PROCEEDINGS
OF
THE LONDON MATHEMATICAL SOCIETY.

VOL. XXXI.

THIRTY-FIFTH SESSION, 1898-99
(since the Formation of the Society, January 16th, 1865).

Thursday, April 13th, 1899.

Lt.-Col. A. J. C. CUNNINGHAM, R.E., Vice-President,
in the Chair.

Ten members present.

The following were elected members of the Society:—Prof. Benjamin F. Finkel, Drury College, Springfield, Missouri, U.S.A.; Henry Thomas Kelsey, M.A. Camb., The Grammar School, Leeds; Prof. Edgar Odell Lovett, M.A., Ph.D., University of Virginia, U.S.A.; Arthur Lionel Pedder, M.A., Fellow and Tutor, Magdalen College, Oxford; and Arthur John Wade-Gery, B.A., Assistant Lecturer in Mathematics, University College, Cardiff.

The Chairman briefly alluded to the recent decease of Prof. Sophus Lie, who was elected an honorary member of the Society, May 9th, 1878.

Mr. Kempe having taken the Chair, Lt.-Col. Cunningham read a note on "Conformal Division." The Chairman, Major MacMahon, and Messrs. Lawrence and Western took part in a discussion of the Note.

The following papers were communicated in abstract:—

Note on the Characteristic Invariants of an Asymmetric Optical System: T. J. P.A. Bromwich.

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B

Concerning the Four Known Simple Linear Groups of Order 25920, with an Introduction to the Hyper-Abelian Linear Groups: Dr. L. E. Dickson.

(1) On the Direct Determination of Stress in an Elastic Solid, with application to the Theory of Plates; (2) On the Stress in a Rotating Lamina; (3) The Uniform Torsion and Flexure of Incomplete Torsos, with application to Helical Springs Mr. J. H. Michell.

The Theorem of Residuation, Noether's Theorem, and the Riemann-Roch Theorem: Dr. F. S. Macaulay.

Impromptu communications were made by Messrs. Hargreaves, Heppel, Roseveare, Western, and Lt.-Col. Cunningham.

The following is Mr. Roseveare's communication:—

Notes on an Elementary Proposition in Geometrical Conics.

1. The following proposition is an easy deduction from Taylor's proof of the diameter property of a general conic:—If O is the middle point of a chord of any conic, and OG is drawn at right angles to the chord, meeting the axis at G , and if OI is perpendicular to the directrix, then $SG = e^2 \cdot OI$.

2. Hence, if N is the projection of O on the axis, and if the suffixes 1, 2 refer to two chords, $G_1 G_2 = e^2 \cdot N_1 N_2$. Now, if the direction of the axis is known, $N_1 N_2$ is known; and, if three points on the conic are given, the relation $G_1 G_2 : G_2 G_3 :: N_1 N_2 : N_2 N_3$, i.e., in a constant ratio, enables us to draw the axis.

3. Hence, if 1, 2, 3, 4, 5 are points on a conic, and a, b, c are the centres of the circles 123, 124, 125, it can be proved that $ab : bc$ in the ratio of the projections of the chords 34, 45 on the line which is inclined to the axis at the same angle as the line abc .

4. Hence, when five points are given, the direction of the axis of the conic through them can be found by an easy geometrical construction; and the axis, focus, &c., follow without difficulty.

The following presents were made to the Library:—

Heusch, F. de.—“Cours d'Analyse,” “Calcul Differentiel,” 8vo; Bruxelles, 1898.

“The Nautical Almanac for 1902,” 8vo; Edinburgh, 1899.

“Comptes Rendus et Mémoires de la Société des Naturalistes à l'Université Impériale de Varsovie” [(Russian.) Biology, 1897; Physics and Chemistry, 1898]; Warsaw, 1898.

Frolov, M.—“La Théorie des Parallèles démontrée rigoureusement,” 8vo ; Paris, &c., 1899.

“Mittheilungen der Mathematischen Gesellschaft,” Bd. III., Heft 9 ; Hamburg.

“Periodico di Matematica,” Serie 2, Vol. I., Fasc. 5 ; “Supplemento al Periodico di Matematica,” Anno 2, Fasc. 5, 6 ; Livorno, 1899.

“Wiadomosci Matematyczne,” Tom III., Zeszyt 1, 2 ; Warsaw, 1899.

“Memorias de la Real Academia de Ciencias,” “Sobre o Desenvolvimento das Funções en Série,” F. Gomes Teixeira, Tomo XVIII., P. 1 ; Madrid.

“Memoirs of the National Academy of Sciences,” Vol. VIII. ; Washington, 1898.

Queen’s College, Galway, “Calendar, 1898–1899,” 8vo ; Dublin, 1899.

“Educational Times,” April, 1899.

“Indian Engineering,” Vol. XXV., Nos. 7–11, Feb. 18–March 18, 1899.

The following off-prints have been presented to the Library by Dr. L. E. Dickson :—

“On the Inscription of Regular Polygons” (*Annals of Mathematics*, 1894).

“A Quadratic Cremona Transformation defined by a Conic” (*Rendiconti del Circolo Matematico di Palermo*, July, 1895 ; the *American Mathematical Monthly*, 1895).

“Analytic Functions suitable to represent Substitutions” (*American Journal of Mathematics*, Vol. XVIII., 3 ; 1895).

From the *Bulletin of the American Mathematical Society* :—

“Systems of Continuous and Discontinuous Simple Groups” (Vol. III., pp. 265–273).

“Higher Irreducible Congruences” (Vol. III., pp. 381–389).

“Orthogonal Group in a Galois Field” (Vol. IV., pp. 196–200).

“Systems of Simple Groups derived from the Orthogonal Group” (Vol. IV., pp. 382–389).

“The Structure of the Hypo-Abelian Groups” (Vol. IV., pp. 495–510).

“Concerning a Linear Homogeneous Group in $C_{m,g}$ Variables Isomorphic to the General Linear Homogeneous Group in m Variables” (Vol. V., pp. 120–135).

“A Triply Infinite System of Simple Groups” (*Quarterly Journal of Mathematics*, No. 114, pp. 169–178).

“The First Hypo-Abelian Group Generalized” (*Quarterly Journal of Mathematics*, No. 117, pp. 1–16).

“The Analytic Representation of Substitutions on a Power of a Prime Number of Letters, with a Discussion of the Linear Group,” with corrections (*Annals of Mathematics*, 1897, pp. 65–143).

“The Quadratic Cremona Transformation” (*Proceedings of California Academy of Sciences*, February, 1898—with this are stitched up “On Rational Quadratic Transformations,” and “On Curvilinear Asymptotes,” M. W. Haskell).

“Systems of Simple Groups derived from the Orthogonal Group” (*Proceedings of California Academy of Sciences*, March, 1898).

The following exchanges were received :—

“Proceedings of the Royal Society,” Vol. LXIV., Nos. 409, 410 ; 1899.

“Beiblätter zu den Annalen der Physik und Chemie,” Bd. XXIII., St. 3 ; Leipzig, 1899.

- "Rendiconti del Circolo Matematico di Palermo," Tomo XIII., Fasc. 1, 2; 1899.
 "Bulletin of the American Mathematical Society," 2nd Series, Vol. v., No. 6; New York, March, 1899.
 "Monatshefte für Mathematik und Physik," Jahrgang x., Pt. 2; Wien, 1898.
 "Bulletin des Sciences Mathématiques," Tome XXIII., Février, 1899; Paris.
 "Rendiconto dell' Accademia delle Scienze Fisiche e Mutematiche," Serie 3, Vol. v., Fasc. 2, 3; Napoli, 1899.
 "Atti della reale Accademia dei Lincei—Rendiconti," Sem. 1, Vol. VIII., Fasc. 4, 5; Roma, 1899.
 "Berichte über die Verhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften zu Leipzig," x., 1899.
 "Nyt Tidsskrift for Matematik," A, Aargang ix., Nr. 6-8; Copenhagen, 1898.
 "Jahrbuch über die Fortschritte der Mathematik," Bd. XXVII., Jahrgang, 1896, Heft 3; Berlin, 1899.
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Note on the Characteristic Invariants of an Asymmetric Optical System. By T. J. P.A. BROMWICH. Received March 15th, 1899. Communicated April 13th, 1899. Received, in revised form, May 17th, 1899.

1. The object of this note is to apply the method of the characteristic function, as treated by Maxwell and Larmor,* to deduce the invariants of an asymmetric optical system. These have been very fully worked out by Prof. Sampson in a recent communication to the Society,† using a continuation of methods due to Gauss.

The direct application of the usual form for the reduced path from one point to another leads to very complicated algebra in calculating the invariants, and it appears advantageous to use a modified form. The form I adopt is found by applying the Hamiltonian method of reciprocation, as employed in the general equations of dynamics; or, rather, Routh's modified form of the transformation, in which some of the coordinates are not transformed. The following is a short account of the theory.

* *Proc. Lond. Math. Soc.*, Vols. IV., XX., XXIII.

† *Ibid.*, Vol. XXIX.