possible, then, and only then, would the graduation of ascent be exactly that of the descent, but in an inverted order, and we give in Table A, the graduation of each of the two motions, in juxtaposition, with a column, to show, by the differences between the two, the actual height attained by the ball.

We extract the following from the lecture alluded to above:—"If we deduct the fall for 25.49 seconds from that for 25.50, the difference will give the space described by the ball in the last hundredth part of a second of its fall. It is as follows, viz. :—

" 25.50 seconds, $25.50^2 \times 13.750 = 8940.9375$
 " 29.49 , $25.49^2 \times 13.751 = 8931.5761151$
 " a difference of 0.01 second, and 6.3613849 feet, showing a
 " velocity of 636.13849 feet per second."

The difference between our calculation and that of Hutton is as follows:—He gives the velocity as 2,000 feet per second; we say it is 636.1285 feet; he gives an actual ascent of 6,424 feet; we say it is 4,056 feet; he divides the time of flight into two unequal parts; we say the two motions are simultaneous.

We have said before, that the graduation of $A Y$, the vertical ascent, varies with the angle of elevation, we shall now show to what an extent this is the case. Suppose the ball fired at elevation to give $1\frac{1}{2}$ seconds time of flight, initial velocity 636.1385 feet per second. We find the elevation thus, say—

as the oblique ascent for 1.5 seconds, 934.5375 feet,
 : is to, 90° ,
 : : so is the vertical descent, for 1.5 seconds, 36.3375 feet,
 : the angle of elevation, $2^\circ..13..40$, logm. 8.58975836.

We obtain the graduation of the vertical ascent by saying as 90°

$$: \left\{ \begin{array}{l} 315.9375 \\ 311.5375 \\ 307.0625 \\ \hline 934.5357 \end{array} \right\} : : 2^\circ..13..40 : \left\{ \begin{array}{l} 12.284558 \\ 12.113471 \\ 11.939471 \\ \hline 36.3375 \end{array} \right.$$

The vertical ascent and vertical descent, equal in magnitude and duration, differ very widely in graduation at this low elevation, as the following table will show:—

In each half-second	12.284558	and the descent is	4.0625
the ascent is	12.113471		12.1875
	11.939471		20.1
	<hr/>		<hr/>
	36.3375	Totals, equal—	36.3375

Table B, *Appendix*, shows how to apply table A to any other elevation, less than 90° , drawing the trajectory without any further trouble

in calculating; let it be supposed that the elevation desired is 35° , we draw a right-angled triangle, its angles 90° , 35° , and 55° , and on the hypotenuse we lay off the 36 graduations marked as "*Ascent Feet*" in table B, then let fall 36 perpendiculars showing the "*Descent Feet*" in the same table; then a line joining the lower extremities of all these perpendiculars is the real trajectory or path of the ball.

Before leaving this velocity we shall show its range at 45° elevation. We find, by repeated trials, that the time of flight will be 20.625 seconds, for the ascent in 20.625 seconds is as follows, viz. :—

From $25.5^2 \times 13.75 = 8940.9375$ the possible ascent for this velocity,
 Take $4.875^2 \times 15.8125 = 375.7939423125$ the fall in 4.875 seconds,
 then

20.625 seconds time, 8565.143546875 is the ascent for this time and $20.625^2 \times 14.2375 = 6056.5$ feet, the descent in the same time. Then we say as $9565.14355 : 90 : 6056.5 : 45^\circ$ the elevation; so that the range at 45° for this velocity is 6056.5 feet, or 2019 yards. We have already shown that the initial velocity for 25.5 seconds vertical flight is 636.1285 feet per second; we shall now show the velocity for the same time of flight at elevation 45° ; it is worked out in the Appendix C. Fired with elevation 45° , if the vertical descent is 8940.9375 feet, the oblique ascent must be $8940.9375 \sqrt{2} = 12644.4$ feet. We find that the velocity to give an ascent of 12644.4 feet in 25.5 seconds is 733.3 feet per second.

In the Appendix, table D, we give a table showing 32 ranges for this velocity. Any artilleryman can now draw the trajectory as described in the last para. on page 378, or he may employ such an instrument as I have now the honour to show, a description of which is given in Appendix F. It is humiliating to think that no book in the English language gives us one service range for elevation, with the initial velocity of the ball. The French are far in advance of us in this respect; their book, the "*Aide-Mémoire à l'usage des Officiers d'Artillerie*," gives, at p. 431, no less than 30 recorded ranges at 45° , with the initial velocity of the ball for each one. In the Appendix E, will be found the calculation by which our theory would assign an initial velocity of 777.1 feet, or 236.8 metres to the ball, which at 45° ranged 3566.4 yards, or 3261 metres. This velocity the French *Aide-Mémoire* says was really 239 metres, being only 2.137 metres more than we make it. Our theory, therefore, agrees moderately well with that of the French artillerymen, judging from results. How widely it differs from that of Woolwich and Addiscombe, as taught by Hutton, has been shown in section VIII (quod vide). Our greatest want now, is a well-recorded table of times of flight, exceeding 40 or 50 seconds. We have nothing beyond 31 seconds in our books, nor have the French. In the Appendix G, will be found the calculations of the trajectory of a ball which, at 30° elevation, ranged 4850 yards. This ball, at 45° elevation, would range 14,000 feet in 32.79 seconds. According to our theory, if fired vertically upwards, it would not return to the ground until the expiration of 41.58 seconds, its initial velocity being 836.8 feet per second.

The model which I now place before you, will prove to any gentleman, that if he will obtain a true table of ranges for elevation and observed time of flight, he can construct an instrument which shall answer all possible questions on gunnery, and render that science as easy and simple as the first rules of arithmetic. My own instrument may not be true, because I have not got a table of ranges that can be depended upon, and I am denied the means of obtaining one.

TABLE A.

Time of Flight 25·5 Seconds.

Initial Velocity, 63613·85 feet per Second.

Seconds of Time.	Vertical Ascent (in feet).		Fall by Gravity, in feet.	Actual Height attained in the whole time.	Space passed through in each $\frac{1}{2}$ second.
	Gradual.	Cumulative.			
0·5	315·9375	315·9375	4·0625	311·875	311·875
1	311·5375	627·475	16·2	611·275	299·4
1·5	307·0625	934·5375	36·3375	898·2	286·925
2	302·5125	1237·05	64·4	1172·65	274·45
2·5	297·8875	1534·9375	100·3125	1434·625	261·975
3	293·1875	1828·125	144·	1684·125	249·5
3·5	288·4125	2116·5375	195·3875	1921·15	237·025
4	283·5625	2400·1	254·4	2145·7	224·55
4·5	278·6375	2678·7375	320·9625	2357·775	212·075
5	273·6375	2952·375	395·	2557·375	199·6
5·5	268·5625	3220·9375	476·4375	2744·5	187·125
6	263·4125	3484·35	565·2	2919·15	174·65
6·5	258·1875	3742·5375	661·2125	3081·325	162·175
7	252·8875	3995·425	764·4	3231·025	149·7
7·5	247·5125	4242·9375	874·6875	3368·25	137·225
8	242·0625	4485·	992·	3493·	124·75
8·5	236·5375	4721·5375	1116·	3605·275	112·275
9	230·9375	4952·475	1247·4	3705·075	99·8
9·5	225·2625	5177·7375	1385·3375	3792·4	87·325
10	219·5125	5397·25	1530·	3867·25	74·85
10·5	213·6875	5610·9375	1681·3125	3929·625	62·375
11	207·7875	5818·725	1839·2	3979·525	49·9
11·5	201·8125	6020·5375	2003·5875	4016·95	37·425
12	195·7625	6216·3	2174·4	4041·9	24·95
12·5	189·6375	6405·9375	2351·5625	4054·375	12·475
12·75	92·4984375	6498·4359375	2412·5015625	4055·934375	1·559375
13	90·9390625	6589·375	2535·	4054·375	1·559375
13·5	177·1625	6766·5375	2724·6375	4041·9	12·475
14	170·8125	6937·35	2920·4	4016·95	24·95
14·5	164·3875	7101·7375	3122·2125	3979·525	37·425
15	157·8875	7259·625	3330·	3929·625	49·9
15·5	151·3125	7410·9375	3543·6875	3867·25	62·375
16	144·6625	7555·6	3763·2	3792·4	74·85
16·5	137·9375	7693·5375	3988·4625	3705·075	87·325
17	131·1375	7824·675	4219·4	3605·275	99·8
17·5	124·2625	7948·9375	4455·9375	3493·	112·275
18	117·3125	8066·25	4698·	3368·25	124·75
18·5	110·2875	8176·5375	4945·5125	3231·025	137·225
19	103·1875	8279·725	5198·4	3081·325	149·7
19·5	96·0125	8375·7375	5456·5875	2919·15	162·175
20	88·7625	8464·5	5720·	2744·5	174·65
20·5	81·4375	8545·9375	5988·5625	2557·375	187·125
21	74·0375	8619·975	6262·2	2357·775	199·6
21·5	66·5625	8686·5375	6540·8375	2145·7	212·075
22	59·0125	8745·55	6824·4	1921·15	224·55
22·5	51·3875	8796·9375	7112·8125	1684·125	237·025
23	43·6875	8840·625	7406·	1434·625	249·5
23·5	35·9125	8876·5375	7703·8875	1172·65	261·975
24	28·0625	8904·6	8006·4	898·2	274·45
24·5	20·1375	8924·7375	8313·4625	611·275	286·925
25	12·1375	8936·875	8625·	311·875	299·4
25·5	4·0625	8940·9375	8940·9375	0·0	311·875

LONDON, 16th April, 1861.

TABLE B.

To Show the Trajectory for Velocity 636·1385 Feet per Second.

Seconds.	Ascent. Feet.	Descent. Feet.	Range. Yards.	Log. Sine.	Elevation.				
					° ' "				
0·5	315·9375	4·0625	105·3	8·1091922	0 44 12·3				
1	627·475	16·2	209	8·4119185	1 28 45				
1·5	934·5375	36·3375	308	8·58975335	2 13 40				
2	1237·05	64·4	411·8	8·7164936	2 59				
2·5	1534·9375	100·3125	510·3	8·8152434	3 45				
3	1828·125	144	598·	8·8963566	4 31				
3·5	2116·5375	195·3875	702·5	8·9652707	5 17 40				
4	2400·1	254·4	795·5	9·0252878	6 05				
4·5	2678·7375	320·9625	886·5	9·07852407	6 53				
5	2952·375	395	973	9·11742555	7 32				
5·5	3220·9375	476·4375	1061·8	9·170023642	8 30				
6	3484·35	565·2	1146	9·2100804	9 20				
6·5	3742·5375	661·2125	1228	9·24712848	10 10				
7	3995·425	764·4	1307	9·2817577	11 01				
7·5	4212·9375	874·6875	1388	9·314186	11 53				
8	4485	9	1458	9·3447493	12 46 40				
8·5	4721·5375	11 325	1529·2	9·3736828	13 40 30				
9	4952·475	124·4	1597·6	9·40118336	14 35 15				
9·5	5177·7375	1385·3375	1656·8	9·427415525	15 31 08				
10	5397·25	1580	1725·3	9·45251883	16 28				
10·5	5610·9375	1681·3125	1784·3	9·47661306	17 26				
11	5818·725	1839·2	1840	9·49380113	18 25				
11·5	6020·5375	2003·5875	1892·4	9·522173	19 26				
12	6216·3	2174·4	1941	9·5438074	20 28 15				
12·5	6405·9375	2351·5625	1986	9·56477385	21 32 12				
13	6589·375	2535	2027·4	9·58513377	22 37				
13·5	6766·5375	2724·6375	2064·6	9·604942308	23 45				
14	6937·35	2920·4	2097·6	9·6242487	24 53 40				
14·5	7107·7375	3122·2125	2128·4	9·64273109	26 03 30				
15	7251·625	3330	2150·	9·66153000	27 18				
15·5	7411·9375	3513·6875	2169·6	9·679582272	28 34				
16	7555·6	3763·2	2184	9·6972883	29 52 20				
16·5	7693·5375	3988·4625	2193	9·71467936	31 13 33				
17	7824·675	4219·4	2196·5	9·73178442	32 38				
17·5	7948·9375	4455·9375	2194	9·74862992	34 06				
18	8066·25	4698	2186	9·7652413	35 37 20				
18·5	8176·5375	4945·5125	2169	9·7818419	37 14 15				
19	8279·725	5198·4	2148	9·7978538	38 53 30				
19·5	8375·7375	5456·5875	2118	9·8138980	40 39 15				
20	8464·5	5720	2080	9·8297947	42 30 45				
20·5	8545·9375	5988·5625	2032	9·8455629	44 29				
21	8619·975	6262·2	1974	9·8612208	46 35 30				
21·5	8686·5375	6540·8375	1905	9·876785	48 51 00				
22	8745·55	6824·4	1823	9·8922823	51 17 30				
22·5	8796·9375	7112·8125	1725·5	9·9077098	53 57				
23	8840·625	7406	1609·3	9·92310075	56 54				
23·5	8876·5375	7703·8875	1470·4	9·93830995	60 12 06				
24	8904·6	8006·4	1299	9·9538229	64 02 40				
24·5	8924·7375	8313·4625	1082	9·96918616	68 40 15				
25	8936·875	8625	780·1	9·98457346	74 49 07·8				

GREAT MALVERN, 10th August, 1864.

APPENDIX C.

A ball, suffered to fall freely from a state of rest, describes a space of 12644·4 feet in the last 25·5 seconds of the time of falling.

To determine the height from which it fell and the velocity acquired by the fall.

1st.	The fall for 31·7 seconds is	$31\cdot7^2 \times 13\cdot13 =$	13194·2057
	6·2	„ $6\cdot2^2 \times 15\cdot68 =$	602·7392
	Differences		
	25·5	„ 25·5 , 2·55 ,	12591·4665
	which is 53·9 feet short.		
2nd.	The fall for 31·8	„ $31\cdot8^2 \times 13\cdot12 =$	17267·4687
	5·3	„ $5\cdot3^2 \times 15\cdot67 =$	621·9423
	25·5	„ 25·5 , 2·55 ,	12645·5265
	which is 1·1266 feet over.		
3rd.	The fall for 31·79	„ $31\cdot79^2 \times 13\cdot121 =$	13260·1363961
	6·29	„ $6\cdot29^2 \times 15\cdot671 =$	620·0090111
	25·5	„ 25·5 , 2·55 =	12640·127385
	which is 4·27 feet short.		
4th.	The fall for 31·798	$31\cdot798^2 \times 13\cdot1202 =$	13266·0022110408
	6·298	$6\cdot298^2 \times 15\cdot6702 =$	621·4565765408
	25		12544·4456345
	which is only two-thirds of an inch more than the magnitude of A B.		
	The final velocity represented by a fall of 31·798 seconds' duration is thus found :		
	the fall in 31·8 seconds, is	$31\cdot8^2 \times 13\cdot12 =$	13267·4688
	31·798 „	$31\cdot798^2 \times 13\cdot1202 =$	13266·0022110408
	the differences are	0·002 , 0·0002 ,	1·46658895
	which indicates a velocity of 733·2944796 feet, say 733·3 feet per second.		

APPENDIX.

TABLE D.

Initial Velocity 733·3 Feet.

Range at 45° = 2980·3125 Yards in 25·5 Seconds.

Secs. of Time.	Graduation of the Ascent. AB.	Graduation of the Descent. BG.	Elevation 90° AB.—BG.	AG = $\sqrt{AB^2 - BG^2}$	Log. Sine. ∠BAG.	∠BAG.
						° ' "
1	726·448	16·2	719·248	726·2	8·3483105	1 16 40
2	1438·776	61·4	1374·376	1437·5	8·6505928	2 33 43
3	2136·384	144	1992·384	2131·6	8·8286832	3 52
4	2818·672	254·4	2564·272	2807	8·9554880	5 11
5	3485·04	395	3090·04	3462·5	9·0543893	6 30 30
6	4134·888	565·2	3569·688	4096	9·1357385	7 51 20
7	4767·616	764·4	4003·216	4706	9·2050194	9 13 30
8	5382·624	992	4390·624	5396·5	9·2645176	10 35 40
9	5979·312	1247·4	4731·912	5847·75	9·3193545	12 02 30
10	6557·08	1530	5027·08	6376·33	9·3669809	13 27 45
11	7115·328	1839·2	5276·128	6873·5	9·4124342	14 58 45
12	7653·456	2174·4	5479·056	7338·3	9·4534817	16 30 20
13	8170·864	2535	5635·864	7767·0	9·4917100	18 04 30
14	8666·952	2920·4	5746·552	8159·5	9·5275759	19 41 30
15	9141·12	3330	5811·12	8513	9·5614448	21 21 48
16	9592·768	3763·2	5829·568	8823·0	9·5936132	23 06
17	10021·296	4219·4	5801·896	9089·8	9·6243266	24 54
18	10426·104	4698	5728·104	9307·6	9·6537909	26 47
19	10806·295	5198·4	5608·192	9473·5	9·6822809	28 45 36
20	11162·16	5720	5442·16	9585·2	9·7096478	30 49 30
21	11492·208	6262·3	5230·008	9636·1	9·7363233	33 01 40
22	11796·186	6824·4	4971·736	9621·6	9·7623247	35 21
23	12073·344	7406	4667·344	9535	9·7877561	37 50
24	12323·232	8006·4	4316·832	9368	9·8127127	40 31
25	12545·2	8625	3920·2	9110	9·8373215	43 26 30
26	12738·648	9261·2	3477·448	8746·5	9·8615440	46 33 15
27	12902·976	9914·4	2988·576	8258	9·8855765	50 12 30
28	13037·584	10584	2453·584	7613	9·9094526	54 16 30
29	13141·872	11269·4	1872·472	6761	9·9332435	59 02 30
30	13215·24	11970	1245·24	5600	9·9570191	64 55 30
31	13257·088	12635·2	571·888	3851·7	9·980849	73 06 30
31·8	13267·2688	13267·4688	0·0		10·0000000	90

APPENDIX E.

The French Aide-Mémoire gives a range at 45° elevation of 3261 metres, with initial velocity 239^m.

1. This range is 10699·341 English feet at 45°, the vertical descent, or fall by gravity is therefore also 10699·341, whence we deduce the time of flight, 28·17 seconds, for $28·17^2 \times 13·483 = 10699·4198187$ feet.

2. The oblique ascent in that time is $10699\cdot34\sqrt{2}$ or $15131\cdot15$ and the mean velocity of this ascent is $\frac{15131\cdot15}{28\cdot17} = \frac{R}{t} = 537$ feet.

3. The time in which a falling body acquires a velocity of 537 feet is 20·2 seconds, if we add to this one-half of the time of flight, $20\cdot2 + 14\cdot085 = 34\cdot285$ we shall have an approximation to the possible time of flight of this ball's initial velocity, therefore an approximate indication of the initial velocity.

4. More exactly we find that the fall by gravity in $35\cdot32$ seconds is $35\cdot32^2 \times 12\cdot768 = 15928\cdot1106432$ and in $7\cdot15$ $7\cdot15^2 \times 15\cdot585 = 796\cdot7430625$ so that in $28\cdot17$ we have $\frac{28\cdot17}{2\cdot817} \frac{15131\cdot3675807}{15131\cdot3675807}$ almost exactly the magnitude and duration of the oblique ascent.

5. The velocity which a ball acquires by a fall of $35\cdot32$ seconds is easily found—

$$\text{From } 35\cdot32^2 \times 12\cdot768 = 15928\cdot1106432$$

$$\text{Take } 35\cdot31^2 \times 12\cdot769 = 15920\cdot3394009, \text{ the differences}$$

$0\cdot01, 0\cdot001, \text{ and } 7\cdot7712423$ indicate a velocity of $777\cdot12423$ feet which is $236\cdot86318$ metres, differing from the French 239 metres by $2\cdot13682$.

APPENDIX F.

Description of an Instrument Constructed to Show the Trajectory of a Ball from given Elevation and Range, together with its Initial Velocity and Force of Striking.

The instrument consists of three straight bars and a set of metal rods; the three bars form a right-angled triangle, upon the hypothenuse of which we hang the metal rods to denote the fall by gravity in successive seconds, the trajectory is then visible; the initial velocity is printed on the hypothenuse, and the final velocity, multiplied by the weight of the ball, is the force of the stroke.

The three straight bars represent:—

I. The oblique ascent in the time of flight, t .

II. The vertical descent in the time of flight, t .

III. The horizontal range resulting from the two simultaneous motions of ascent and descent.

The two bars which represent the ascent and the descent are joined by a hinge, forming the apex of the right-angled triangle; these two bars are graduated exactly alike. Upon the vertical bar the graduation represents the fall by gravity, as modified by the resistance of the atmosphere; upon the front of the vertical bar are printed the figures 1, 2, 3, and so onwards to denote the time; and on the side of it, the figures 16·1, 64·4, 144, and so on to denote the fall by gravity. Upon the oblique bar is printed the velocity which a ball would acquire by falling vertically through a space equal to the distance from the apex of the triangle to each point of the graduation. The horizontal

bar is graduated by merely marking off equal spaces to represent successively 100, 200, 300 yards' range. At one end of the horizontal bar is a pentagraph joint, fixed; through the side tube of this passes the bar of oblique ascent; this side tube represents the gun. Upon the horizontal bar is a moveable pentagraph joint; through the side tube of this passes the bar of vertical descent.

We now proceed to show how to use the instrument. Set the oblique bar to the required angle with the horizontal bar, and run it out until the vertical bar intersects the horizontal bar at the required distance, the given range, you have now the triangle, within which is to be inscribed the trajectory. Read the time of flight upon the vertical bar, and select from the metal rods all those marked for less time than this time of flight, and one more, upon which you mark the exact length of the vertical bar. Upon the third mark of graduation of the oblique bar, hang the metal rod marked 3, upon the next that marked 4, and so on, till all are hanging from the oblique bar, when it will be seen that the last few hang below the horizontal bar, and the last comes short of the given range. Now run out the bar of oblique ascent one step of the graduation marked upon it, transferring each of the pendulums one step in succession, keeping the elevation the same; move the vertical bar further from the gun, increasing its length at the same time; this bar ceases now to be of any use except as a pillar to support the bar of oblique ascent with all its pendulums. If the longest pendulum still shows below the horizontal bar, the oblique ascent must again be run out, and all the pendulums shifted, till the mark upon the longest pendulum as stated above exactly coincides with the given range of the ball, then the trajectory is shown, and the initial velocity read off.

APPENDIX G. SHOEBOURNESS, NOVEMBER 24th, 1854.

10-inch Howitzer of 125 cwt. Shot 140 lbs. Charge 16 lbs., $\frac{1}{14}$ = 0.112357142857.
Elevation 30°. Range 4,850 yards = 14,550 feet. Oblique Ascent 16,800.8 feet.
Vertical Descent 8100.4. Time 24.64. Initial Velocity, 836.8.

Seconds.	Oblique Ascent, Feet.			Simultaneous Vertical Descent	Range in Yards.	Elevation.	Demonstration.
	Cumulative.	Gradual.	Differences.				
1	832.91508	832.91508	8.252	Feet.			
2	1657.57416	24.66103	852	16.2	277.6	1 05 20	
3	2473.38324	15.809	9.4	64.1	552	2 13 40	
4	3279.74072	06.357	10.0	114	823	3 20 15	
5	4076.04510	796.305	6	257.4	1050	4 27	
6	4861.69318	85.653	2	395	1352	5 33 33	
7	5636.69526	74.401	8	505.2	1610	6 40 36	
8	6399.64874	62.549	4	761.4	1861	7 47 40	
9	7149.74572	50.697	13.0	902	2107	8 55 07	
10	7855.79050	37.045	6	1247.4	2346	10 03	
11	8609.18338	23.393	12	1530	2579	11 11 11	
12	9318.32196	09.141	8	1839.2	2803	12 20 07	
13	10012.61464	694.259	4	2174.4	3020	13 29 40	
14	10691.45112	78.837	16.0	2535	3229	14 39 58	
15	11354.27620	62.765	6.7	2920.4	3403	15 57 45	
16	12000.36928	46.133	2	3330	3618	17 03 15	
17	12629.25076	28.681	8	3763.2	3791	18 16 70	
18	13249.27944	11.029	4	4219.4	3965	19 31 02	
19	13853.85652	592.577	19.0	4698	4126	20 47	
20	14406.33160	73.525	6	5193.4	4272	22 04 26	
21	14960.25168	53.873	2	5720	4407	23 23 42	
22	15493.67576	33.621	8	6262.2	529	24 44 40	
23	16006.64484	12.769	4	6824.1	637	26 08	
24	16497.98192	491.317	22.0	7406	730	27 33 40	
25	16977.227	69.265	6	8006.4	807	29 01 54	
26	17413.84008	46.613	2	8625	872	30 33 10	
27	17837.20116	23.361	8	9261.2	915	32 07 40	
28	18256.71024	399.609	4	9914.1	43	33 46 04	
29	18617.6732	75.057	25.0	10584	50	35 28 31	
30	18967.7240	50.005	6	11269.4	37	37 15 51	
31	19286.12548	21.353	2	11970	03	39 08 40	
32	19584.22656	29.101	8	12655.2	857	40 55 43	
33	19857.47764	71.249	4	13314.1	759	43 13 30	
34	20099.27272	43.797	25.0	14157	610	45 28 45	
35	20315.01780	15.745	6	14912.4	490	47 53 46	
36	20502.11080	187.033	2	15680	174	50 31 20	
37	20659.95196	157.811	8	16459.2	075	53 24	
38	20787.94104	127.959	4	17249.1	790	56 36 30	
39	20883.47812	97.537	31.0	18050	3437	60 15 45	
40	20951.96320	66.155	6	18860.4	2990	64 33 40	
41	20987.96628	34.833	32.252	19650	2396	69 56	
41.58	20992.26009	5.46381		20503.2	1455	77 41 30	
				20992.26	0	90	

AB=16900.8
BC=8100.4 therefore t=24.64 for 24.64 x 13.836=8100.451456 ft.
AG=11350

Then $41.58^2 \times 12.112 = 20992.260088$
 $16.94^2 \times 14.606 = 4191.9903116$

$24.64^2 \times 10680.8637472 = AB.$
And $41.58^2 \times 12.112 = 20992.260088$
And $11.57^2 \times 12.143 = 2093.8920607$

0.01 x 0.001 8.368061 shows V = 836.8

MR. OLIVER BYRNE: When great minds conspire to perpetuate a fallacy, it has always been a difficult matter to clear that fallacy away. I know of no subject capable of being submitted to mathematical investigation that has received a greater amount of fallacious treatment, and that too, by great minds, than the motion of projectiles. Initial velocities have been little more than guessed at, the resistance of the air overrated, and the force of gravity mis-stated. The system introduced by Gen. Anstruther, which is practical, and easily applied, must give correct results within the range of his experiments, without offering any special theory about initial velocities, the resistance of the air, or the force of gravity. Indeed in his system are collected all those elements. Let us examine the theories put forward with respect to the resistance of the atmosphere to the motion of a ball passing through it. The resistance of a fluid to the motion of a body is said to vary as the square of the velocity; the hypothesis

upon which this law rests may be thus considered; the fluid displaced must have the same motion given to it as that of the moving body, hence on this supposition, the units of work destroyed by the fluid will be equal to the accumulated work in the fluid.

FIG. 1.
Velocity = 7 feet a second.

For example, let the cross-section of the body perpendicular to the direction in which the body moves = 3 sq. ft. the weight of a cubic foot of the fluid = 62 lbs., and the velocity of the body 7 ft. a second.

The weight of the fluid moved at a velocity of 7 ft. a second = $7 \times 3 \times 62$ lbs.

The units of work expended

in the displacement will be—

$$\frac{7^2 \times 7 \times 3 \times 62}{2g}$$

In which g is put = $32\frac{1}{2}$; $32\frac{1}{2}$; 32; &c.; and is said to vary inversely, as the square of the distance from the centre of the earth.

Then the body in moving through 7 ft. destroys—

$$7 \times \frac{7^2 \times 3 \times 62}{64} \text{ units of work.}$$

If T be the resistance of the fluid in pounds, the units of work destroyed will also be represented by $T \times 7$;

$$\therefore T \times 7 = 7 \times \frac{7^2 \times 3 \times 62}{64}$$

$$\text{or } T = \frac{7^2 \times 3 \times 62}{64}$$

Hence it is concluded that the resistance increases with the square of the velocity, as well as with the area of the cross section of the body presented to the fluid. It is easily perceived, especially with high velocities, that this law is not strictly true, and very far from the truth when the motion takes place in air. It has been found from experiments on railways, that the resistance of the atmosphere to the motion of the train, depends chiefly on the length of the train, and not upon the frontage of the carriages. The resistance resembles more that of friction than the moving of a long parallelopiped of the fluid in which the body moves.

Taking this latter view of the motion of a ball through the atmosphere from one given point to another, the units of work will always be the same, whatever may be the form of the curve described.

Let W , Fig. 2, be a body moved through the air, then by the resolution of pressures, the pressure of W , perpendicular to the tangent = $W \cos \alpha$.

$$\therefore \text{The resistance of friction} = fW \cos \alpha,$$

putting f for the co-efficient of the friction of the air on the ball. Hence the total work on the plane $\alpha\beta$ = $z f W \cos \alpha + W y$; but $z \cos \alpha = x$,

$$\therefore \text{The work on each plane} = fW x + W y.$$

This expression being independent of the inclination of the plane. It points out the work employed to move the body over the horizontal distance $\alpha\gamma$, added to the work due to the gravity in elevating the body the vertical space $\beta\gamma$. Since a curve may be supposed made up of an infinite number of straight lines, therefore the work upon the whole curve will be equal to the work done upon the horizontal projection AB , added to the work done in opposition to gravity in raising the body from B to C , no matter what form be given to the curve $AWWWC$. This demonstration shows

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$$\begin{aligned} v dt &= ds \\ \therefore \int v dt &= s = \frac{ds}{dt} \\ \therefore s &= \frac{vt^2}{2} = Nm = Mp. \end{aligned}$$

$mM = vt = \text{uniform velocity} \times \text{time}.$

Again suppose the time so short that the arc is equal to the chord, or arc $mM = \text{chord in } M.$

$$\text{Then } Mp \times MA = (\text{chord } Mm)^2 = (\text{arc } Mm)^2 = (vt)^2;$$

$$\text{but, } Mp = \frac{ft^2}{2} \text{ and } MA = 2r,$$

$$\therefore Mp \times MA = \frac{ft^2}{2} \times 2r = v^2 t^2$$

$$\therefore f = \frac{v^2}{r}; \quad (A).$$

If T be the time of an entire revolution, then

$$Tv = 2\pi r \quad \therefore v^2 = \frac{4\pi^2 r^2}{T^2}$$

$$\text{and } f = \frac{4\pi^2 r}{T^2}; \quad (B).$$

Let another body E , Fig. 3, describe the circumference of a circle EPQ in the time T_1 ; and let F represent the accelerating force of gravity at E . Put $R = EC$, then, from (B)

$$F = \frac{4\pi^2 R}{T_1^2}$$

$$\therefore f : F :: \frac{r}{T^2} : \frac{R}{T_1^2}; \quad (C).$$

Now the third law established by the experiments of John Kepler shows that the squares of the times are as the cubes of the distances from the centre C ; hence if we put r^3 for T^2 , and R^3 for T_1^2 , then (C) becomes

$$f : F :: \frac{1}{r^2} : \frac{1}{R^2} \quad (D).$$

and thus it is shown by the experiments of Kepler, that the force of gravity varies inversely as the squares of the distances. Sir Isaac Newton had little to discover in this matter when Kepler's laws are admitted. The experiments at General Anstruther's command are as accurate as those upon which the law indicated by (D) is founded. The influence of gravity on the motion of the moon round the earth, or of the earth round the sun, is then compared with the falling of a heavy body near the earth's surface, and near the earth's surface f is represented by g , which is said to be found by experiments. Astronomers apply a variety of corrections to uphold (D), and the cooked up value they have given to g . When g is trimmed up to suit the motions of the heavenly bodies, it is out of gear to apply it to the falling of heavy bodies in air near the earth's surface. In changing the value of g , General Anstruther is right, for his system corrects the constant quantities, and secures correct results within the range of his limiting experiments.

To find whether the resistance of the air or any other fluid medium is proportional to the square of the velocity³ (V) or not, and also to find whether the value usually given to (g), the force of gravity near the surface of the earth is under or over-estimated.

Generally g is put = $31\frac{1}{2}$; $32\frac{1}{2}$; 32 , &c.

Let v^2 multiplied by some constant coefficient express the retarding force; and to simplify the investigation put this coefficient under the form n^2g .

Then the retarding force will be expressed by—

$$g - gn^2v^2; \text{ (I).}$$

In this investigation s is put for the space passed over in the time t .

In general terms if F be any accelerating force, U the velocity, S the space, and T the time, the relations between these quantities are found from the two equations—

$$U = \frac{dS}{dT} \text{ and } F = \frac{dU}{dT}; \text{ (II).}$$

When T is eliminated from these equations, then

$$UdU = FdS; \text{ (III).}$$

Again eliminate U , then we have

$$FdT = \frac{dS}{dT}; \text{ (IV).}$$

Then from (I) substitute $g(1 - n^2v^2)$ for F in (II), then,

$$gdt = \frac{dv}{1 - n^2v^2}; \text{ (V).}$$

$$\text{From (III), } gds = \frac{v dv}{1 - n^2v^2}; \text{ (VI).}$$

When the equations (V) and (VI) are integrated and corrected, on the supposition that when $t = 0$; $v = 0$ and $s = 0$; the resulting equations will be

$$v = \frac{1}{n} \frac{\varepsilon^{2gn^2t} - 1}{\varepsilon^{2gn^2t} + 1} \text{ and } s = \frac{1}{2gn^2} \log \frac{1}{1 - n^2v^2}$$

Eliminate v and equation (VII) is obtained

$$s = \frac{1}{n^2g} \log \frac{1}{2} \left\{ \varepsilon^{n^2gt} + \frac{1}{\varepsilon^{n^2gt}} \right\} \text{ (VII).}$$

The reduction may be effected as follows:—

$$v^2 = \frac{1}{n^2} \frac{(\varepsilon^{2n^2gt} - 1)^2}{(\varepsilon^{2n^2gt} + 1)^2} \therefore n^2v^2 = \frac{(\varepsilon^{n^2gt} - 1)^2}{(\varepsilon^{n^2gt} + 1)^2}$$

$$1 - n^2v^2 = \frac{4\varepsilon^{n^2gt}}{(\varepsilon^{n^2gt} + 1)^2}$$

$$\begin{aligned} \text{Then } \frac{1}{1 - n^2v^2} &= \frac{(\varepsilon^{n^2gt} + 1)^2}{4\varepsilon^{n^2gt}} = \left(\frac{1}{2}\right)^2 \frac{(\varepsilon^{n^2gt} + 1)^2}{\varepsilon^{n^2gt}} \\ &= \left\{ \frac{1}{2} \frac{\varepsilon^{n^2gt} + 1}{\varepsilon^{n^2gt}} \right\}^2 \\ &= \left\{ \frac{1}{2} \left(\varepsilon^{n^2t} + \frac{1}{\varepsilon^{n^2t}} \right) \right\}^2 \end{aligned}$$

$$\therefore s = \frac{1}{2n^2g} \log \frac{1}{1 - n^2v^2} = \frac{1}{n^2g} \log \frac{1}{2} \left(\varepsilon^{n^2t} + \frac{1}{\varepsilon^{n^2t}} \right).$$

(VII) may be put under the form

$$2\varepsilon^{n^2gt} = \varepsilon^{n^2t} + \frac{1}{\varepsilon^{n^2t}} \text{ (VIII).}$$

Now from (VIII) the values of n and g may be found by dropping cannon shot from a balloon, at different known heights, the time occupied in falling being measured. Two different values of s and two corresponding values of t being thus

determined (VIII) presents two independent equations, from which u and g may be eliminated by the dual method of solving equations. I have but little doubt that the liberty taken by the General to change the reputed force of gravity near the surface of the earth, to meet the practical necessity, will be warranted when the value of g and the resistance of the atmosphere are independently and accurately determined. The force of gravity is pulled in all directions by astronomers, to make it fit the motions of the heavenly bodies, yet after all the corrections and allowances are made, the moon and asteroids will not obey. The practical results given by the system of General Anstruther also show that it is right to change the initial velocities previously conjectured, for it is easily demonstrated that the initial velocities given, both by the ballistic pendulum and the pendulum of Navez, are erroneous. These instruments may be employed to indicate any conjectured initial velocity, for the time incorporated in the calculation is made to depend on a body very much restrained, falling in air a few inches. To say the least of the process, it is very ridiculous; the results may be corrected until the time anticipated is obtained. The value of g in or out of air is unknown, and yet it is made to tell all about the pendulum, and in return the pendulum is made to tell all about g . The General points out the errors of other systems in bold relief, and arrives at practical results within the range of his guiding experiments, without assuming initial velocity, a particular value for g , or including any wild theory about the wonderful resistance of the atmosphere to cover shortcomings.

Captain HEATH, R.N., C.B. : It is not easy to discuss a lecture of this nature at a moment's notice, but I should like to put one or two test questions to the General. 1st. Do I understand you (addressing the General) to say, that if a shot weighing 3 lbs., and another weighing 100 lbs., are discharged in the air at the same elevation, and with charges such that they shall both have the same range, they must then, according to your theory, have started with the same initial velocity?

General ANSTRUTHER: Yes, provided they are spherical shot.

Captain HEATH: Secondly, do I also understand you to uphold that if those same shot, viz., one of 3 lbs., and one of 100 lbs., are dropped simultaneously from a balloon two miles high, they will both, falling through the air, reach the ground at the same moment?

General ANSTRUTHER: Yes.

Captain HEATH: Then I have no more questions to ask, the answers you have given clearly prove to my mind that your theory is erroneous. With respect to the disbelief avowed by the gentleman who has just made remarks on the paper, in the correctness of Major Navez's instrument for measuring initial velocities, I wish to point out to him that if he will take the observed initial velocities recorded in the book in his hand, and multiply them by 2 or 3, he will find the results a little larger than the observed ranges for a two or three seconds time of flight, which is what those who believe in "resistance of the air" would expect, and this is, I think, a rough and ready proof of the correctness of the observed initial velocities.

J. REDDIE, Esq.: Mr. Chairman, as an old member of this Institution, and although a civilian, I beg leave to make a few observations on the interesting and important subject under discussion; it being one to which I have given considerable attention. I regret that I have not been able completely to master General Anstruther's system, and do not understand upon what principle the vertical parallels in his diagrams are made to vary in their relative distances, and gradually to decrease from the beginning to the end of the trajectory. It is of course plain, that if these parallels were equidistant throughout, the construction would resolve itself into a parabola; than which—though it happens to be the theoretical path of a projectile *in vacuo*—nothing can be further from the real path of a projectile in the air. But I am told that these parallels are diminished in distance, according to a principle, though to me they appeared to be so, only by a kind of rule of thumb. Of one thing, however, I feel perfectly certain, namely, that the deviation of projectiles so immensely from the theoretical parabolic path, is not entirely—or, indeed, to any great extent—due to the resistance of the air. This deviation, I may observe, is so very great, that even Dr. Whewell, as a Cambridge mathematician (implicitly accepting the theoretical accuracy of the parabolic trajectory *in vacuo*), admits that the old trajectories of Robins

and other practical gunners, are far more nearly accurate, according to our actual experience whenever a ball is fired. It is, in fact, only when balls are fired at very slight elevations that their trajectory has the least resemblance to a parabolic curve; and from that fact I argue,—1st. That it is not “the resistance of the air” that throws out the theory; for the air is equally present when trajectories are low as when they are high; and consequently, 2nd, that the parabolic theory is at fault, because we don’t know the real value of “ g ,” or the true accelerative force of a falling body. I regret that my friend Mr. Byrne has not been able to give us a corrected value of g , or a true theory of the ratio of the accelerative force with which a body falls; though he has given us a formula by which to discover this ratio, when furnished with a few facts as to the time of falling from a great height,—facts which there is now no excuse for our not ascertaining. Those who adhere to the traditional theory of the air’s great resistance, ought to tell us why the air should, as it were, conspire (as upon their theory it does) against a body moving onwards, and not equally against its downward course, or in other directions? If “the terminal velocity of a falling body cannot exceed 419 feet per second” (as in the quotation in General Anstruther’s paper), how could the initial velocity of a ball when fired, exceed that rate? But, again, we know that there is a greater curvature towards the earth in the latter part of a projectile’s flight than at the beginning; from which it may be inferred that this bending towards the earth is caused by gravity having very much more influence than the current theory gives, and the air very much less. Instead of the ball having its descent retarded, it is very clear that its downward course must rather be hastened, towards the end of its path, or its trajectory would take a different form. As the air will act equally against the ball in rising and in falling, its chief influence must be against the onward path of a body when fired obliquely; and were resistance relatively less when the velocity is greater (contrary to the current notions), that might account to some extent for the greater curvature downwards, as the onward velocity of the body becomes reduced. But, I must say, that that alone would not account for the suddenness and greatness of such curvature. That, I feel certain, will be found to be due to the fact, that the accelerative force of gravity, now said to be equal to 32 feet per second, has been greatly underestimated. This ratio may be approximately true for a fall of one or two seconds (and hence the agreement of the theory with the actual flight of projectiles at slight elevations); but it cannot be the true ratio of the effect of the constant force of gravity pressing downwards, after a body begins to descend. I may observe that I think there has been some misapprehension as to current views regarding the course of a projectile. Granting the ratio of the fall of a body in equal times to be, as 1, 3, 5, 7, &c., or as the squares of the whole times, 1, 4, 9, 16, &c., there is no error in the usual mode of constructing the parabolic trajectory, as *in vacuo*. This is usually done by representing the projectile force, according to the first law of motion, as equal in equal times, or as the ordinates $AB'B''$ in figure 1, annexed, and the force of gravity as the abscissæ $AC'c''$, as equal to the squares of the times. These two forces combined give the parabola ADd' .

But there is a fallacy in this mode of treating the two forces as

FIG. 1.