COMPOUND STRESS EXPERIMENTS.

BY MR. C. A. M. SMITH, B.Sc., *Associate Member*,
ASSISTANT PROFESSOR OF THE EAST LONDON COLLEGE (UNIVERSITY OF LONDON).

(I.) *Theories of Elastic Failure*.—The object of this Paper is to give certain experimental results which have been obtained upon solid steel bars subjected to compound or combined stress. An attempt is made to indicate briefly the methods used during the experiments.

The practical results of such experiments are chiefly connected with the design of crankshafts and that matter is therefore discussed.

The following systems of loading the specimen were adopted, and in each case the maximum stress on the material at elastic failure was noted: (1) Compression only; (2) torsion only; (3) combined compression and torsion; (4) tension only; (5) combined tension and torsion. The notation for such stresses, used throughout the Paper, is, for compression $+p$; for tension $-p$; for torsion q .

There are four theories of elastic failure of materials:—

4 o 2

(1) *The Maximum Principal Stress Theory.*—This is known as the Rankine theory, and may be calculated by the formula

$$\frac{p}{2} + \sqrt{\frac{p^2}{4} + q^2} = C_1.$$

(2) *The Maximum Principal Strain Theory.*—This is known as St. Venant's theory and may be calculated by the formula

$$\frac{1}{E} \left\{ \frac{p}{2} + \sqrt{\frac{p^2}{4} + q^2} - \frac{1}{m} \left(\frac{p}{2} - \sqrt{\frac{p^2}{4} + q^2} \right) \right\} = C_2.$$

(3) *The Maximum Shear-Stress Theory.*—This has been called, for mild steel, Guest's law, and may be calculated by the formula

$$\sqrt{\frac{p^2}{4} + q^2} = C_3.$$

(4) *The Internal-Friction Theory.*—This is known as Navier's theory, and may be calculated from a formula

$$\sqrt{\frac{p^2}{4} + q^2} \sec \phi + \frac{p}{2} \tan \phi = C_4.$$

The first two formulae are to be found in *all* text-books on the strength of materials.

The results of the author's experiments show that, for mild steel, elastic failure takes place by reason of the maximum shear-stress, under all conditions of loading.

(II.) *Previous Experiments.*—In order that the object of the investigations may be fully understood, it is thought advisable to mention a few facts of especial interest to Members of this Institution. In the year 1900 Mr. J. J. Guest¹ published the results of some experiments made upon tubes subjected to tension, torsion, and internal fluid pressure. Nine steel tubes were used. These experiments did not, for some time, receive very much recognition, probably for two reasons: (1) a great deal of space was occupied, in the Paper embodying the results, with a description of the instruments used; (2) engineers are sceptical

¹ Philosophical Magazine, 1900.

about results, as to elastic strength of materials, obtained upon tubes.² Mr. Guest's experiments³ and the practical results of the deductions to be made from them⁴ have been discussed by the author. The correspondence which followed the publication of these analyses of Mr. Guest's work in *Engineering* was maintained for several months.

Reference will be made to probable sources of error in these experiments which may have considerably affected the results. Mr. Guest showed that the shear stress at elastic failure was more constant than the principal stress or the principal strain when the material was loaded in tension and torsion. In 1906 Mr. Walter Scoble attempted to solve the problem by conducting tests upon material subjected to combined bending and twisting.⁵ These experiments seemed to confirm the above. Although a system of loading by combined bending and torsion was adopted, there was no evidence to show that the material did not fail by reason of a shear stress due to a combined stress of tension and torsion. A difficulty was experienced in locating elastic breakdown. For the tests published the average variation from the mean is, for the shear-stress theory 3.72 per cent. and for the maximum principal stress theory 18.3 per cent. In America Professor E. L. Hancock⁶ attacked the problem, but he obtained contradictory results. The author has dealt with the work of Mr. Scoble and Professor Hancock elsewhere.⁷ The experiments of Professor Hancock appear to refute the shear-stress theory, but the variations in the results are so great that it cannot be said with certainty that they confirm or reject any of the theories of elastic failure.

² See letter to *Engineering*, 19 February 1909, page 258, by W. A. Scoble.

³ "Guest's Law on Combined Stress," *Engineering*, 10th July 1908.

⁴ "Guest's Law of Combined Stress—the case of the Crankshaft," *Engineering*, 23rd April 1909, page 545.

⁵ *Philosophical Magazine*, vol. xii, page 535.

⁶ *Philosophical Magazine*, vol. xii, page 418.

⁷ "Some Experiments on Solid Steel Bars under Combined Stress," *Engineering*, 20th August 1909.

It may be mentioned that Mr. Laurence B. Turner has compared the elastic failure of steel tubes subjected to simple tension and simple torsion.⁸ He says "Nineteen comparisons in simple pull and simple torque having been made, it is found that the difference between the shears is generally small." The author has published results on copper tubes,⁹ solid steel specimens,⁷ hollow machined steel specimens¹⁰ and various non-ferrous materials.¹¹

All of these experiments have been made by independent investigators and have caused considerable discussion in the technical journals. From the very nature of the problem it was inevitable that any new formula recommended would need experimental results for its support before it would be accepted. Such experimental work involves new apparatus, and requires the consideration of many details which, at first, appear extraneous to the work and are in themselves in the nature of preliminary researches or experiments. More than 300 tests have been made at the East London College—most of them included about seventy readings of extensions. It is impossible to include a full account of such work in any one Paper, but throughout the author has endeavoured to give references to enable those interested to investigate such details.

After a careful study of the Proceedings of the technical societies, it was evident that the engineers demanded further evidence concerning the behaviour of mild steel under combined stress. In June 1903 Mr. William H. Maw, in the "James Forrest" Lecture, commented upon the need of further research work on the subject.¹² In 1906 Mr. C. E. Stromeyer said that "he

⁸ "Elastic Breakdown of Materials subject to Compound Stress," *Engineering*, 5th February 1909, page 169.

⁹ "Strength of Pipes and Cylinders," *Engineering*, 5th March 1909, page 327.

¹⁰ *Journal*, Junior Institution of Engineers, July 1909.

¹¹ "Elastic Breakdown of Non-Ferrous Metals." *Paper* read before the Institute of Metals, October 1909.

¹² "Some Unsolved Problems of Engineering." *Proceedings*, Institution of Civil Engineers, 1903, vol. cliv, page 238.

trusted that the question of compound stresses would, if possible, be taken up by the Institution.¹³

(III.) *Solid Steel Bars used.*—This was the problem which offered itself for solution when the research work was commenced some years ago by the author. Mr. Guest's experiments, while fully appreciated by a few people, were either not very well known or not sufficiently convincing to influence the general body of engineers. Moreover there was only evidence that the law held for a combination of stresses due to tension and torsion. Lord Kelvin had suggested that a compression load, in combination with a torque on mild steel, would raise the yield-point of the material. If this were proved to be the case the shear-stress theory of elastic failure would seem to break down. It was, therefore, necessary to make experiments upon steel, not only for a combination of tension and torsion, but especially for a combination of compression and torsion.

Preliminary experiments caused the author to adopt solid steel specimens. He found that tubes were unsatisfactory for the following reasons:—

(1) It is only possible to apply a tension load by grips which cause distortion, or by means of solder which necessitates heating the ends of the tubes.

(2) The material is not isotropic. Annealing reduces this evil, but it seems to be doubtful whether it entirely removes it.¹⁴

(3) Difficulty of making accurate measurement of areas subjected to the loads.

(4) With thin tubes, strain-measuring instruments are not easily secured to the specimen.

¹³ Proceedings, Institution of Mechanical Engineers, 1906, Part 1, page 38.

¹⁴ In this connection it is worth noting that the matter is discussed in *Science Abstracts*, No. 1225 (1908) and No. 965 (1909). Experiments were made to determine the effect of extension, flexure, and torsional strain on iron wires. In the latter abstract we read "metals are only rendered isotropic to a very approximate extent by annealing."

(5) When loaded in compression, the tubes may fail by reason of secondary flexure.

(6) Engineers do not seem to have confidence in the elastic breakdown tests made upon tubes.

The great advantage to be gained by using tubular specimens is that the stress distribution, during a torsion test, is the same over all of the material.

FIG. 31.

General View of Combined Tension and Torsion Apparatus showing the Specimen with Torsion Bars and Pulleys.

Preliminary experiments were made upon both solid and tubular mild-steel specimens, and it was then decided to use solid bars.

(IV.) *Tension and Compression Tests.*—In most of the author's experiments, quoted in this Paper, a specimen of 1 inch diameter was used. The length between the shoulders was $4\frac{3}{4}$ inches. The reason why similar specimens were used for torsion, tension, compression and combined stress tests was that it was desired to

make them interchangeable for elastic range tests.¹⁵ The ratio of the length to the diameter was sufficient to ensure that, unless the loading during a compression test was placed with considerable eccentricity, the yield-point in compression would be reached before the specimen failed as a strut. If the loading is directly through the axis of the specimen, it is safe to make this ratio twenty for a mild-steel specimen.

Previous experiments with various grips were made, and as a result the specimens were turned with screwed ends, gas-threads being used. The general shape of the specimen and the arrangement of the apparatus is indicated in Fig. 31 and Fig. 48 (page 1272). The screwed ends were carefully fitted to the tension grips and the compression caps. The centre $4\frac{1}{2}$ inches of the specimen was turned parallel to 1 in 2,000. Torque bars were fastened by set-screws which fitted directly on to the specimen, at the top and bottom, as shown in Fig. 31.

A 50-ton Wicksteed testing-machine was used for the application of tension and compression loads. Ball-bearings were fitted so that the specimen was free to revolve when tested in torsion.

The torque bars were flexibly coupled together and moved simultaneously, the friction of the ball-bearings being thus eliminated before a reading was taken.

For measuring strains produced by tension or compression stresses, the sphingometer was used. An earlier form of the instrument is possibly known to Members of this Institution,¹⁶ and the improved type has been described in detail recently by the author.¹⁷ It was necessary to make a number of preliminary experiments upon columns in order to determine the most suitable proportions for the specimens used in the compression tests. It was anticipated that there would be difficulty in securing axial

¹⁵ Journal, Junior Institution of Engineers, pages 562, 573.

¹⁶ "A Method of Detecting the Bending of Columns," Proceedings, Institution of Mechanical Engineers, 1908, Part 3, page 699.

¹⁷ Journal, Institute of Metals, 1909.

loading for the compression tests, and for that reason some new method of measuring strains was considered desirable. Since the accuracy of the experiments recorded depend upon the instrument a brief description is given, and an isometric projection and photograph are shown in Figs. 33 and 34, Plate 32.

Two carriers are fastened to the specimen, each by three set-screws. The distance apart of the set-screws is the gauge length, and can be accurately measured. Three strain-recorders, in planes of 120 degrees, indicate the "stretch" of the specimen in these three planes. Each strain-recorder is calibrated separately and at the commencement of every test. The sensitiveness of the three strips may be varied as rapidly as the springs of a steam-engine indicator. Appendices II and III (pages 1263-1271) give proofs that the mean extension of the three strips is equal to the extension of the specimen as a whole, and that, given the readings from the three strips, the *maximum stress*, as well as the mean stress on the specimen, can be determined. The whole object of the sphingometer is to enable the maximum stress at elastic breakdown upon a specimen in tension or compression to be calculated.

(V.) *Torsion Tests*.—Fig. 31 (page 1242) gives a general idea of how the torque was applied to the specimen. The torque arms were actually secured by means of set-screws let into countersunk holes in the specimen. From the end of each torque-bar a cord was carried over a bicycle-wheel and fastened to a knife-edge upon which rested a hollow bar. A tray was hung by a knife-edge on the centre of this bar, and weights, marked W, were placed in the tray in order to apply a torque.

From the arrangement of the pulleys shown in the sketch it will be seen that every effort has been made to eliminate bending during the application of the torque. Owing to the longer distance between the points of support of the specimen, the stress caused by bending—due to bad application of the torsion load—is greater during a tension than during a compression test.

A probable source of error in Mr. Guest's experiments may now be explained. He used long torque-bars at the top end of

the specimen, and two shorter bars at the bottom end. These two lower bars were brought against stops. The advantage of this arrangement was that it was only necessary to use one tray and one set of weights; but it is doubtful whether the two shorter bars were stopped in a satisfactory fashion. If only one lever had been resting on the stop, considerable bending resulted. It is almost certain that unequal springing took place, which caused unequal forces at the end of the torque-bars. This in turn caused bending of the specimen, and it is probable that the stress caused by such bending was 4 or 5 per cent. of the total stress. An example, selected from the author's data, may make this point clearer. During the torsion tests the weight in the scale-pan at elastic failure was about 100 lb. Let us assume that the difference between the forces at the end of the torque arms was 30 lb. Then, since the distance between the spherical seats of the tension grips was 21 inches and the torque-bars were secured 8 inches from each spherical seat, we should have had a bending moment on the specimen caused by the unbalanced force of 30 lb. This force would produce a stress of approximately 1,500 lb. per square inch on the specimens used. The stress on such a specimen, at yield-point, obtained by a pure torque was 18,000 lb. per square inch. Thus an error of 10 per cent. could be easily caused by using stops. This method of loading was therefore abandoned by the author, and the arrangement shown in Fig. 31 was adopted.

In the author's experiments the torsional strain was measured by means of a sphingometer strip shown at T, Figs. 33 and 34, Plate 32. This was placed at right angles to the strips used in the tension and compression tests. The strip was calibrated for each test. A full description of this additional fitting, together with a proof of the accuracy of the method employed for calibration, has been published.¹⁸

By the use of ball-bearings in the testing machine and the pulleys, the effect of friction was reduced to a minimum and was negligible.

¹⁸ "The Torsion Sphingometer." Institute of Metals, 1909, Appendix II.

The sphingometer strips rendered it a simple matter to record very small strains.

(VI.) *The Yield-Point.*—In the experiments recorded (SS specimens) the elastic limit and the yield-point coincided. In the case of perfectly homogeneous specimens loaded in tension and compression in such a manner that the load passes through the axis of the specimen—and consequently there is no bending—the yield-point is marked by a rapid increase of stretch which takes place for a small addition of load. The author's experiments showed that the load never is acting exactly through the axis of the specimen. Some of the material used (SS specimens) was remarkably homogeneous,* but the load was always slightly non-axial. In such a case there will be an apparent “elastic limit effect” if an extensometer is used which merely measures a mean extension of the specimen. With non-axial loading, and a homogeneous material, the yield-point is reached at one portion of the specimen before the whole of it yields. With an extensometer which gives the distribution of stress, there will be an apparent rapid change in the eccentricity of loading directly any portion reaches the yield-point. Then will follow the “elastic limit effect” (during which the mean extension of the specimen is slowly increasing for equal increments of load) due to eccentric loading, and then will come the total yield shown by a great increase in strain. What has been recorded in these experiments is the maximum stress on the specimen when the first portion of it reaches the yield-point.

Until the load is reached at which this first portion yields there seems to be no “time effect,” but at this point that phenomenon becomes most marked.

If the material is not homogeneous the problem is more complex, and need not now be discussed: (1) because the material used, especially that of the SS specimens, was almost homogeneous; (2) and further, such material cannot be expected

* Throughout the Paper the term “homogeneity” is intended to include isotropy and uniformity.

to give much evidence concerning the theories of elastic failure, since the specimens vary in elastic properties.¹⁹

Every specimen tested, except those of mild steel, had an elastic limit as distinct from a yield-point. This shows that the material was not homogeneous. It would appear that mild steel, as now manufactured, is very much more homogeneous than other materials used by the engineer. When several specimens were cut off the SS mild steel bar, they seemed to be almost exactly similar with regard to elastic properties. The specimens of special steels and non-ferrous materials tested did not possess this property so essential for the experiments discussed in this Paper.

During the torsion tests the loads were added by small increments. In Appendix I (page 1260) the full record of one of these tests is given. An examination of figures under the columns "Torque load" and "Deflection of torsion strip" will show that the readings were extremely regular until a certain critical load was reached. At that load the beam of light on the scale, which recorded the strain, began to travel rapidly and the load was at once removed.

One advantage claimed for tubes, in compound stress experiments, is that the yield-point of the material during a torsion test is well defined. In the torsion tests on these solid specimens of SS steel there was no difficulty in noting the yield-point. It is believed that the reasons are: (1) the arrangement to obtain a pure torque and no bending; (2) the material was remarkably homogeneous. It should be mentioned that the torsional-strain instrument was so sensitive that the first indication of yielding could be detected, and an interval was allowed so that it was certain that the material would not strain further at each increment of the load. Within the torsional yield-point there was no "time effect" upon this material, although

¹⁹ This point is especially revealed in the tests recorded in the author's Paper to the Institute of Metals, previously mentioned. The elastic properties of certain specimens of arsenic copper, high conductivity copper, rolled aluminium, and brass tubes are discussed at length in that Paper.

that phenomenon had been observed to take place with other specimens—notably aluminium.²⁰

(VII.) *The Results with the Mild-Steel Specimens.*—The accompanying Table 1 (pages 1250–1) and the illustration, Fig. 36,

FIG. 36.—Experimental results showing the relation of Direct to Shear-Stress at failure.

The shear-stress is here plotted to twice the scale employed for the direct stress, the Guest ellipse thus becoming a circle.

show the results obtained with the SS steel. The structure is shown in Fig. 35, Plate 32. Table 2 (page 1252) gives the results of tests on HT steel, which are shown graphically on Fig. 38.

²⁰ "The Elastic Breakdown of Non-Ferrous Materials." Institute of Metals, 1909.

(Continued on page 1254.)

FIG. 37.—*Summary of Results obtained on Mild-Steel Specimens (Tables 1 and 3) at the East London College.*

Analysis of the Mild Steels.

Specimen.	Carbon.	Silicon.	Sulphur.	Phosphorus.	Manganese.
	per cent.	per cent.	per cent.	per cent.	per cent.
S.C. . .	0.20	0.08	0.14	0.032	0.50
S.S. . .	0.09	0.02	0.036	0.020	0.44
A.D. . .	0.30	0.08	0.024	0.031	0.53
S.B. . .	0.48	0.03	0.059	0.071	0.76

TABLE 1.—*The Results of SS Steel (tests made during June, 1908, and February and March, 1909).*

Specimen and Test.	Diameter.	Direct load-tons + p compression, - p tension.	Torsion load lb.	Ratio* of Maximum Mean Longitudinal Strain.	p . lb. per sq. in.	q . lb. per sq. in.	Max. principal stress $\frac{p}{2} + \sqrt{\frac{p^2}{4} + q^2}$	Max. shear stress $\sqrt{\frac{p^2}{4} + q^2}$	Max. principal strain $\times 10^3$.	Torsion slip approx.
SS Ia	1.003	+0.5	96	1*	1,420	18,500	+19,250	18,540	0.813	1½ p.c.
SS IIa	1.001,	+0.5	98	1	1,420	18,960	+19,710	19,000	0.833	2 „
SS IIb	1.001,	+0.5	96	1	1,420	18,580	+19,330	18,620	0.817	1 „
SS IIc	1.001,	+8.0	74	1†	22,800	14,310	+29,700	18,300	1.052	6 „
SS IIIa	1.000	+8.0	72	1	22,800	13,920	+29,400	18,000	1.038	3 „
SS Ic	1.003	+0.5	96	1	1,420	18,500	+19,250	18,540	0.813	20 „
SS Id	1.003	+0.5	94	1	1,420	18,110	+18,860	18,150	0.796	2 „
SS Ie	1.003	+5.0	90	1	14,200	17,350	+25,850	18,750	0.970	20 „
SS IVa	1.002	+0.5	96	1	1,420	18,540	+19,290	18,580	0.816	2 „
SS IVb	1.002	+2.0	90	1.975	11,200	17,390	+23,860	18,260	0.916	2½ „
SS IVc	1.002	+4.0	88	1.15	13,080	16,990	+24,760	18,220	0.935	2½ „
SS IVd	1.002	+6.0	82	1.125	19,200	15,830	+28,100	18,500	1.017	2½ „
SS IVe	1.002	+8.0	65	1.089	24,750	12,540	+30,030	17,650	1.046	3 „

SS If	1.003	+0.5	95	1	1,420	18,300	+19,050	18,340	0.805	9	„
SS Va	1.002 ₅	+10.2	0	Bent load removed before yield.							
SS Vb	1.002 ₅	+0.4	98	1	1,140	18,910	+19,450	18,940	0.828	2	„
SS Vc	1.002 ₅	+3.0	94	1.134	9,650	18,110	+23,545	18,720	0.917	2	„
SS Vd	1.002 ₅	+6.0	86	1.111	18,920	16,590	+28,550	19,080	1.040	—	
SS VIIa	1.002	-0.5	95	1	-1,420	18,350	-19,080	18,390	0.817	2	„
SS VIIb	1.002	Between 10 and 10.5 tons bent badly and failed.									
SS VIa	1.003	Taken to 9.7—no yield in tension—grips slipped.									
SS VIb	1.003	„ 9.5 „ „ „ „									
SS VIc	1.003	-7.0	68	1.192	-23,770	13,140	-29,600	17,720	1.088	6	„
SS VIIIa	1.002	-7.0	76	1.036	-20,570	14,660	-28,210	17,920	1.010	8	„
SP 4A	1.026	-12.7	0	1.129	-38,800	0	-38,800	19,400	1.284	—	
SP 4B	1.026	+12.5	0	1.099	+37,250	0	+37,250	18,620	1.233	—	
†MP 4	0.999	-12.7	0	1.065	-38,600	0	-38,600	19,300	1.278	—	
MP 6	1.026	+13.5	0	1.050	+38,450	0	+38,450	19,220	1.273	—	

Torsion slip is given as a percentage of total elastic strain. Curves shown in Appendix I.

p = Maximum direct stress on horizontal plane.

q = Maximum shear-stress on horizontal plane.

* When given as unity exact value not known.

† In these early tests the stress distribution was not calculated, but a fair average value is taken now, in which case the shear-stresses for Tests SS IIc and SS IIIa are 18,900 and 18,700. Average variation from mean of maximum shear-stress is 1.96 per cent.

‡ Curves for these tests are shown in "Method of detecting the Bending of Columns," Proceedings, 1908, Part 3, Figs. 4 and 6 (pages 700-701).

4 p

TABLE 2.
Results of H.T. Steel Tests made in July, 1909.

Specimen.	Diameter.	Direct Load [Tons] Compression + Tension -	Torsion.	Ratio of Maximum	p lb. per sq. in.	q lb. per sq. in.	Maximum Principal Stress	Maximum Shear-Stress
				Mean Longitu- dinal Strain.			$\frac{p}{2} + \sqrt{\frac{p^2}{4} + q^2}$	$\sqrt{\frac{p^2}{4} + q^2}$
H.T. 1a	0.750	+0.2	41	1	1,014	18,900	+19,407	18,900
H.T. 2b	0.750	+3.0	34	1.394	21,200	15,690	+29,500	18,900
H.T. 2c	0.750	+4.0	31	1.017	20,640	14,300	+27,930	17,610
H.T. 3a	0.750	+3.0	28	1.276	19,400	12,910	+25,840	16,140
H.T. 3b	0.750	+5.2	—	1.221	32,000	—	+32,000	16,000
H.T. 4a	0.750	+5.2	—	1.270	33,550	—	+33,645	16,750
H.T. 4b	0.750	+0.2	38	1	1,014	17,530	+17,827	17,320

TABLE 3.
Tests on Mild-Steel Specimens.

Specimen and Test.	Ratio of Maximum Mean Longitudinal Strain.	Maximum Principal Stress* $\frac{p}{2} + \sqrt{\frac{p^2}{4} + q^2}$	Maximum Shear-Stress $\sqrt{\frac{p^2}{4} + q^2}$	Specimen and Test.	Ratio of Maximum Mean Longitudinal Strain.	Maximum Principal Stress $\frac{p}{2} + \sqrt{\frac{p^2}{4} + q^2}$	Maximum Shear-Stress. $\sqrt{\frac{p^2}{4} + q^2}$
S.C. 3a	1.000	+19,800	19,500	A.D. 2b	1.083	-31,810	21,000
S.C. 3b	1.000	+19,800	19,500	A.D. 3a	1.088	-31,570	20,760
S.C. 3c	1.070	+26,100	20,000	A.D. 3b	1.114	-35,620	17,810
S.C. 3d	1.058	+29,500	20,400	† A.D. 4a	2.958	-38,180	19,190
S.C. 3e	1.036	+32,300	20,400	A.D. 4b	1.000	-20,750	20,610
S.C. 3f	1.089	+34,900	19,900	A.D. 5a	1.069	+27,950	20,320
S.C. 3g	1.062	+36,500	18,900	A.D. 5b	1.043	+30,140	19,710
Average Variation, from Mean, of Maximum Shear-Stress on S.C. Specimens = 2.16 per cent.				A.D. 6a	1.029	+26,370	20,500
S.B. 1a	1.000	+24,900	23,700	A.D. 6b	1.008	+29,050	20,420
S.B. 1b	1.000	+24,900	23,700	A.D. 7a	1.102	+32,300	21,300
S.B. 1c	1.114	+47,600	23,800	A.D. 8a	1.100	+36,420	18,210
Average Variation, from Mean, of Maximum Shear-Stress on S.B. Specimens = 0.21 per cent.				A.D. 10a	1.000	+20,560	20,420
A.D. 1a	1.000	-20,170	19,450	A.D. 11a	1.000	-19,190	19,050
A.D. 2a	1.000	-20,560	20,420	A.D. 11b	1.375	-26,210	20,320
				Average Variation, from Mean, of Maximum Shear-Stress on A.D. Specimens = 4.02 per cent.			
				C = 11,660,000 lb. per square inch.			
				E = 29,250,000.			
				$\sigma = 0.256.$			

* Where + the direct load was compression; and where - it was tension.

† Note the stress distribution for this test.

This material was not so homogeneous as the SS mild-steel, and the results are therefore not so regular.

In addition to the SS and HT tests, numerous other experiments have been made upon different specimens. Several of these have been published,* but it is thought desirable to give

FIG. 38.
Results of Tests on High Tensile Steel.

here a complete synopsis of all the work done upon steel. Table 3 and Fig. 37 (page 1249) enable the results to be seen. The usual notation has been used.

In Test AD 4a, Table 3 (page 1253), one of the balls was out of the race. This was not discovered till the end of the test. Great eccentricity of the load was thus caused, and the special seats did not take this up. The ratio of maximum to mean stress was nearly 3 to 1.

Tests on other Steels.—A large number of tests have been made

* See previous foot-notes, 7, 8, and 10.

upon special steels, and the author regrets to be compelled to state that the results were extremely unsatisfactory. He refrains from a complete analysis of these tests, although many instructive points have been learnt about the properties of the materials tested. The two great troubles experienced were: (1) Different specimens from the same bar tested in simple torsion or simple direct stress did not give repeated results; (2) the elastic breakdown was "veiled," and the "time effect" so great that no complete result concerning combined stress can be accepted which depends upon such variable material. A few examples will make this clear.

Self-Hard Steel.—The results of tests made upon this material contrasted sharply with those made upon a good quality mild-steel. It was found that, with each of the specimens tested, the strain and stress were not proportional at moderate values of the stress, yet considerable increase in the applied load produced only a sensible increase in the strain. The stress-strain diagram for the material gradually bent over. Specimen self-hard No. 1 was tested in pure torsion, and the elastic breakdown occurred when the maximum shear-stress reached 16,120 lb. per square inch. Specimen self-hard No. 2 was tested in compression and the maximum direct stress at elastic failure was 55,750 lb. per square inch. Specimen self-hard No. 3 was tested in tension; the direct stress at elastic failure was 55,650 lb. per square inch. Specimen self-hard No. 4 was tested in torsion, and the stress at elastic failure was 23,000 lb. per square inch. It would be unwise to build any formula upon material whose elastic properties vary so much.

Nickel Steel.—This material, like the self-hard steel, was very indefinite with respect to the yield-point. The values given are for elastic breakdown. Three tests seemed to suggest that the material obeyed Guest's law. The results are as follows:—NS 1 was tested in compression. Direct stress at elastic breakdown 45,100 lb. per square inch. NS 4 tested in tension. Direct stress at elastic breakdown 46,300 lb. per square inch. NS 3 tested in torsion. Maximum shear-stress at elastic breakdown 23,000 lb.

per square inch. NS 2 gave unsatisfactory results. It is hoped to investigate the elastic properties of this interesting material further. It may be said at once that the point of elastic breakdown can be considerably raised if the material is overstrained in the direction in which it is again tested. It is worth noting that the elastic breakdown of these nickel steel specimens took place at a stress about 25 per cent. in excess of that required to ensure elastic breakdown of the SS specimens. In Appendix III (page 1269) curves are shown of a test on this NS material.

A series of experiments were made upon some specimens of mild steel of very low elastic limit. Two sets were supplied, but there was such variation in the material that the author hesitates to include them.

Several special steels have been tested, but on the whole insufficient material of any one type of such steels has been supplied to produce useful data. Very great difficulty was experienced in securing the instruments to some of the special steels; the specimens must have been most difficult to prepare. The stress at tensile elastic breakdown was 87,000 lb. per square inch on a forged material tested in tension.

(VIII.) *Experimental Results.*—The experiments lead to the following general conclusions:—

(1) The elastic failure of mild steel takes place when a definite shear-stress is reached, and does not depend upon the value of the maximum principal stress nor the maximum principal strain.

(2) The law, which Guest's work on tubes showed was approximately correct for mild-steel tubes loaded in tension and torsion, is confirmed and applies also to the same material loaded in *compression and torsion*.

(3) The "elastic limit" effect noticeable during tension, torsion and compression tests on mild steel which has not been previously stressed, is probably due to a fault in applying the load, and to the method of measuring strains.

(4) The ratio of maximum to mean direct stress during elastic tests is greater than is usually assumed to be the case (*see* Appendix III, page 1267).

(5) The use of spherical seats does not ensure axial loading. This point is here mentioned as it probably accounts for the discrepant results obtained by experimenters who have measured the elastic limit instead of the yield-point.

(6) The ratio of the working stresses for mild steel in tension

FIG. 39.—Curves showing the relation of Direct to Shear Stress at failure according to the three theories of elastic failure.

Both stresses to the same scale.

and shear should be 2:1, and not, as is usually quoted in text-books, 5:4.

The curves, Figs. 39 and 40, show the relationship between torque and direct load at the yield-point according to the three theories of elastic failure.

(IX.) *The Practical Result.* — Although several cases of combined stress upon mild steel occur in engineering practice, the two chief cases are those of the boiler and the crankshaft. Both

cases have been fully discussed by the author.²¹ It may be useful to record that if L be the equivalent bending-moment, then, according to the four theories of elastic breakdown enumerated above, we have:—

- (1) Rankine . . $L = \frac{1}{2} M + \frac{1}{2} \sqrt{M^2 + T^2}$
- (2) St. Venant . $L = \frac{3}{8} M + \frac{5}{8} \sqrt{M^2 + T^2}$
- (3) Guest . . . $L = \sqrt{M^2 + T^2}$
- (4) Navier * . $L = \frac{1}{1 + \sin \phi} (\sqrt{M^2 + T^2} - M \sin \phi)$

FIG. 40.—Curve showing the relation of Bending-Moment and Twisting-Moment for a constant equivalent Bending-Moment.

$$OQ = 0.30 A.$$

The diagram, Fig. 40, shows the curves for the first three theories. The fourth theory is comparatively new, and has never been advanced for the purpose of crankshaft design, and it is omitted from the curves as it would possibly confuse the diagram. It will be noticed that it reduces to Guest's law if $\tan \phi = 0$, that is, if the coefficient of internal friction for steel is zero. All of the curves agree at A. But if any ordinate OQ be drawn, it will be

²¹ See references (3) and (4) on page 1239.

* - Recent experiments on mild-steel specimens have given results approximating so closely to Guest's law, that Navier's theory is not considered for the design of crankshafts.

seen that, with an equivalent bending-moment of value L , the permissible torques at that point shown on the diagram are, according to the three theories $Q = 0.954 L$; $Q = 1.390 L$; $Q = 1.673 L$. The two latter are 46 per cent. and 76 per cent. higher than the first. In other words, crankshafts designed according to the principal stress or principal strain theory of elastic failure have had a false factor of safety.

There has been evidently, since Mr. Guest published his experiments, some hesitation in accepting the practical conclusions which follow from the shear stress-theory of elastic failure. The author trusts that the discussion arising out of this Paper and the results enumerated above will end the controversy which has taken place concerning the design of crankshafts and structures subjected to combined stress.

The author desires to thank the authorities of the East London College for the facilities extended to him for carrying out this work, also Sir William H. White, K.C.B., Past-President, Mr. H. F. Donaldson, C.B., Member of Council, and Mr. W. J. Luke, of Messrs. John Brown and Co., for their personal interest in the experiments. He acknowledges the many opportunities afforded by Professor D. A. Low; and recognizes the valuable assistance given by:—Mr. A. G. Warren and several students of the East London College; Mr. E. Tyndall Cook, instrument maker, of Leyton; Messrs. Hadfield's Steel Works, the locomotive superintendent of the Great Eastern Railway, the Hoffman Ball Bearing Company and other firms, for supplying material.

The Paper is illustrated by Plate 32 and 6 Figs. in the letterpress, and is accompanied by 4 Appendices with 13 Figs.

[*The Discussion on this Paper was combined with that on Mr. William Mason's Paper, and commences on page 1278.*]

APPENDIX I.

A Combined-Stress Test fully worked out.

Specimen SS V, Test c.—Length of specimen between gauge points = 3.903 inches. Diameter 1.0025 inch. Length of torque-bars = 76.4 inches. Distance of sphingometer strips from axis of specimen = 2.58 inches.

Calculations for Tension or Compression Test.—The following readings and calculations were made:—

	Strip No. 1.	Strip No. 2.	Strip No. 3.
Scale Readings. Compression load } 0.5 ton }	238	281	332
Scale Readings. Compression load } 3 tons }	130	234	134
Calibration equivalent of scale } divisions in inches . . . }	0.00000776	0.00000917	0.00000752
Extension of strips in inches for } load of 2.5 tons . . . }	0.000838	0.000431	0.001489
Surface extension in inches . . .	0.000903	0.000824	0.001030

From the above the mean extension = 0.000919 inch, whence $E = 30.2 \times 10^6$ lb. per square inch.

The maximum surface extension (calculated as in Appendix III) = 0.001041 inch, whence ratio $\frac{\text{maximum stress}}{\text{mean stress}}$ on surface perpendicular to axis = $\frac{1041}{919} = 1.134$.

Torsion Test.—Calibration of torsion strip gave 1 division = 0.00001986 radians.

A compression-load of 3 tons was kept on the specimen and the torque applied by adding weights to the trays.

Torque Load.	Deflection of Torsion Strips.	Torque Load.	Deflection of Torsion Strips.
20	0	80	383
30	64	82	396
40	128	84	409
50	193	86	422
60	256	88	434
70	319	90	447
75	351	92	461
78	370	94	474*

* Slipped rapidly to 480 and load immediately removed.

Fig. 41 shows the curve for this test, on which the yield-point is evident.

FIG. 41.

Curve showing the relation of Torsional Stress to Strain.

FIG. 42.—Yield-Points for all the Torsion Tests on the SS Steels (See Table 1).

NOTE.—It will be observed that the slopes of the curves differ. This is due to scale distance not being constant. The curves are plotted with *scale-divisions*, and no correction is made for varying calibration over the scale. In each case it was considered only necessary to note the yield-point. Numerous other points on the curves, well below the yield, were noted, but are not marked on the curves. In many cases the load was removed before very much slip had taken place.

It will be seen that an increase of 70 lb. caused a deflection of 447 divisions.

$$\text{Whence strain} = \frac{0.501 \times 447 \times 0.00001986}{3.903}$$

$$\text{Torque} = \frac{70 \times 76.4}{2} = \frac{\pi}{16} f d^3$$

whence $f = 13,510$ lb. per square inch.

$$C = 11.87 \times 10^6 \text{ lb. per square inch.}$$

Now since $(1 + \sigma) 2 C = E \therefore \sigma = 0.298$.

Stresses at Yield-Point.

$$p = \frac{3 \times 2240 \times 4}{\pi \times 11005} \times 1.134 = 9,650 \text{ lb. per square inch.}$$

$$q = 18,110 \text{ lb. per square inch.}$$

$$\therefore \text{Maximum shear-stress} = 18,720.$$

$$\text{Maximum principal stress} = 23,540.$$

$$\text{Maximum principal strain} = 0.917 \times 10^{-3}.$$

Fig. 42 shows the yield-point for all the torsion tests. Typical curves of the elastic breakdown for direct stress tests are to be seen as stated on page 1251.

APPENDIX II.

Proof that the mean extension of the three strips of the Sphingometer is equal to extension of the specimen as a whole. Further, that the variations from the mean of the three readings are the variations of the corresponding generating lines on the specimen, magnified in the ratio of the diameter of the cylinder, passing through the strips, to the diameter of the specimen. Also to find the amount of bending.

From the extension of the three sphingometer strips equally spaced round and from the axis of the specimen and parallel to it, we obtain three measurements.

If there were no bending on the specimen and the material were perfectly homogeneous, these extensions would all be equal.

That the variations are due to a bending-moment (probably because the load is placed eccentrically) is evidenced by the fact that the mean of all the readings, up to the elastic limit of the material, obeys a linear law.

The sphingometer carriers and strips are shown diagrammatically in Fig. 43, to which the following symbols belong :—

FIG. 43.

Let $\alpha \beta \gamma$, $\alpha' \beta' \gamma'$ represent the fixed ends of the sphingometer strips. Let O and O' be the centres of the triangles $\alpha \beta \gamma$ and $\alpha' \beta' \gamma'$. Then the displacement of $\alpha \beta \gamma$ with respect to $\alpha' \beta' \gamma'$ may be resolved into a motion of O to or from O' and a rotation of the plane $\alpha \beta \gamma$ about some axis such as PQ .

As all motions are relative, there is no need to consider the points $\alpha' \beta' \gamma'$, and for the purpose of investigation these may be considered fixed. The points $\alpha \beta \gamma$ move by a combination of two motions. There is the increase (positive or negative) of OO' —the extension—which is part of the increase of length of strip, and

there is also the change in length due to an alteration in the inclination of the planes $\alpha \beta \gamma$, $\alpha' \beta' \gamma'$ to each other. The line PQ in the plane $\alpha \beta \gamma$ is called the axis of bending, and the angle between the planes divided by OO'—the radian bend per unit length—is the amount of bending.

The Mean Extension.—

Let the angle α OQ be θ

then the angle β OQ is $\left(\theta + \frac{2\pi}{3}\right)$,

and the angle γ OQ is $\left(\theta + \frac{4\pi}{3}\right)$.

Let the angle between the two planes, which were originally parallel, be ϕ after straining. Let x be the true extension of the specimen. Then we have, if

A = extension of strip

$r = Oa = Ob = Oc$,

the angle ϕ is measured in radians and is considered small.

We may take $A = x + \phi r \sin \theta$.

Similarly for the other strips we have

$$B = x + \phi r \sin \left(\theta + \frac{2\pi}{3}\right)$$

and

$$C = x + \phi r \sin \left(\theta + \frac{4\pi}{3}\right).$$

Then the arithmetic mean of the reading of the strips is

$$\begin{aligned} x + \frac{\phi r}{3} \left(\sin \theta + \sin \left(\theta + \frac{2\pi}{3}\right) + \sin \left(\theta + \frac{4\pi}{3}\right) \right) = \\ = x + \frac{\phi r}{3} \left\{ \sin \left(\theta + \frac{2\pi}{3}\right) \right\} \left\{ 1 + 2 \cos \frac{2\pi}{3} \right\}. \end{aligned}$$

But

$$2 \cos \frac{2\pi}{3} = -1.$$

Therefore the mean reading of the strips is x ; that is to say, the extension of the axis of the specimen is obtained by taking the mean of the three strip readings.

APPENDIX III.

A method of determining the maximum stress when given the displacements of three points spaced equally around the specimen.

In any stressed specimen the mean stress can be easily calculated. It remains to find the ratio of the maximum to the mean stress in order to determine the maximum stress to which the specimen is subjected.

Referring to Fig. 43 (page 1264), let A B and C be the stresses at the points on the specimen corresponding to α β and γ . Let S be the maximum stress on the specimen.

Then, writing stresses for strains in the equations (ii), (iii), and (iv) on page 1266, Appendix II, we obtain the corresponding relations, namely:—

$$A = \frac{A+B+C}{3} + \left(S - \frac{A+B+C}{3}\right) \sin \theta \quad . \quad . \quad . \quad (i)$$

$$B = \frac{A+B+C}{3} + \left(S - \frac{A+B+C}{3}\right) \sin (\theta + 120^\circ) \quad . \quad (ii)$$

$$C = \frac{A+B+C}{3} + \left(S - \frac{A+B+C}{3}\right) \sin (\theta + 240^\circ) \quad . \quad (iii)$$

Then expanding and eliminating between the first two expressions above we have:—

$$\left(S - \frac{A+B+C}{3}\right)^2 = \frac{4}{3} \left[\left(A - \frac{A+B+C}{3}\right)^2 + \left(A - \frac{A+B+C}{3}\right) \left(B - \frac{A+B+C}{3}\right) + \left(B - \frac{A+B+C}{3}\right)^2 \right].$$

The application of this method is first of all shown with hypothetical figures, which are simpler than those actually obtained in practice.

Suppose that the distance of the centre of the strip from the centre of the specimen is four times the radius of the specimen. Suppose the sphingometer readings to be 6, 8 and 10. Then the mean extension of the strips and of the specimen will be 8, and the surface strains will be the mean, together with the differences

4 q

from the mean of the sphingometer readings reduced in the ratio of four to one, that is,

$$8 + \left(-\frac{2}{4}\right); 8 + \frac{0}{4}; 8 + \frac{2}{4} = 7\frac{1}{2}; 8 \text{ and } 8\frac{1}{2}.$$

Then we have from the above, if S is the maximum strain,

$$\begin{aligned}(S-8)^2 &= \frac{4}{3} \left[(7\frac{1}{2} - 8)^2 + (7\frac{1}{2} - 8)(8 - 8) + (8 - 8)^2 \right] \\ &= \frac{4}{3} \left[\frac{1}{4} \right] = \frac{1}{3} \\ \therefore S &= 8 + \frac{1}{\sqrt{3}} = 8.577.\end{aligned}$$

Therefore ratio of maximum to mean strain—also the maximum to mean stress, since by Hooke's law, these are proportional to strains—is $8.577 : 8 = 1.072$. This is about the order of stress distribution for a good compression test, namely, 7 or 8 per cent. above the mean.

An actual example of stress distribution is now given. The curves, Figs. 44 and 45, show the load-strip curves and the stress-strain diagrams for this case.

Nickel Steel. Stress Distribution. Load 8.3 tons.

Strip readings in $\frac{1}{1000}$ ths of an inch:—

Strip 1	.	.	.	.	.	1.90
„ 2	.	.	.	.	.	2.31
„ 3	.	.	.	.	.	6.09

The ratio of the radius of action of the strip to the radius of the specimen was measured and found to be $= 6.89$.

The mean extension of the strips was 3.43 inches.

The differences from the mean are:—

(1)	.	.	.	.	.	- 1.53
(2)	.	.	.	.	.	- 1.12
(3)	.	.	.	.	.	+ 2.56

The surface extensions are :—

$$(1) 3.43 + \frac{-1.53}{6.89} = 3.43 - 0.222$$

$$(2) 3.43 + \frac{-1.12}{6.89} = 3.43 - 0.163$$

$$(3) 3.43 + \frac{2.66}{6.89} = 3.43 + 0.386$$

FIG. 46.

Showing method of obtaining ratio of Maximum to Mean Stress across a section perpendicular to axis.

Then

$$(S - 3.43)^2 = \frac{4}{3} \{ (-0.222)^2 + (-0.222 \times -0.163) + (-0.163)^2 \}$$

$$,, = \frac{4}{3} (0.0493 + 0.0362 + 0.0266)$$

$$,, = \frac{4}{3} \times 0.1121 = 0.1496$$

$$S = 3.43 + 0.387.$$

$$\text{Therefore } \frac{\text{Maximum stress}}{\text{Mean stress}} = \frac{3.43 + 0.387}{3.43} = 1.1127.$$

Fig. 46 shows a method of estimating the plane of maximum stress. The three strip readings are set off to scale from a point and a circle drawn in to pass through these lines. This method has not been used by the author, but it seems to be simple. Substituting in the formula is very easy.

APPENDIX IV.

On the Method of Gripping Specimens and a test of ordinary "Vee" grips.

It was, as explained earlier in the Paper, decided to use spherical seats for the tension tests. A sketch of the arrangements for the tension and tension with torsion tests are shown in Fig. 47; the arrangement for compression and torsion in Fig. 48.

With this arrangement of spherical seats much better alignment is secured than when "Vee" grips are used. The effect of eccentric loading is not, as is often assumed to be the case, entirely eliminated. The causes of eccentric loading are chiefly due to the friction of the spherical seats and the inexact alignment of the screwed portions of the specimens and grips; of these the friction of the spherical seats is the most serious. The effect of this friction is by no means negligible. The theoretical analysis was confirmed by the practical results obtained with the sphingometer. In order to test, by an independent method, the eccentricity of the loading, tests were made at the East London College and (by kind permission of Mr. John Duncan) at the West Ham Technical Institute. The method devised is shown in Fig. 49.

Two pipe-flanges were secured from each end of the specimen. Small gauge-points were fitted to the pipe-flanges at about 120 degrees. The distance between the gauge-points was carefully measured by means of an ordinary micrometer for each increment

FIG. 47.

*Tension Grip with Ball Race for
Torsion Tests used in Experiments
at East London College.*

FIG. 48.
*Compression and
Torsion
Experiments.*

of loading. The actual micrometer readings obtained during the test are plotted in Fig. 50 (page 1274). The mean curve obtained is a straight line.

In the tests made by the author calculations of the ratio of the maximum stress to the mean stress have been tabulated.

The actual value varies, of course, from test to test. A fair average is 1.1. It is generally greater at the smaller loads. This will be seen from the published curves. Figs. 51 and 52 (page 1275)

show a representative tension test. Fig. 53 (page 1276) shows the variation in the ratio of maximum to mean stress during a test. In Fig. 51 the three curves obtained from the sphingometer strip readings are plotted. In Fig. 52 the same curves are referred to the edge of the specimen. The maximum stress on the specimen is calculated from these curves.

FIG. 49.
*Apparatus used to Measure
Bending in Tension.
East London College.*

The ratio of maximum to mean stress is generally greater during tension tests than during tests in compression. This will be seen on reference to the Tables, leaving out the one extreme case A.D. 4a, Table 3, caused by a ball becoming displaced from its seat, and those in which the stress distribution was not calculated. The average ratio for tension tests on the A.D. specimens was 1.165 in tension, and 1.059 for tests in compression. The mean ratio over all was 1.101.

There are two reasons for the above-mentioned variation. The means of applying the load in compression are simpler, and with equal workmanship greater accuracy of necessity results. Also, for compression tests, a ball 1 inch in diameter was used instead of a 3-inch diameter spherical seat.

FIG. 50.—*Stress Distribution Tests made at West Ham Technical Institute.*

A study of Fig. 53 reveals the fact that the stress distribution is better with heavy loads. This is as would be expected, for with greater tensions the greater is the tendency to pull into line and bed down on the seats.

It is probable that the variation of values for limits of elasticity, which is usually noticeable in tension tests, is due to this varying stress distribution.

FIG. 51.—*Extension of Springometer Strips.* FIG. 52.—*Extension of Specimen in 3 Planes.*

It has been remarked that, although ultimate strengths of various test-pieces may be constant, elastic limits vary.

Until this rate of stress distribution is obtained, we cannot obtain very accurate estimates of the elastic failure of materials.

In order to determine the effect of the ordinary "Vee" grips in producing eccentric loading, and so to compare that method of securing specimens with the spherical seats and screwed specimens, two experiments were made upon a long mild-steel

FIG. 53.—*Curve showing Variation of Ratio of Maximum to Mean Stress with the Load.*

specimen. The overall length of the specimen was about 15 inches and the diameter of the test portion 0.846 inch. Sphingometer readings were taken upon the middle 4 inches. The angle θ is the angle between the line of minimum displacement and strip 1. Between the tests the specimen was removed from the grips and replaced in the same position in the machine. This was done in order to determine if the "Vee" grips would line up similarly in two consecutive tests. It will be seen that such was not the case, as the two tests differ considerably.

Test 1.—At a load of 5 tons on the specimen, the strip extensions, recorded in one-thousandths of an inch, were:—(strip 1) — 0·737; (strip 2) 4·145; (strip 3) 3·592; which gave a ratio of maximum to mean stress of 1·211. $\theta = 10\cdot3^\circ$.

Test 2.—At a load of 5 tons on the specimen, the strip extensions, recorded as above, were:—(strip 1) — 3·934; (strip 2) 11·248; (strip 3) — 0·649; which gave a ratio of maximum to mean stress of 1·575. $\theta = 39\cdot5^\circ$.

The axes of bending were not widely different, but the stress distribution varied greatly. It therefore appears that this method of gripping the specimen is unsatisfactory for elastic failure tests.

Discussion.

On the motion of the PRESIDENT, a hearty votes of thanks was accorded by acclamation to the authors for their Papers.

Mr. J. J. GUEST, in opening the discussion, said the members had had the pleasure of listening to two exceedingly interesting Papers, but he wished by way of criticism to say that the authors had not exactly explained what they were aiming at. He made that remark because it was best to assume that some in an audience had not had time to digest the Papers presented, and would appreciate the discussion more fully for a sketch of the general trend of the research. He hoped the authors would therefore excuse him if he said a few words upon that point.

If a specimen in an ordinary testing-machine were pulled, it was possible to ascertain that it broke at so many pounds per square inch. Taking a member in a bridge which was in tension or compression, it could be compared directly with the result of the experiment. In the next place take a piece of mild steel and put it into a boiler which was unstayed. There was a hoop stress round the circumference, and any rectangular part of the material (cut along the length and circumference) was subjected to a corresponding pull on its two ends. At the same time the two ends of the boiler tended to go off, and that made another pull of the material at right angles to the first pull; so that that piece of the material was subjected to tension in two directions at right angles. The ordinary testing-machine experiment gave no definite indication of the strength of the material under such circumstances, it only gave an approximation. Was it possible to ascertain what the effect of this second tension was on the resistance of the material; would it lower the strength of the material subjected to the first stress? If the material yielded at 21,000 lb. per square inch in the testing-machine, and there were 20,000 lb. one way, and 10,000 lb. the other way, was that 10,000 lb. going to make the material yield? That was the question they were trying to answer.

The important case of the crankshaft, which Professor Smith had referred to, was similar but opposite. The second tension was then a compression. He commenced about ten or twelve years ago to try and find out the effect of the second stress, and, for the purpose, conducted a certain range of experiments. They did not answer everything, but they satisfied him that the shearing stress was the governing factor. The authors had extended that work in certain ways, Fig. 54.

FIG. 54.
Diagram of Simple and Compound Stresses.

FIG. 55.
Diagram showing defects caused by very heavy draught of a tube.

He wished to criticize Mr. Mason's diagram on pages 1220-1, but only because he did not think the author had been quite fair in putting the figure in the manner in which he did. The author (Mr. Mason) had tested the material in tension and found that it yielded at an axial stress represented by the point A, Fig. 54. Then he did the boiler experiment, only more so. As much tension was obtained one way as the other, and the material yielded at stresses represented by the point B. So much tension axially and the same tension circumferentially were applied and a yield obtained under equal axial and hoop tension. That told one about the boiler case, but

(Mr. J. J. Guest.)

at the point B. It was possible to put on the material a tension up to C in a testing-machine, and it had a margin of safety, or a certain factor of safety. Other tensions at right angles to it, as represented by CD, could be added on (not in a testing-machine), but it would not make the material any more likely to yield, that is, the second tension would not effect the yielding of the material under the first at all; Mr. Mason then put the material in compression, and obtained a yield-point represented by the point E (or more properly K). Then he took so much tension and an equal amount of compression, but he could not in this case apply nearly as much stress. As he applied the stresses the material yielded at a point represented by F. That was the case of a shaft in torsion. With a compression instead of a tension the second stress did have an effect on the power of the material to resist.

Mr. Mason then subjected the material to two compressive stresses and found that it then yielded under stresses represented by the point H. This was the exactly opposite case to the two tensions case B. The experimental determination of this point H was very difficult, owing to the high stresses involved in material which tended to buckle; this part of the work was quite new, and Mr. Mason was to be congratulated on his clever piece of work. It showed that the second compressive stress had no effect on the capability of the material to withstand the first compressive stress.

Now Mr. Mason had obtained points A, B, F, E (or rather the corresponding point K) and H, on the diagram, Fig. 54, but the speaker did not think that Mr. Mason should duplicate the diagram by reflexion in the line BH—the point L should be reserved for hoop tension, which Mr. Mason did not experiment on; with his apparatus, Mr. Mason could obtain this point and then could justifiably mark it on the diagram, and it would afford some information as to whether the material of the tubes were isotropic. The diagram represented the whole case for a material subjected to two stresses—the important range for engineering. It would be desirable to have experiments on material subjected to three stresses, and that, he supposed, was the next step to be taken.

Turning to Professor Smith's Paper he would first reply to Professor Smith's criticism of his use of tubes. Professor Smith strongly objected to tubular specimens, but personally he thought that tubes were very suitable for some experiments. In his own experiments he had obtained a sequence of points running from A to B, and then along B towards L, and also from A to F. Mr. Mason had added the point H, and Professor Smith had made sure of the additional piece FE by obtaining a number of points along it. In attacking the subject, he had naturally selected the easiest route—the part BAF was easier than FE or the point H, the parts investigated by Professor Smith and Mr. Mason, for these latter parts involved difficulties due to the instability of the specimen.

In his own experiments, one of his difficulties was the accurate control of the pressure of the fluid inside the tubes (probably due to the small size of the secondary intensifier), but he did manage to get the points in which the tensions were equal in the two directions and went round the corner from B towards L. It, however, took some doing, and to load up using fluid pressure in apparently the way Mr. Mason had done was a difficult matter. The object of such experimental work, to find the effect of the second stress upon the yield-point of a material already subjected to one stress, was represented by the determination of the line on the diagram which the research had shown to be BAFEH. And this line had been fairly well determined along BAFE and the point H also found.

Complete knowledge of the law of the yield would include the case for three stresses at right angles. In certain cases small third stresses had occurred, and he had stated in his Papers his conclusion from his experiments. To investigate properly the case for three dimensions would be very difficult, though there was an easy case or two. When one considered the cases met with in actual practice, it would almost always be found that the yield occurred first on the surface and that there one of the three principal stresses was zero or small.

(Mr. J. J. Guest.)

Mr. Mason and Professor Smith, working independently, had fortunately taken two different parts of the curve to experiment upon; it would have been very unfortunate if two men had put in such an immense amount of work upon exactly the same thing, and some collaboration might be suggested. If the results and curves in the Papers were referred to, it would be noticed that Mr. Mason's points did not come so exactly in a line as Professor Smith's did. This was partly accounted for by the fact that Professor Smith had elaborated his apparatus, and adopted the refinement of taking account of how far the axis of stress was from the axis of the specimen. There was no doubt that it never was exactly coincident, but it could be arranged to be fairly near. However, a testing-machine was not a precision tool; it was a very accurate weighing-machine. Too much was not to be expected from it.

Professor Smith had criticized the amount of bending that occurred in some of his (Mr. Guest's) tests. The author pointed out that he (Mr. Guest) applied a pure torque at one end of the specimen scientifically, but that at the other end he allowed anything it liked to go off, torques pure, or bending, or anything. He did not know whether Professor Smith made that statement in order to rouse him, but he did not think that it was quite reasonable to suppose that, when a great deal of care was taken on so many points, that one would be quite neglected. As a matter of fact, set-screws were carefully used for that purpose, and he had little doubt that pure torque came off all right at the bottom as well as at the top of the specimen. Almost the same arrangement of bicycle pulleys was used that Professor Smith adopted. There was another reason why he did not think there was very much bending, namely, that he used instruments for measuring the strains. It was a great advantage to be able to watch just what happened in a specimen, and he, therefore, designed single-reading instruments both for the purpose of measuring the elongation and the twist of the specimens.

The twist was measured in the following way: to the specimen he fastened two mirrors, inclined at 45 degrees, to the axis of the specimen—the exact method of fastening did not matter. The

prolongation of the optic axis of a telescope, which was horizontal, when reflected in the first mirror became vertical, and after reflexion at the second mirror again became horizontal, and a horizontal scale was placed some distance off in its path. The twisting of the specimen between the mirrors caused change of scale reading. If there was any bending of the specimen, the horizontal cross-hair of the telescope would appear to rise and fall across the scale or to become sloped, so that one could watch any bending in the specimen as it occurred. He recorded the occurrence of any decided bending in his experiments. Of course, it was not a perfect method, but it worked fairly well.

In the next place, he wished to make a few remarks with regard to the tubes on which Professor Smith had a long array of points. The author stated, in the first place (page 1241), "It is only possible to apply a tension load by grips which cause distortion, or by means of solder, which necessitates heating the ends of the tubes." He did not think that was quite right. He thought it was possible to apply a tension load by grips which did not cause distortion. One could make very good fits by the collet type of mechanism, but personally he used solder. He never found that it injured the tube, and the ends were well away from all the strain-measuring apparatus. He had experienced no difficulty in applying tension loads.

The author stated, in the second place, "The material is not isotropic. Annealing reduces this evil, but it seems to be doubtful whether it entirely removes it." Personally he was very careful about the tubes he used. So far from the tubes not being isotropic, he thought they had an advantage in that respect. A tube could be made in one or two ways: either it was drawn through a die, or else it was rolled down by the Mannesmann process. Commercial considerations tended to greater drafts on the tube material, and to the use of tools until so worn that the commercial accuracy was impaired. The defects caused by over-drawing were well known, and the accuracy of the tubes used could be ascertained, and those with serious errors rejected before further work was done upon them. His tubes were smaller in diameter than Mr. Mason's, but he had never found the tubes seven-thousandths of an inch out, as

(Mr. J. J. Guest.)

Mr. Mason had the misfortune to do; they were very close indeed to being accurate. He supposed all the dies were ground. If one wished to draw a tube or wire with the least number of passes, and make a very heavy draft on it, a certain series of defects occurred in the tube or wire. Fig. 55 (page 1279) was a magnified section of the defects. Sometimes they were so big that they could be seen. Such a tube was not isotropic by any means, and it would be better not to experiment with it. It was better to make sure, first of all, that the tube was isotropic. A wire was exactly the same; sometimes it was possible to break the wire off, and get a conical section. If a piece of pianoforte wire were taken which had been drawn very hard, and if it were pulled and pushed, forming a loop, two or three times, it could be split down the middle because of those defects; but such tubes were not being dealt with, only carefully-made tubes. When the tube was obtained, they had, first of all, the power of applying a stress to it by a direct pull, and then there was the power of adding a circumferential stress by internal pressure. By containing these two it was possible to see whether the material was isotropic or not, because if the two interchanged and the same results were not obtained, then it was known that the material was not isotropic; and although if the same results were obtained, then, while it was not a complete proof that the material of the tube was isotropic, it certainly gave a great degree of confidence. He did not see how that could be done on a solid specimen with the same certainty of getting the result.

If a right-hand torsion were applied to the tube, and a certain result were obtained, the results of applying a left-hand torsion would give a good hint whether the material was isotropic or not. A tension could also be put on it simultaneously with a torque, which practically had the effect of twisting the whole thing round and altering the ratios. It could be tried at a different angle. He tried all those, and found that the tubes were satisfactory.

Professor Smith said, in the third place, "Difficulty of making accurate measurement of areas subjected to the loads." It was a difficult thing to do if it were looked at straight away, and it was possible to get round the corner. He weighed the tubes in water

and in air, which gave the volume of the tubes and the density. He was very anxious to get the density as well as the volume. The volume did not matter so much for his purpose, and it would be noticed that Mr. Mason did not say much about it. It really did not come in; it all cancelled out, and was a matter of comparatively little importance. But if a low density were obtained, the investigator would suspect that the flaws he had sketched existed. If the density were high, those flaws could not exist. That was really what he was looking for, and not so much the area which the author obtained so exactly. Personally he was looking for the density in order to see whether there were any flaws in the tubes. If there were flaws in the tubes, the density would have been low. He was also careful that the ordinary tension test gave a high result. If the ordinary tension test did not give a high result, flaws might have existed.

Professor Smith said, in the fourth place, "With thin tubes, strain-measuring instruments are not easily secured to the specimen." Of course, it would have to be a very thin tube for that, but the tubes need not be so excessively thin. Perhaps in some extreme case the physicists, as well as the engineers, would raise objections—something based on molecular dimensions. The author said, fifthly, "When loaded in compression, the tubes may fail by reason of secondary flexure." Mr. Mason had experienced one or two such cases. The great point was to keep as near to the secondary flexure as possible without incurring it. In the sixth place, the author said, "Engineers do not seem to have confidence in the elastic breakdown tests made upon tubes." He supposed engineers knew what the chances of such little flaws were, but personally he did not take tubes with little flaws in them.

He congratulated the authors on the piece of work they had done. Their experiments had covered the two-stress range, but it had meant to them, as he knew from his own experience, a very great deal of work and of pluck. Although he did not know positively, he had no doubt that they both had a great many setbacks. Professor Smith had developed his apparatus to measure to pretty nearly $\frac{1}{100,000}$ inch on specimens which were under

(Mr. J. J. Guest.)

very high stresses; it was not easy work, and a great deal of trouble must be experienced in making such tests. When an audience saw one particular point on a curve, they did not think it meant seventy observations or so to get it, and they did not know that it perhaps meant three or four months' work before the apparatus could be got right in order to make such tests. Professor Smith had stated that he would like a large grant made for the purpose of erecting a large testing-machine.

Professor SMITH said that what he wanted was a new alternating-stress machine, not a large testing-machine.

Mr. GUEST, continuing, said he thought personally the money could be spent in a better way. Professor Smith's results proved that he could do very accurate work with the apparatus he had, and he foresaw that Professor Smith would be led on to develop his research and need further apparatus. When he (the speaker) did his own experiments, some of the requisite money had to come out of his own pocket, which was a very disagreeable state of affairs. He hoped the University with which the author was connected would help him. He noticed that the University of Liverpool were not letting the work stand still, because, although the particular work to which he had referred had been covered, it had only been done for very mild steel. It was known that cast-iron and hard steel usually went in tension, but how did the other steels midway between go? Engineers at present had no knowledge at all on that subject. Was there a sudden change-point, or did they gradually change? He suspected the former. There were years of work still to be done on the subject, and he only hoped that the University of Liverpool and the East London College, and other similar bodies would continue their support.

Mr. C. E. STROMEYER said that Mr. Guest had amongst other things explained what he (Mr. Stromeier) also had wished to demonstrate by means of the wall-diagram which he had prepared (see Fig. 56). It was designed on the same lines as the diagram

which he drew during the discussion on Mr. Izod's Paper,* except for the modification, which had also occurred to Mr. Guest, that one of the axes should represent the simple axial tension and compression

FIG. 56.

Diagram of Simple and Compound Stresses.

stresses, while the other should represent the circumferential stresses, and the compound stresses occupied the several quadrants. The present diagram showed only three experiments, the first one

* Proceedings 1906, Part 1, page 34.

(Mr. C. E. Stromeayer.)

marked G, summarized IV, VII and VIII of Mr. Guest's experiments, and extended over about one and one-quarter quadrant, and M represented Mr. Mason's experiment V, which extended over one quadrant on one side and half a quadrant on the other, and SC represented Professor Smith's experiments SC, which extended over about one-half quadrant. It was of course gratifying, as Mr. Guest had pointed out, to find that the combined diagrams nearly completed the loop, but it was a pity that at any rate Mr. Mason had not experimented on simple hoop-tension, on simple hoop-compression, and on hoop-tension and axial compression combined, for which his apparatus seemed admirably adapted. Had this been done, then instead of bits of curves of three different materials being obtained, the complete curve for one material would have been available, and, as mentioned in the discussion on Mr. Izod's Paper, he thought that each material would ultimately be found to have a characteristic curve of its own. Professor Smith had omitted to deal with simple tension or compression stresses, which seemed to be a rather serious omission, for it had to be borne in mind that the complicated nature of the tests here described prevented their being carried out except very occasionally, and it was of the utmost importance to establish by a set of all-embracing tests in how far the compound stresses could be deduced from the three simpler ones which could be made with any testing-machine.

Professor C. A. M. SMITH said that on Table SS (page 1250) the simple compression and simple tension were both given.

Mr. STROMEYER replied that in that experiment Professor Smith had not measured the ratio of maximum to minimum stresses for all the tests.

Professor SMITH said that that was given in the last four readings on page 1251.

Mr. STROMEYER, continuing, said that that was so, but he had not been able to use that curve as the points were too few and the results

too irregular. Professor Smith had drawn far-reaching conclusions, which, in his opinion, the shortness of the curve SC did not warrant. In the Rankine formula the two principal stresses were

$$S_1 = \frac{p}{2} + \sqrt{\frac{p^2}{4} + q}, \text{ and } \frac{p}{2} - \sqrt{\frac{p^2}{4} + q};$$

and Professor Smith wished to prove that materials gave way whenever the difference between the maximum and the minimum stresses reached a certain value. That meant, as long as both stresses were kept at the same intensity, they might be increased to infinity.

Professor SMITH said that there was a third plane which was unstressed.

Mr. STROMEYER agreed that that was the case, but Professor Smith's experiments only carried conviction over the field which they covered; if one went as far as Mr. Mason's experiments carried one, Professor Smith's deductions would not be borne out by the results. For instance, the axial compression and circumferential compression combined, and circumferential and axial tension combined, would be quite wrong according to Professor Smith's theory. He had perhaps misunderstood Professor Smith, but he was afraid that all engineers would get the same impression.

He also desired to make a few remarks with regard to the extensometer. It had always been his view, ever since he had worked with such an instrument—since about 1882—that several instruments should be attached to a test-piece. He remembered that the first time he used an extensometer of his own make, he found that when the elastic limit was reached the pointer went back instead of going on. At first he could not understand it, but subsequently found it was due to excentric loading. Ever since then, a matter of thirty years, he had always used at least two instruments, and during some experiments which he made on the Blackfriars Bridge and on the swing-bridge at Glasgow he used three. In fact, in the latter case, he used six, three at the top of each member and three at

(Mr. C. E. Stromeyer.)

the bottom. By those means he was able to obtain not only the mean stress in the members, but also the bending and the irregularities of the thrust lines at the riveted joints. He liked the author's instrument very much, but there were a few objections to the coiled metal-strip. First of all, it was of a different material to the test-bar. He thought, in any case, that the coils should have been of steel if the test-bar was steel, and it should be brass if the test-bar was brass. This would have obviated the possibility of introducing temperature effects which might be very serious, as each experiment lasted a considerable time. Then, also, the coils were strained, and in moving one way or the other there was bound to be a little elastic lag, and this would mask any elastic lag in the test-piece; therefore, this property of the material would not make itself felt. He quite appreciated that the author's instrument was a most beautiful one for giving the elastic limit and the yield, but he did not think it would quite answer the purpose for some of the elastic determinations.

The last complaint he desired to make was that he wished both the authors had given fuller details of their experiments. Professor Smith had only given one torsion-curve. He did not think the Institution would have objected if all the curves had been given in the Papers, and even now he thought no objection would be raised if the actual readings were sent in a manuscript form to the library for purposes of reference. Although Mr. Mason had given twenty curves he had omitted thirty-three, and Professor Smith had mentioned that he had made many experiments, but he had not even given the figures relating to them.

In conclusion, he was highly delighted with the fact that the experiments on compound stresses were progressing. It was a matter he had advocated ever since he mentioned it in his book on "Boilers," published in 1893, and he was very glad to find that both the authors and Mr. Guest were now largely experimenting on the subject. The names which he had then proposed for the duplex compound stresses dealt with in this Paper were: Shearing Stress; Drum Tension, consisting of two tensions in one plane; Negative Drum Tension or Strangling Stress, consisting of two compression

stresses in one plane. The triplex compound stresses, with which no one had yet dealt, he had called Fluid Pressure, consisting of three compressions, and Fluid Tension, consisting of three tensions. Strange to say, fluids could resist both these compound stresses, but no others. Then there was a Draw Stress, consisting of a simple tension and a strangling stress; all materials which could be drawn into wire broke down easily under this stress. The last stress he had called a Bomb Stress, because it existed at the inner surface of an exploding bomb. It consisted of a radial pressure and a drum tension.

Communications.

Mr. GEORGE C. DOUGLAS wrote that it was interesting to note that the authors from their experience confirmed Guest's shear-stress theory of elastic failure. He believed he was correct in saying that most of the breakdowns in gas-engines had happened through the fracture of their crankshafts, and if the authors' experiments did nothing more than confirm the ideas which now obtained that gas-engine crankshafts should be made stronger than was thought necessary a few years ago, their work would not have been in vain. But work such as theirs and work by others as evidenced in previous Papers went much further, in so far that they provided data which could be safely used by themselves or others in determining, say, for instance, the ultimate state of matter. However, before saying anything about what might be done in the future, he would like to ask if a note was taken of the time during which each specimen was subjected to stress, and of the temperature. He had no doubt but that in their experiments the time devoted to each type of specimen was approximately the same and the temperature normal atmospheric. He had reason to believe that

(Mr. George C. Douglas.)

if two identical specimens were subjected to the same test, but that the one test was completed in the shortest possible time and the other extended over a very much longer period and with the temperatures widely different, that very dissimilar results would be obtained.

His reason for suggesting variable time and temperature tests was that engineers, like others who were interested in kindred sciences, were inclined to take concentrated views, with the result that frequently something vital was missed. For example: In these two Papers there were exact records of the behaviour of mild steel under combined stresses, in which was shown that the determining

FIG. 57.
Professor Ewing's H-loops.

p shows a minor loop due
to electrical stresses.

factor was shear. In Professor Ewing's classical writings on hysteresis there was the phenomenon of residual induction, which in a sense, and so far as the writer knew, might be in nature similar to shear-stress. The interesting loop *p*, Fig. 57, was analogous probably to loops he had obtained when testing cast-iron beams to breaking point, of which mention was made below.

Then again, Professor Dewar had found that electrical resistance decreased to 200° C. and then the curve changed; and, according to the writer's theory, at that temperature there probably would be less shear-stress and less hysteresis. Professor T. Claxton Fidler informed him (Mr. Douglas) lately that when experimenting with non-magnetic rotating overhung shafts, weighted so that the end was sensibly deflected, he found that the shaft vibrated in a straight

line across the axis, that is to say, the shaft when rotating at a high speed behaved as if it was a flexible shaft stiffened across a diameter by, say, a thin strip of tempered steel. From this it would again be seen that the question of shear-stress, residual induction, or the like, arose.

Some years ago—in 1885—the writer designed a shop machine for roughly testing to the breaking point cast-iron beams 36 inches by 2 inches by 1 inch, supported at both ends and loaded at the centre. The load was applied by means of a lever and a traversing weight. A cylinder, moving proportionally as the weight, carried an indicator cord, and a pencil operated by the bar marked the magnified deflection. With these bars the combined movement of the weight and the deflector pencil gave an approximately straight line to the breaking point. If, however, the gradual application of the weight was stopped before the breaking point, the deflection still continued to increase. If the weight was after a time increased, the pencil came back and continued the straight line, an open loop being formed. Fig. 58 (page 1294) showed the diagram. Figs. 59, 60 and 61 were tracings of other diagrams showing loops which, in a way, were like Professor Ewing's H-loops *p*. From these various electrical, rotational gradually applied and re-applied loads and the authors' combined stress experiments, it seemed as if there were some common relationship, and he believed if this question of internal stress were thoroughly investigated by the Institution, it would be of the utmost value to not only the engineering profession but to science generally. He would like to ask the authors, for the benefit of engineers, to give calculations and dimensions of a crankshaft for, say, a 250 h.p. gas-engine, the safe-stress per square inch being deduced from their observations. The dimensions of crankshafts for gas-engines, as made by a leading firm, were as follows: 140 h.p. single cylinder, main bearings, 9 inches diameter by $17\frac{3}{4}$ inches length, crank-pin, 10 inches diameter by 10 inches length; and 320 h.p. double cylinder, 160 h.p. each cylinder, main bearings 10 inches diameter by $19\frac{3}{8}$ inches length, crank-pins each 11 inches diameter by 11 inches length. He would like to ask the authors whether, in their opinion, these crankshafts were sufficiently strong.

(Mr. George C. Douglas.)

FIG. 58.

Transverse-Stress Diagram of a cast-iron beam subjected to a gradually applied load.

*At a the weight ceased travelling for 10 mins.
and at b for 80 mins.*

Breaking load 2804 lb.

Deflection 0.3"

FIG. 59.

Shows the Mechanical Hysteresis loops due to the travelling weight being made to move backwards and forwards, the bar breaking with a less weight than was previously applied.

Breaking load 3204 lb.

*First d o was marked, stress 3496 lb.
def. 0.425." Then loop of a, g o, then
of a b, k g o, then taken back to c,
and finally from c to breaking point
along dotted line.*

The Institution could effect a great work if a Committee composed of engineers (mechanical and electrical), physicists,

FIG. 60.
Shows one H-loop.

FIG. 61.
Shows two H-loops.

metallurgists, and the like, were formed to investigate the behaviour of metals under widely different conditions. For

(Mr. George C. Douglas.)

example: Suppose a testing-machine of the most approved type were selected and modified in one important particular, that is, instead of being placed on an ordinary foundation let it be set on a foundation which was insulated electrically, and let experiments be made with mild-steel bars all of the same quality. Then, let a set of bars be subjected to simple tensional rupture: (*a*) at a fairly high temperature, the bars being kept hot by a heated surrounding medium; (*b*) at a normal temperature as at present, the machine being thoroughly demagnetized; (*c*) at a low temperature by surrounding the bar with liquid air; (*d*) when magnetized, which could be done by surrounding the bar with a coil through which a powerful current was passed; (*e*) similar compressive and combined stress experiments. Work such as indicated above would give results interesting by themselves and which would inevitably lead to further developments.

Professor JOHN GOODMAN wrote that, although he did not entirely agree with all the conclusions arrived at by the authors of these Papers, yet he considered that they had done great service in calling the attention of engineers to the fact that when materials were subjected to combined stress they might, as indicated by Guest's law, fail by reason of the maximum shear-stress which was actually lower in intensity than the maximum tensile stress in the material, as indicated by Rankine's theory. He trusted that the authors would continue their researches upon other materials, since the results they had obtained probably only held with very soft ductile materials, such as mild steel; he suggested that they should work on three or four different grades of harder steel including a high carbon steel, since there was distinct evidence in support of the view that Guest's law did not hold with brittle materials such as cast-iron and probably hard steel, and that Rankine's law did very closely hold for such materials. One difficulty in getting stress data from cast-iron was that its strength was very irregular, and, moreover, it had no elastic limit. The nominal stress at the breaking load was usually termed the "modulus of rupture" when referring to bending or to torsion; if the ratio of the modulus of rupture to the real

breaking stress were the same in torsion as it was in bending, then there would be no great difficulty in determining approximately whether the stress conditions agreed with the Rankine or with the Guest hypothesis.

In another respect, however, cast-iron was a better material for combined stress experiments than a ductile material, in that it was easy to fracture and the angle of fracture was readily determined. One of the routine experiments carried out by students at the Leeds University was the testing of cast-iron bars in pure torsion, pure bending, and combined torsion and bending; in addition to calculating the "modulus of rupture," the angle of fracture was measured and within narrow limits it was found to agree closely with the angle calculated by the Rankine theory, but it was very different from that required by the Guest law, which was certainly strong evidence in favour of the Rankine hypothesis for this material. Unfortunately, this method could not be so readily applied to ductile materials, mainly because they became almost plastic before fracture and the stress distribution was no longer that indicated by the theory of elasticity.

The method of procedure used by the authors of the Papers now under discussion, namely, that of accurately determining the stress at which the elastic limit was reached when the bars were subjected to compound stresses, appeared to be quite satisfactory. The writer knew of two instances of failure in crankshafts and another instance of fracture in the main axle of a bicycle, all of which were broken by combined bending and torsion and which fractured at angles practically in accordance with the Rankine theory. If those who came across fractured crankshafts would either work out the results themselves or get others to do so, and send them to the Institution with a view to publication, the data would be of the greatest possible value in throwing light upon this important subject. Even if it could be proved that crankshafts did fracture along the Rankine spiral, it did not necessarily prove that the Guest law was wrong for even these identical crankshafts, because it was well known that when materials were broken by repeated stresses, the fracture always resembled that of a piece of brittle material,

(Professor John Goodman.)

although the same material when tested in tension showed great ductility—but far be it from the writer to suggest that there was a particle of truth in the old fallacy, that ductile materials turned crystalline by repeated stress or by vibration.

The question naturally arose with designers, "How can we, when designing machine parts which will be subjected to compound stress, determine whether we are to use the Guest equation or the Rankine equation? Unfortunately, we did not know, but the following suggestion might serve as a guide: When the shearing strength of a material was greater than the tensile strength, the Rankine equation should be used, but when the shearing strength was less than the tensile strength, the Guest equation held. The published values of the ratio between the shearing strength and the tensile strength were liable to mislead, for the simple reason that when a shear test was made the section was not reduced in area at the maximum load, but in a tension test of a ductile material the section was very greatly reduced; hence the true tensile stress was much greater than the nominal. When this correction was made, we got a much lower value for the ratio of the shearing strength to the tensile strength. Such values were given in the correspondence upon Mr. Izod's Paper.* Advocates of the Guest theory had for long urged and had backed up their opinions by experiments that the shear elastic limit of ductile materials was in the neighbourhood of one half of that in tension—the figures in the Table referred to appeared to show that the same relation approximately held even at the maximum load.

Professor Smith showed that with the spherical holders used by him the pull was not truly axial and in one instance, Fig. 45 (page 1269), the strain was about 0.0005 inch greater on the one side of his test-bar than on the other on a length of 2.05 inches, or about 0.0025 inch difference on a 10-inch length. The writer had made several tests with four extensometers on a test-bar 10 inches long, and had never recorded more than 0.0003 inch difference between any two sides. The writer did not wish to suggest that Professor

* Proceedings, 1906, Part 1, page 41.

Smith's results or tests were inaccurate, but out of justice to the makers of the testing-machine (both Professor Smith's and the writer's were by the same firm) he wished to point out that such a great divergence as Professor Smith obtained from a truly axial pull was not common to all such testing-machines. With regard to Vee grips, no one would ever think of using them for highly refined scientific testing, but for ordinary commercial testing they were incomparably superior to screwed ends as regards rapidity and convenience in handling.

The following results of tests made by combined torsion and bending on cast-iron bars might be of interest, since they showed a very close agreement between the Rankine angle and the angle as found by experiments:—

Angle of Fracture.

CALCULATED (Rankine hypothesis).	EXPERIMENTAL.	CALCULATED (Rankine hypothesis).	EXPERIMENTAL.
Degrees.	Degrees. (Pure torsion)	Degrees.	Degrees.
45	43	21	22
42	40	20	19
41	37	19	16
40	42	18	19
39	40, 42	17	15
38	37, 40, 39, 41	16	18, 17
37	38, 38, 35, 36	15	16, 15, 15
36	33	14	14
33	31	13	17
32	35	12	11
31	33	11	12
29	33	0	(Pure bending)
28	27		0

(Professor John Goodman.)

The stress results did not hold nearly so well, largely due to irregularities in the strength of the material. Some bars tested in pure torsion gave a "modulus of rupture" of 12 tons per square inch, and some as high as 22 tons per square inch; likewise those tested in combined torsion and bending varied from 13 to 23 tons per square inch, but the variation in the strength of the bars did not affect the angle of fracture.

Mr. G. H. GULLIVER wrote that the very interesting results obtained by Mr. Mason and Professor Smith, when taken in conjunction with the previously published work of the latter and of other writers, had settled definitely an important point. There could be no more doubt that the yielding of ductile mild-steel occurred when the maximum shearing-stress reached a certain value, to an approximation amply close for modern engineering calculations. The authors were to be congratulated upon this important confirmation of the earlier work of Guest, which was foreshadowed to some extent by that of Darwin and Carus-Wilson.

From the physical point of view, it was a question how far one was justified in applying a theory, in which perfectly homogeneous and isotropic bodies were contemplated, to a piece of metal which was an assemblage of crystal aggregates of irregular size and shape, and of random orientation. From the experiments of Ewing and Rosenhain, and of others, it was known that the permanent deformation of a ductile metal was due chiefly to sliding along the crystalline cleavages. The surfaces of sliding were therefore irregular and discontinuous. But, since the resistance of the mass to deformation, except under some special circumstances, was sensibly the same in all directions, it was not unreasonable to expect that the yield-point would be a function of the calculated maximum shear-stress, and that, if microscopic irregularities were neglected, the general surfaces of sliding would be inclined at 45° to the directions of maximum and of minimum principal stress, and would be parallel to the intermediate principal stress. According to Hartmann and the present writer, however, the general surfaces of sliding were inclined to the maximum principal stress (tension) at an

angle greater than 45° , and to the minimum principal stress at one correspondingly less than 45° , while remaining parallel to the intermediate principal stress. The inclination of the general surfaces of sliding to those of theoretical maximum shear-stress, though usually small, was quite definite, and was large enough to show that the simple shear-stress theory, however satisfactory from the practical standpoint, was not a perfectly accurate representation of the facts.

Some further explanation of these facts was required. The somewhat crude internal-friction theory did not give a satisfactory solution, and it seemed unlikely that a more complete theory on the same lines would do so either. Remembering the random orientation of the crystal aggregates, it was clear that but few could be so placed as to have even one set of cleavage planes in the direction of maximum sliding tendency. Sliding along a cleavage plane with the most favourable direction was resisted by neighbouring aggregates in which the cleavages had less favourable directions, so that the yield-point of a mass of metal was higher than that of an isolated crystal aggregate with the most favourable orientation of cleavage planes. If the actual direction of maximum sliding tendency were inclined at approximately 45° to the direction of maximum principal tension, the orientation of the various sets of cleavage planes would deviate from this direction in roughly the same manner on each side. The calculated shearing stress was the same for planes having the inclinations $(45^\circ + \theta)$ and $(45^\circ - \theta)$ to the direction of maximum principal tension, but the normal tension on the former was greater than that on the latter in the ratio of $\sin^2 (45^\circ + \theta) / \sin^2 (45^\circ - \theta)$. If there were any frictional resistance to motion along the cleavage planes—and there must be some such resistance—sliding would take place more easily along cleavages inclined at an angle greater than 45° than along those inclined at less than 45° to the direction of maximum principal tension. The general direction of sliding would give an indication of the average deviation from the direction of maximum sliding tendency of the most favourably situated cleavages in each crystal aggregate, if the direction of maximum sliding tendency differed but little from that of maximum shear-stress.

(Mr. G. H. Gulliver.)

Mr. Mason's observations of the lines of Lüders which appeared upon his specimens were interesting. The writer would point out, however, that a helical line, such as one of those shown in Figs. 10 to 13, Plate 31, was not, mathematically speaking, the trace of a helical surface, but was the extreme member of a family of helices, all of which had the same inclination to lines parallel to the axis of the tube. The pitch of each helix varied directly as the radius of the cylindrical surface upon which it lay, and the pitch therefore decreased from the outside to the inside of the tube. The helical lines of Lüders upon the outside and the inside surfaces of the tube should have the same inclination, but should differ in pitch, and should not be, therefore, exactly opposite to each other throughout their length. Would Mr. Mason examine his specimens more carefully in order to see how nearly this condition was realized?

The formula for the internal-friction theory, given on page 1238 of Professor Smith's Paper, was that which would apply to the compression side of a crankshaft. According to the theory, the tension side of the shaft would yield under a less load than the compression side, other conditions being the same. To determine the safe values of p and q , the minus sign of the formula should be changed to a plus sign, C_4 being a constant for any particular material. Professor Smith's use of the word homogeneous, though intelligible enough, was not exactly in accordance with its scientific meaning.

Mr. WILLIAM MASON wrote that he thought it would have added greatly to the usefulness of Professor Smith's Paper if he had given a number of the stress-strain curves for his tests, instead of merely three, only one of which belonged to the specimens tabulated. In nearly all of Professor Smith's tests there was considerable variation of stress in the specimen; in the torsion tests of course the shear-stress on horizontal planes varied from 0 to a maximum at the surface. Thus, to use Professor Smith's words, the yield-point was "reached at one portion of the specimen before the whole of it yielded." What was recorded was the maximum stress on the specimen "when the first portion of it reached the yield-point." Even with the most delicate of strain measurers, it surely must be

a difficult matter to locate the point when perfectly elastic strain ended and partial non-elastic strain commenced. Indeed, two independent observers might very probably fix upon points at a considerable stress-difference apart. Even if the first appearance of a time-effect were carefully looked for, the effect took place primarily on such a minute volume of the specimen, the remainder being elastically strained, that it was very doubtful if the first observed appearance would coincide with the elastic breakdown at the skin of the specimen. If the material were not homogeneous (it did not appear that Professor Smith annealed his specimens), it was obvious that the above remarks applied with greater force.

With regard to those tests in which part or the whole of the stress was directly compressive, Professor Smith said, "With an extensometer which gives the distribution of stress, there will be an apparent rapid change in the eccentricity of loading directly any portion reaches the yield-point." Supposing that were so, then during this apparent rapid change of eccentricity, the stress at any point (except, perhaps, at those localities which had yielded) was varying rapidly although the load was the same; again, how was the exact point ascertained at which to calculate the yield-stress at the "skin"? If Professor Smith would give a sufficient number of curves as samples, the limits of error of his yield-stresses might be estimated, and his results could then be applied the more confidently to test Navier's friction theory.

In combined stress experiments, it appeared to him (Mr. Mason) more fair, and to promise results of greater accuracy, to compare stresses at the general yield of a whole specimen (as could be done with tubes as specimens) than to trust to yield-stresses over very small volumes or at the "skin" of a specimen.

[Since the above was written, Professor Smith had published (*Engineering*, 7 January 1910, page 11) the torque-strain curves of certain of his specimens. If these, with samples of his curves for compression and tension, were appended to Professor Smith's Paper, greater convenience for future reference would be gained.]

M. F. OSMOND wrote that he had no observations to offer on the substance of Professor Smith's Paper. He only wished to make

(M. F. Osmond.)

a few remarks on the subject of the limit of elasticity. The author said (page 1247) that all metals, with the exception of mild steel, had an elastic limit distinct from a yield-point, which was true, and he concluded from it that mild steel was particularly homogeneous. In fact, mild steel was the most heterogeneous of the common metals. Not only was it normally formed of two very different constituents, ferrite and pearlite, but it also contained variable proportions of sulphides, oxides, and silicates, while coppers and sheet brasses, for example, were often of great purity. It was necessary, therefore, in order to explain the experimental facts, to seek a different cause.

All metals, including mild steel, were composed principally of crystalline grains, each of which possessed its own orientation. Now, if a smooth test-piece were strained, it was seen, by the appearance of the slip-bands, that all the grains were not strained under the same load. This was still clearer, if one experimented, as M. Fremont proposed, on a conical smooth test-piece, and if one submitted this specimen to a load so that the resulting roughening of the deformation did not reach the greatest diameter. It was then found, with the aid of a microscope, that there was a rather large distance between the circle which showed the first rough isolated grains and that in which all the grains were strained. The value of the elastic limit depended, then, in crystalline bodies, upon the crystalline orientation in comparison with the direction of the stress and, if one had crystals large enough from which a test-piece could be cut off, one would certainly find, provided the load was directed exactly following the axis, a well-defined elastic limit for a given orientation and variable with the orientation.

Mild steel which was, like the other metals, an aggregate of crystalline grains (omitting pearlite) should behave like them and give an indistinct elastic limit, since the grains were oriented at random. Since it was not so, one was led to conclude that the yield-point of mild steel was a phenomenon peculiar to iron and quite different from the elastic limit of other metals; it might be attributed to a partial allotropic transformation. A part of

α iron would be transformed, under a given load, into γ iron whose elastic limit would be exceeded at the moment in which it was formed. But, whatever the explanation might be, the fact remained, and it was corroborated by this other fact that the slip-bands were not rectilinear on the iron as on other metals, and were not due, in consequence, to deformations of a crystalline nature. The crystalline limit of elasticity of iron existed none the less; only, it was higher than the yield-point. Now, the crystalline deformations were instantaneous, while the yield-point was subject to retardation. It happened therefore that, in making the experiment more rapidly, the load was increased under which the yield-point was produced. One could even suppress this yield-point, if the application of the load was exceedingly rapid: which might happen in the case of shocks and vibrations. Then the crystalline elastic limit of iron was attained: the slip-bands became straight (Ewing and Humfrey); but, as the crystalline iron had a plane of cleavage, the deformation entailed sudden rupture.

One would perhaps object that the rupture was produced in these conditions when the maximum load had remained constantly below the yield-point. But this was only apparent. The calculation of the load was made from the ordinary formulae of statics, and these formulae were not applicable here. Nobody knew exactly what instantaneous stress corresponded to a shock, and nobody could foresee the local maximum resulting from the meeting of mechanical waves. When the rupture was produced, one must remember that the load of rupture had been exceeded.

Mr. WALTER A. SCOBLE wrote that he considered Mr. Mason's Paper to be extremely valuable, but he suggested that the principal stresses should be given in the Tables of results. Mr. Mason's work had confirmed his conclusion that the shearing strength varied in different directions, and that this fact explained the differences in the shear-stress at yield, which were formerly attributed to internal friction. Mr. Mason's method of testing approximated to the practical examples of brittle materials under combined stress; it was, therefore, to be hoped that he would extend his tests to include brittle materials.

(Mr. Walter A. Scoble.)

Professor Smith appeared to believe that ductile materials exactly followed the shear-stress law, and he had set out to confirm this. Undoubtedly the shear-stress law represented the average behaviour of ductile materials, such as mild steel, and, as a first approximation, was an hypothesis from which formulae might be deduced for the designer. No doubt a material might be found which would exactly follow the law, but it would be exceptional. The writer emphasized this point, because Professor Smith had dealt with the case of the crankshaft. So far as he (Mr. Scoble) was aware, he was the only observer who had employed bending and torsion, and the bending moment was always greater than the torque at the elastic breakdown in the simple tests. On the solid steel specimens, tests were made under bending alone, simple torsion, and under combined loadings to fill uniformly the gap between these extremes. The shear-stress at failure increased steadily from 29,170 lb. per square inch in torsion to 33,500 lb. per square inch under bending, and similarly the maximum principal stress increased from 29,170 to 64,600 lb. per square inch. Professor Smith estimated the average variation from the mean to be, for the shear-stress theory 3.72 per cent. and for the maximum principal stress theory 18.3 per cent., figures which must have misled his readers. In a note, Professor Smith stated that recent results from mild-steel specimens approximated so closely to Guest's law, that it might be stated that Navier's theory need not be considered for the design of crankshafts. He (Mr. Scoble) had shown that this theory was wrong.*

Professor Smith had remarked that he (Mr. Scoble) had suggested that independent experiment was desirable to test the magnitude of the error due to eccentric loading. He wished to make it clear that he did not suggest the method described by Professor Smith.

Mr. A. G. WARREN wrote that most of the previous work which had been done in connection with this subject had been on specimens subjected to shear and tension. The authors rightly devoted considerable time to the consideration of the failure of materials

* *Phil. Mag.*, December 1906, and *Phys. Soc.*, November 26.

under shear and compression. Their methods of loading differed considerably. Mr. Mason had used tubes, and adopted direct stress and internal pressure as the method of loading; Professor Smith had used solid specimens under direct stress and torsion. Why internal pressure should be so popular he (the writer) could not quite understand. First, it could not be easier to apply than torsion, and, secondly, it necessitated the use of hollow specimens. In those used by Mr. Mason he very much doubted whether the direct stress was uniformly distributed over the section. Further, tubes were not isotropic and could not be rendered so by annealing.

Mr. Mason described a method (page 1208) which he adopted for uniformly distributing the load. It was pleasing to note that he recognized the necessity for doing this, but at the same time it must be stated that the method used was a great refinement. Had every other precaution been taken, it would have been unwise to have neglected this, but with the apparatus shown in Fig. 3, Plate 30, it could hardly be expected that the load would be axial. The conical point shown could transmit a considerable bending moment, and the author's curves would also indicate non-uniform stressing. This led to another very interesting point. The instant at which the first portion of a material yielded must be noted. It required a larger stress to cause "slipping" to start than it did to continue plastic flow: hence the yield-point as determined by a sudden increase in the strain was incorrect if the stress was not uniformly distributed. This fact also allowed a definite yield-point in a solid specimen, even though the load might be very varying. Another point in favour of solid specimens was that, with varying stress, lateral plane sections remained plane, but not so in the case of tubes. This meant that for solid specimens the stress distribution could be calculated if suitable measurements were taken. For tubes this was practically impossible.

Mr. Mason wrote, in reply to the discussion, that he agreed with Mr. Guest's criticism (page 1279) of the diagrams on pages 1220-2, that the point L (*see* Mr. Guest's Fig. 54, page 1279) ought properly to be reserved for *hoop* tension, and the point E for *hoop* compression;

(Mr. Mason.)

and the quadrant A E for axial tension and hoop compression. But he did not think that a false impression of the work actually done was conveyed by the diagrams, since it was clearly stated on the diagrams what the points marked actually represented. He had attempted to obtain the yield-point in hoop tension, but he had, so far, been unsuccessful in obtaining satisfactory determinations. He had in hand at the present time the determination of the yield-points for the above-mentioned conditions of stress. These experiments were not easy to make; but the results were important inasmuch as they would, with similar results to those already obtained, give evidence of the state of isotropy of the specimens. Since he had been unable to obtain this evidence, he had had to rely upon a thorough examination of the specimens in order to detect any of the flaws of the large size mentioned by Mr. Guest. The appearance of the specimens after rupture in tension gave no indication of flaws of any kind.

As mentioned in the Paper, he had been unable to obtain tubes of the large size required of uniform thickness. Some did not vary more than $\frac{1}{1000}$ inch, but the majority varied $\frac{4}{1000}$ or $\frac{5}{1000}$, and the maximum variation was $\frac{7}{1000}$ inch (3.5 thousandths each way from the mean). The method of applying the compressive load allowed this eccentricity to be compensated, so that uniform distribution of stress was obtained. In the tension experiments the greatest variation of $\frac{5}{1000}$ inch (2.5 thousandths from the mean) occurred in specimen No. 1b; the resulting eccentricity of pull was 0.024 inch, and the corresponding variation of stress 3.2 per cent. from the mean. If Mr. Guest's tubes Nos. IV to IX* had varied $\frac{1}{1000}$ inch in thickness, the stress variation would have been 2 per cent. from the mean, this percentage being more in proportion on account of the smaller diameter of the tubes.

Mr. Stromeyer expressed his regret that sufficient data for a complete curve (see Fig. 56) had not been obtained (page 1288). It was his (Mr. Mason's) intention to continue the work, and if possible to get some complete curves, though the experiments already reported had been very laborious.

* *Phil. Mag.*, July 1902.

In reply to Mr. G. C. Douglas's enquiry (page 1291), the duration of the experiments was not noted, nor the temperature. The time of making a test varied from twenty to thirty minutes, more than half of this time always being occupied by observations near the expected yield-point.

Professor Goodman's Table (page 1299) of angles of fracture of cast-iron under torque and bending was very remarkable and most interesting. It gave confirmation of Mr. W. A. Scoble's results of similar experiments,* and was a proof that the fracture of cast-iron under such stressing was due either to direct tensile stress or strain.

A point was raised by Mr. G. H. Gulliver (page 1302) concerning the inclination of certain of the Lüders lines which, appearing on both the inner and exterior surfaces of a tube, were apparently traces of the same surface of slipping. It was mentioned in the Paper that, in several cases, these lines were on the same spiral surface. This statement was based upon the result of measurement of the angles by wrapping paper on the inner and outer surfaces. As nearly as could be ascertained, these inner and outer lines were spirals of the same pitch and of different inclination. He (Mr. Mason) wished to supplement a statement he had made in the Paper (pages 1216 and 1217). It was stated that if there was any change in the angles of the Lüders lines when the ratio of the longitudinal compressive and hoop tensile stresses was altered from unity to 2 and 0.5 respectively, the change was very small. Further experiment had shown a small though distinct change even for the ratio 2; with ratios increasing from 2, the inclination to a generator became greater, until with a simple compression the inclination was about 51° . This result was of course opposed to Navier's theory, and also to the constant shear-stress theory of elastic breakdown.

In response to Mr. Scoble's suggestion (page 1305) the tensile and compressive stresses given in the Tables were the greatest and least principal stresses at the inside of the tube, the part most

* Proceedings Physical Society, Vol. 20, Part V.

(Mr. Mason.)

severely stressed. The other principal stress, equal at this locality to the hydrostatic pressure, if any, was, of course, very small.

Mr. Warren (page 1307) was of the opinion that a bending moment would be transmitted through the area of the cone used for obtaining the compressive stresses, even though the cone were set axially coincident with the centroid of the sectional area of a tube. Such moment could only be introduced by a tendency to lateral movement, unresisted in the machine itself, between the upper and lower compression blocks of the testing machine. Several times, when making compression tests, he (Mr. Mason) took pains to ascertain whether the conical cap remained in exact position during the squeezing down of the thin lead ring inserted between the cap and the tube; the cap always kept its exact position. Mr. Warren was also of opinion that the yield-points would be affected by an error due to the alleged fact that (expressed in Mr. Warren's words) "it required a larger stress to cause slipping to start than it did to continue plastic flow." He (Mr. Mason) considered this statement to be incorrect. In commercial testing it was often necessary to reduce the load on a mild-steel tension specimen, in order to keep the beam floating during the large straining at yield. But he had found no drop at yield in the stress-strain curves for several specimens in which the extensometer was kept in operation until the end of the yield-stage; indeed, this experience was a most common one. In a commercial test the loading was done quickly, and it was most probable that the small extra stress (over and above that which continued the plastic flow) did what was accomplished by the longer time in the more exact and slower tests.

"Mr. Warren remarked vaguely that a "point in favour of solid specimens was that, with varying stress, lateral plane sections remained plane, but not so in the case of tubes." He (Mr. Mason) would have been glad if the point had been definitely stated; it was difficult to understand what Mr. Warren had in contemplation when he spoke of lateral plane sections.

Professor C. A. M. SMITH wrote that, in reference to Mr. Guest's remarks (page 1278), he would point out that the diagram, Fig. 54

(page 1279), drawn by Mr. Guest, did not have any real application to the experiments made at the East London College. In the loading adopted by Mr. Mason the axes of principal stress were constant and were geometric axes. This was not the case with the loading employed with torsion and direct load. In the tests made by Mr. Mason there were only two sets of surfaces of maximum shear, namely, two families of 45 degree helices and two families of 45 degree cones. It would be noticed that in his (Professor Smith's) case the surfaces were different for different ratios of the loads.

It must not be assumed that Mr. Guest's check for isotropy was complete, because the planes of maximum shear for the tests remained the same. This check was considerably discounted, if it were once established that the material used approximately obeyed Guest's law. The nearer the material approximated to Rankine's law the better was this test for isotropy. Therefore Mr. Guest must not claim that he obtained a complete check for isotropy. Mr. Mason did not mention his method of checking.

Mr. Guest was evidently slightly confused on one or two points in the Paper; but at the meeting time did not permit the author going over points discussed in other Papers, references to which were supplied all through the text of the Paper. Mr. Guest stated that the members might not understand the purpose of the experiments, but he (the author) considered that was fully explained in the Paper. Suffice to say, now, that his object was to prove definitely the case for the crankshaft. Mr. Guest had made experiments on tubes which suggested that the design formula for crankshafts should be changed. Lord Kelvin, on the other hand, made a statement which really meant that the shear-stress theory of elastic failure was not correct. The experiments carried out by the author and described in the Paper, if accepted as accurate, proved, not that the shear-stress theory was *approximately correct*, but that it was, for the S S mild steel used, correct to within very narrow limits. Mr. Guest's approximate results obtained with tubes were confirmed, and results obtained on specimens under combined compression and torsion proved the shear-stress theory.

(Professor C. A. M. Smith.)

Mr. Guest was favourable to the use of tubes, as they were essential to the scheme of his own experiments. The author had made many rough experiments with tubes, and found the drawbacks enumerated in the Paper. The tubes used by Mr. Guest were made in America, and Mr. Guest acknowledged that the English-made tubes contained—only for the purposes of such compound-stress experiments—blemishes (such as a variation of $\frac{7}{1000}$ inch in thickness), from which American-made tubes were free. Tubes were probably satisfactory for quite approximate results—say, within 10 per cent.—but he (Professor Smith) had aimed at as great an accuracy as possible in order to test Navier's friction theory, which approximate results did not test. Many of the objections to tubes disappeared entirely if it were considered that within 10 per cent. was good enough. Considering the labour and expense of any experimental work on compound stress, he thought it was worth aiming at much greater accuracy. For accurate work would reveal any tendency to follow Navier's theory. The tests on the SS specimens gave experimental results, all of which almost coincided with the shear-stress curve.

Whilst on the subject of tubes, he would mention that as for all experiments on tubes certain results had to be discarded, that fact alone pointed to a variation in the material and lack of homogeneity; that the elastic breakdown of a tube in compression could not be located with any certainty (Mr. Guest's work did not include the compression test, so essential for the crankshaft formula), because of secondary flexure; that heat treatment of a portion of the tube also slightly affected the elastic properties of any other portion of the tube; that when Mr. Guest happened by chance to hear of the work at the East London College, he was emphatic in his praise for solid specimens, instead of tubes, for the compression-torsion tests. It might be further stated that, as it was desired to compare the elastic properties of non-ferrous metals with those of steel (*see* Institute of Metals Paper, 1909) it was useless to experiment with tubes. He (the author) knew quite well the limitations and the difficulties involved by using solid specimens. But they offered a solution of the

practical example—the crankshaft—in a manner which results with tubes could not supply.

There were certain cases of compound stress which could be approximately checked if tubes were used. Such checking was of scientific value, and he admired—and would venture to emphasize—the careful and able manner in which such work had been done by Mr. Guest and others. The author's experiments did not touch that problem at all, because he was anxious to experiment on the practical issues. Concerning the bending in torsion, Mr. Guest stated at the meeting that he took precautions with the adjustment of the stops to prevent this bending, and yet many of his experiments were marked to indicate that bending was noticed. It was a matter of great satisfaction to find Mr. Guest, after a somewhat lengthy silence, again taking an active part in the discussion of this subject. It was to be hoped that he would continue to take an interest in such research work. It must be understood that any experimental work done now, or in the future, could not detract from the genius revealed by the now historic tests made by Mr. Guest. He would probably be the first to acknowledge that it was essential to obtain the compression-torsion results to be quite sure that the crankshaft formula $L = \sqrt{M^2 + T^2}$ was the correct one to use. Until his (Professor Smith's) rough tests on compression combined with torsion, made some time ago, showed him that Lord Kelvin's theory did not apply, he was not an advocate of Guest's law. Since then he had been convinced of its approximate correctness, and now believed it almost strictly accurate for the S S steel used in the experiments. The statements concerning tubes were made partially with the object of extracting a contribution to the discussion from so eminent an experimenter as Mr. Guest, and also because it was of some importance in the matter of all tests.

Mr. C. E. Stromeyer's diagram, Fig. 56 (page 1287), was difficult to follow. Surely the curves should be straight lines. Further, it was assuming a good deal to draw in a curve (as M) when only two points were known. However, the method of plotting was most ingenious, and Mr. Stromeyer deserved admiration for inventing it. All of his (the author's) results would be so plotted in future. Mr. Stromeyer

(Professor C. A. M. Smith.)

appeared to think that the "far-reaching conclusions" drawn in the Paper were arrived at from the S.C. tests. They were, on the contrary, obtained chiefly from the SS tests, but also from all the tests mentioned. Assuming that the shear-stress theory was true for three-dimensioned stresses (as Mr. Mason's tests seemed to indicate), it was merely the difference between the algebraic maximum and minimum principal stresses which mattered. This, of course, led to the conclusion that a hydrostatic stress would not cause a body to yield. That point would be elaborated shortly in *Engineering*. The range of two-dimensioned stresses was covered by the results in his (Professor Smith's) Paper, and the deductions made by Mr. Stromeier when he said that the theory was wrong did not obtain, if the three planes of principal stress were considered.

Concerning the sphingometer, Mr. Stromeier's remarks were a direct confirmation of his own experience. Mr. Stromeier would see that the curves were now included in the Paper, and that, in the Paper, the author mentioned how he (Mr. Stromeier) had urged experimental work.

In reply to Mr. Douglas (page 1291), the author would refer him to the various footnotes in the Paper. He had discussed in detail the design of crankshafts, and the references showed where the details were to be found. The time and temperature effects were most important, but they demanded special experiments such as Mr. Douglas suggested. His suggestion for a Committee was the first printed statement of a wish that he (Professor Smith) had been working (with the hope of realizing) for some time.

Professor Goodman had made a capital suggestion (page 1297) that every fractured crankshaft should have the angle measured. Surely, however, when the material was overstrained, it hardened and became a different type of material. The designer of machines would always be on the safe side by using the Guest equation, and, therefore, if any doubt existed as to whether a material was ductile or brittle, it was best to use Guest's law and to be on the safe side. Unless further details of Professor Goodman's experiments were forthcoming, it was not possible to say why he did not obtain more eccentricity in loading. That there could be considerable eccentricity

(even with spherical seats) from a theoretical deduction was, he hoped, proved.* Concerning Vee-grips for commercial purposes, why should not such grips be carried on spherical seats? The results of tests on cast-iron bars were indeed of great value.

Mr. Gulliver's remarks (page 1300) opened up a field of great possibilities. His suggestions on the work published by the author had been stimulating. The best way of replying to this most useful contribution was to embody the suggestions in more experiments. Most of the statements were lucid and original.

Mr. Mason (page 1302) would see that the curves had since been added to the Paper. With mild steel there was a very definite slipping when the first point reached the yield. This was only possible because of the fact that the stress immediately after yield was considerably reduced. With regard to the point on the curve which showed most accurately elastic breakdown, that was a matter of opinion. It was of interest to note, however, that Mr. Scoble had recently revised his method of determining the point of elastic breakdown. It was probable that this new breakdown point or Mr. Scoble's coincided with that shown for the SS specimens.

Mr. Osmond, it would be seen, accepted the fact that mild steel had a distinct elastic limit. His communication (page 1304) was regarded by the author as a great compliment. The prominent part which this world-renowned investigator had taken in the discussion of the early Reports of the Alloys Research Committee had cleared up many obscure points and had been of great value to English engineering. The cone experiment was very ingenious, and the statements concerning the causes of yield-point for ductile materials and fracture for brittle materials were most valuable.

In reply to Mr. Walter A. Scoble (page 1306) he was aware that the latter's valuable experiments had, of course, disproved Navier's theory, but he (Professor Smith) desired to test the matter by an independent method. Mr. Scoble complained that "Professor Smith appeared to believe that ductile materials exactly followed the shear-stress law, and he had set out to confirm this." On the

* *Engineering*, 10th December 1909.

(Professor C. A. M. Smith.)

contrary, no such idea concerning ductile materials obsessed his mind. He believed (as a result of his own and other people's experiments) it true for mild steel for all practical purposes. Experiments were required for other materials. The author's attitude to other observers apparently required to be defined. He had, wittingly, only depreciated one set of experiments made in America. Mr. Scoble had never defended these experiments, and he (Professor Smith) challenged him to do so. All of the work done by other investigators contained most trustworthy results, which were but slightly affected by the various matters mentioned by the author at various times, and which he considered were small blemishes. It was probable that his own work contained similar blemishes, and criticism on such details assisted them to be eliminated in future tests.

In conclusion, he wished to thank those who had so kindly taken part in the discussion. He was especially sensible of the kindness of the authorities of the East London College, who had provided the facilities for the research work.

Fig. 32.
Section of
Sphingometer
strip.

- R. _ _ Twisted Strip.
M. _ _ Mirror.
K. _ _ Micrometer.
T & T'. Casings.
L. _ _ Sliding bush to
which strip is fastened.
U. _ _ Set-screws.
O. _ _ Feather key.
N. _ _ Spring.

- A. Specimen.
C. Carriers.
D. Sphingometer strips.
T. Torsion
M. Micrometer heads.
Z. Zero mirror.

Figs. 33 and 34. Improved Sphingometer
filled with torsion as well as tension strips.

Fig. 35. Photomicrograph Pl. 32.
of Steel (SS). $\times 100$ diams.