V.—**Spectroscopic Observations of the Rotation of the Sun.** By Dr J. Halm,
Assistant Astronomer at the Royal Observatory, and Lecturer in Astronomy at
the University, Edinburgh. *Communicated by* THE ASTRONOMER ROYAL FOR
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The causes of the peculiarities of the solar rotation exhibited by the superficial layers of the sun's body must still be considered unknown, notwithstanding recent interesting attempts at explanation. This is no doubt partly due to the difficulties of the hydrodynamical problem placed before the mathematician in an investigation of such complexity as the movements of particles in a rotating fluid subject to energetic convection. But it must also be conceded that the observational data available for a basis of mathematical, or even speculative, research are still so scanty, that for this reason alone we may perhaps not feel surprised at the failure so far of theoretical attempts.

The first empirical demonstration of the peculiar law which apparently governs the rotation of our luminary was given by CARRINGTON, whose important investigations were taken up and extended by SPOERER. Their observations brought to light the main character of the peculiarity of solar rotation, viz., the decrease of the angular velocity from the equator towards the poles. An objection has, however, been raised against the value of their results with regard to the general rotation of the sun's photosphere, on the ground that they were obtained from the observed movements of solar spots. It was justly urged that spots have "proper motions" which preclude their adoption as points of reference. Besides, we must remember that these spots were visible only within an equatorial zone of about $\pm 50^\circ$ or 40° latitude, and that therefore the polar regions remained inaccessible by this method. This difficulty and limitation was overcome by the ingenious application of the spectroscope to the problem, which we owe to Professor DUNÉR. His conclusions are based on the displacements shown by the Fraunhofer lines at the solar limb, where the gases producing these absorptions are carried by the rotation either towards us or from us. His results are, it is true, not directly comparable to those derived from the movements of the spots, because both refer most probably to different levels, and therefore perhaps to different conditions of motion; but the great advantage of the spectroscopic method seems to me to lie in the fact that we always measure at the same level, wherever this level may be—a point on which we are by no means certain in the case of the spots. Besides, we are independent of the uncontrollable vicissitudes of proper motions, and we are able to extend the investigations from the equator to the immediate vicinity of the poles. The results obtained by Professor DUNÉR may be summarised by the statement that the retardation of the angular velocity discovered by CARRINGTON and SPOERER was found to be also shared by the photospheric layer emitting the Fraunhofer lines, and that the amount of this

"lag" appeared to increase continually towards the poles. So far, then, as the general character of this peculiarity goes, the question appears to be empirically settled. But there remains still another, and, as I think, not less important question. Are we allowed to suppose that the surface rotation of the sun remains unaffected by the periodic changes of solar activity? Judging the question from a purely logical point of view, we are almost bound to answer it in the negative. It seems to me difficult to imagine that such violent disturbances of the normal conditions of convection as we perceive in solar eruptions and spots, and the consequent displacements of matter in the solar globe, should have no influence on the distribution of the rotational velocities at the surface. A careful study of the behaviour of the solar rotation during a cycle of activity may probably teach us far more about the causes and the seat of these solar

FIG. 1.

Group of lines as seen in the viewing telescope. (1 and 3 are telluric, 2 and 4 solar lines.)

disturbances than the whole array of statistical facts regarding the periodic displays of dynamical phenomena at the surface which are now in our possession. Professor DUNÉR's observations, covering a period of three years, during which next to no change took place in the activity of the sun, cannot give an answer to this question. I therefore thought it a promising venture, the success of which seemed to me in some way guaranteed by the great accuracy and consistency of DUNÉR's results, to extend these observations over a time of more pronounced changes of solar activity. The following contains a description of the results so far obtained, and of the instrument employed in my investigations.

The observations were begun in August 1901, and so far carried on to the end of 1903. Although this interval of time is not greater than that covered by DUNÉR's work, there is this essential difference, that the year 1903 was characterised by an abrupt and violent increase of solar disturbances after a pronounced and persistent calm during 1901-2. This fact, as we shall see, has an important bearing on the results, which

seem to me so remarkable and unexpected, that I trust this early publication may induce other observatories to take part in the work, at this opportune time of the beginning of what seems to be an energetic sunspot cycle.

As I intended to make the following observations directly comparable to those of Professor DUNÉR, I have used the same group of lines. The wave-lengths of these are given on page 55 of his treatise, "*Recherches sur la rotation du Soleil.*"* The measurements therefore refer to the same photospheric level. Fig. 1 represents this group of lines as seen in the viewing telescope. The displacements of the solar lines shown in the figure correspond approximately to the shift at the solar equator.

With regard to the instrument, some deviations from DUNÉR's arrangement have been suggested by the apparatus at my disposal. Since they increase the stability of the instrumental plant, these alterations may be considered as essential improvements. The most important of them was attained by the use of a siderostat. DUNÉR's spectroscope was mounted on the great refractor of the Lund Observatory. The extraordinary dimensions of the apparatus gave rise to flexures which were bound to lessen the accuracy of the observations. The influence of such flexures is, of course, avoided if the whole spectral apparatus, including the front telescope, is mounted on fixed and insulated tables, and if the solar light is thrown upon the object-glass of the telescope by means of a siderostat. The considerable advantage of such an arrangement is indeed shown by the fact that the probable error of a single observation appears to be only half the probable error of one of Professor DUNÉR's measurements. This increase of accuracy has to be ascribed chiefly to the greater stability of the instrument, and perhaps also to the easy and comfortable position of the observer during the observations.

A second essential departure from the design of the apparatus used by Professor DUNÉR is to be found in the arrangement by which the focal images of the two opposite limbs of the sun are thrown upon the slit of the spectroscope. DUNÉR employed a system of right-angled prisms arranged according to a device previously suggested by LANGLEY. By successive inner reflections from the hypotenuse surfaces of these prisms the light of the solar limbs can be thrown upon neighbouring points near the centre of the slit, which lies in the optical axis of the telescope. In a much simpler way, however, this same purpose can be attained by using as a front telescope a heliometer of sufficient optical power. By separating the halves of the object-glass, we can at once bring opposite points of the solar limb into contact, and these images may be thrown upon the centre of the slit without any further auxiliary apparatus. Besides, by altering the position angle of the heliometer, all the opposite points of the solar disc can be successively brought into contact. Thus we are enabled to determine the rotational velocity for any desired heliographic latitude simply by turning the heliometer into a position which corresponds to that latitude.

It seemed advantageous to throw the two solar images at each observation into such a position that the line joining their centres coincided with the slit. This could be

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effected by bringing a large-sized, right-angled prism into the cone of light between the object-glass of the heliometer and the focus. By turning the prism, the solar images projected upon the slit-plate could be brought into the desired position, which is represented in the accompanying fig. 2. I may say that in order to attain sufficient accuracy of this adjustment, two lines parallel to the slit and at equal distances from it had been engraved upon the slit-plate. The solar images were brought into such a position that the two lines cut off equal segments. This arrangement was quite sufficient to guarantee the heliographic latitudes of the points measured within a fraction of a degree. Such small errors in the adjustment have, however, no appreciable effect on the observations, because they displace the measured point on the one limb exactly as much towards the equator as they displace the point measured on the other limb towards the pole. The total displacement therefore still agrees practically with that we should have obtained if no such error had been present. This consideration, however, does not apply to points exactly on the equator.

FIG. 2.

The heliometer employed in these observations has been kindly lent to this Observatory by the Hon. Lord M'LAREN, Judge of the High Court of Session of Scotland. It is the instrument used by Sir DAVID GILL in the Mauritius Expedition, 1874, and a full description of it may be found in vol. ii. of the Dunecht Observatory publications. Its aperture is 4.2 inches, and the focal length about 64 inches. The optical quality of the glass is exceedingly good. For the purpose of the observations the eye-end part of the heliometer had to be removed, so that the focus came to fall *outside* the main tube. The instrument is mounted upon a cast-iron table, adjustable in two horizontal directions at right angles, with the optical axis parallel to the line of the meridian, the object-glass towards the north. As a collimator, a telescope is used of 4 inches aperture and 50.5 inches focal length. The eye-end tube, which carries the slit, is adjustable by means of a focussing screw. The cylinder of rays, emanating from the object-glass of the collimator, is thrown upon a Rowland plane grating of speculum metal, 5 inches long, $3\frac{1}{2}$ inches broad, with 14,438 lines to the inch. The surface of the grating is perpendicular to the axis of the collimator. I have used the spectrum of the third order, which, upon the one side of this grating, is remarkably bright and well-defined. The disturbing effect of the overlapping violet spectrum of the higher order was found to be sufficiently elim-

inated by a plane glass of purple colour fixed in front of the eye-piece. The grating is mounted upon a stand, turning on a vertical axis in such a way that the axis of rotation coincides exactly with the plane of the grating. This stand carries a horizontal circle, divided from 5 to 5 minutes, which can be read by two diametrically opposite microscopes. By this arrangement a differential determination of wave-lengths is made possible.

The viewing telescope, which is horizontally mounted on another isolated stand, has an object-glass of 4.1 inches aperture and 60 inches focal length. A Cooke wire micrometer is used for measuring the distances between the lines. After various preliminary trials with crossed and parallel wires, the pointing on the spectral lines by means of sufficiently close parallel wires seemed to me to yield the best results. The observation consists in bringing, by turning the micrometer screw, the centre of the space between two close wires exactly upon the line to be measured, in the same way as the meridian circle observer sets the division of the circle between the wires of the microscope. Care has to be taken, however, that the setting is made exactly on the solar limb. I have found that this may be done without the least difficulty. The errors of the micrometer screw have been repeatedly determined, with the result that the progressive error may be considered as negligible, but that there are indications of a small periodic error. In order to eliminate the effect of the latter directly from the observations, I have invariably observed the lines with *two* pairs of wires which were at a distance of exactly $1\frac{1}{2}$ turns of the screw. As is well known, the periodic error is practically eliminated from the arithmetical mean of the observations made with two sets of wires thus arranged. The value of a revolution of the screw, expressed in wave-lengths, has been determined by measurements of the distances of neighbouring spectral lines, by a method which is fully described in Professor DUNÉR's paper. Such measurements will be continued in the future, but there can be no doubt already that the provisional value here adopted must be sufficiently correct, and that therefore the computed values of the velocities may not have to be altered in the final discussion. The value of the screw is subject to small variations, chiefly owing to temperature fluctuations. These have, however, been taken into account in the reductions in the following way:—Each set of observations yields a very accurate determination of the normal distances between the four lines of the group measured. Since the wave-lengths of these lines are known, the value of a revolution of the screw expressed in wave-lengths may be found by dividing the measured distances of each pair of lines into the difference of their wave-lengths. Thus values of a turn of the screw are found for different points within the group, and from these the value to be applied in the reduction can be easily derived.

The computation of the position angles of the various points measured along the solar limb requires the knowledge of the rotation of the field of the siderostat at the time of the observation. Obviously, what is required is the diurnal rotation of the hour circle of the sun, and therefore also of the pole of the heavens, round the centre of the field. The data required for this computation have been supplied by CORNU in his

paper on the "Law of Diurnal Rotation of the Optical Field of the Siderostat and Heliostat" (*Astrophysical Journal*, vol. xi., 1900, pp. 148-162). It appears from his calculations that in our case, where a siderostat oriented in the meridian has been used, the angle $\mathbf{Y}$ at the centre of the field between the reflected image of the meridian and that of the hour circle of the sun is expressed by the formula,

$$\tan \frac{1}{2} \mathbf{Y} = \mathbf{K} \tan \frac{1}{2} t,$$

where t is the hour angle, reckoned positive towards the west, and

$$\mathbf{K} = \frac{\sin \frac{1}{2} (\phi - \rho)}{\sin \frac{1}{2} (\phi + \rho)},$$

ϕ being the latitude of the place and ρ being the polar distance of the sun. These equations determine the amount and the direction of the rotation. We are thereby enabled to fix the position of the north point of the solar disc as it appears in the field of the siderostat for any time of the day. The values $\mathbf{Y}$ can be tabulated with the arguments t and declination of the sun. The position circle of the heliometer is so oriented that the diameter 0° - 180° falls in the plane of the meridian. The angular distance of the point under observation from the north point of the solar disc, that is, the position angle $\mathbf{P}$ of the point, is then found by the expression

$$\mathbf{P} = 90^\circ - \mathbf{\Pi} + \mathbf{Y},$$

where $\mathbf{\Pi}$ is the reading of the heliometer position circle. The value of $\mathbf{P}$ being found, the heliographic latitude of the point observed can be directly computed from the formulæ given by Professor DUNÉR in his treatise, where he also exhibits extensive tables, greatly facilitating these computations. I need not, therefore, enter more fully upon this part of the reductions.

I shall now briefly describe the way in which observations were performed with this instrument. The sunlight reflected from the siderostat mounted on the main platform of the Observatory is thrown into a meridional direction upon a window of 6 inches aperture in the north wall of the great optical room, and falls upon the object-glass of the heliometer placed immediately behind this window. The halves of the object-glass are screwed apart until the two solar images are nearly in contact, but still separated by a narrow space, which in the viewing telescope appears as a dark horizontal band between the spectra of the two limbs. The right-angled prism mentioned above is then turned and the heliometer adjusted so that the solar images fall upon the slit in the position indicated in fig. 2. After reading the position circle of the heliometer, the observer commences the measurements by pointing the first pair of the micrometer wires on the four lines of the upper of the two spectra seen in the viewing telescope. He begins, say, with the left-hand line of the group, and proceeds towards the right. He then measures with the same pair the lines of the lower of the two spectra in the direction from right to left. The observations are afterwards repeated with the second pair of wires; this time, however, they are begun on the lower spectrum from left to right, and finally the upper spectrum is measured from right to left. This arrangement, though it may

seem perhaps somewhat pedantic, has certainly the advantage of referring the mean of the observations on each limb to one and the same moment of time. Besides, approaching a line alternately in opposite directions must decrease the amount of the personal error of the observer in his judgment of the bisection. After the completion of these measurements, the halves of the heliometer object-glass are reversed and the time noted. The observations are then repeated in the same order as before. A complete set therefore consists of 32 pointings, 16 on each limb. When the set is finished, the heliometer is turned 10° in position angle, and the solar images are again brought into the position of fig. 2. The observation of a full set, including the necessary adjustments of the instrument, requires on the average from 10 to 15 minutes.

The observations of Professor DUNÉR refer to six equidistant points on the solar limb between the equator and 75° latitude. The selection of these fixed heliographic latitudes was perhaps necessitated by the arrangement of his instrument, which required a previous computation of the difference of declination between the north limb of the sun and the point to be observed. Since, however, a previous knowledge of the heliographic latitude is not required in our case, I have preferred to proceed from 10 to 10 degrees on the position circle of the heliometer, without considering at all the heliographic position of the points thrown upon the slit. The readings of the position circle, together with the time of the observation, are sufficient to evaluate afterwards the true heliographic latitude. In this way, a uniform and continuous distribution of observations over the whole quadrant, from the equator to the pole, may be secured, and we are, I think, in a better position to ascertain the character of the velocity-curve by this method than by the observations of six single points of the quadrant.

After this general description of the instrument and the method of observation, I shall now turn to the results. The measurements were commenced on 13th August 1901 and extended to 6th November 1903. During this time 564 determinations of the rotational velocity were made. The values obtained, expressed in kilometres per second, were divided into two groups; the first group comprising the measurements made during 1901-2, the second those of the year 1903. The reason for this division is that the period 1901-2 was characterised by a low and protracted minimum of solar activity, while in the early part of 1903 the commencement of a new solar cycle was vigorously manifested by the appearance of large spots. It was therefore to be expected that, if indeed solar activity has an influence on the rotation of the sun, such a division into groups as I have made would show this influence more clearly than any other. In each group the individual values were arranged according to their heliographic latitudes, and from the materials thus collected normal values were formed by adequate combinations of single values into arithmetic means. The figures thus obtained are exhibited in the following table.

TABLE I.

1901-2.			1903.		
Heliographic Latitude.	Linear Velocity per sec.	No. of Obs.	Heliographic Latitude.	Linear Velocity per sec.	No. of Obs.
	km.			km.	
1.6°	1.908	33	3.7°	1.898	16
5.2	1.894	30	9.1	1.883	15
8.3	1.871	30	15.5	1.831	16
12.8	1.802	30	21.1	1.753	15
18.0	1.720	30	27.7	1.631	16
23.8	1.594	30	34.0	1.512	16
30.6	1.488	30	40.3	1.365	15
38.7	1.265	30	47.1	1.201	16
47.3	1.061	30	54.7	1.014	16
55.8	0.840	30	62.3	0.797	15
65.1	0.560	30	69.2	0.564	16
75.5	0.307	30	76.0	0.408	15
			83.0	0.187	14

The values of the linear velocities were now plotted upon squared paper as ordinates, with the heliographic latitudes as abscissæ, and in each group a curve was drawn representing, as closely as possible, the observed ordinates. In this graphical form the results are represented in the accompanying fig. 3. It is seen at first glance that the two groups differ materially from each other. They agree fairly well only at the equator, but in higher latitudes the observations of 1903 show considerably larger values than those of the two preceding years. There is not a single exception to this rule. The evidence of these curves leaves therefore no doubt as to the reality of this systematic difference, which is also apparent from a consideration of the probable error. An investigation of the observations of 1903 shows the probable error of a single observation to be ± 0.070 km., a value which appears to be very nearly constant for all heliographic latitudes. From this we find the probable error of a mean value for the first group ± 0.013 km., and for the second group ± 0.018 km., while the observed systematic differences between the groups attain the considerable value 0.16 km. in middle latitudes.

Professor DUNÉR has also computed the probable error of his observations. I find from his figures that of a single observation to be ± 0.138 km., also nearly constant for all latitudes. Considering that his single observations comprise twelve to twenty-four pointings of each line, as compared with only eight in my observations, we may conclude that the latter show at least double the accuracy of those of Professor DUNÉR. As already pointed out, this favourable result is in my opinion only due to the greater stability and the more comfortable management of the instrumental plant.

The influence of a possible error in the position angles shown by the heliometer has

also been closely investigated. The careful mounting and orientation of the instruments, it is true, did not warrant the assumption that such an error was to be seriously apprehended. Besides, the arrangement of the observations was such that an error of this kind, even though it existed, should practically disappear from the mean values. This

Fig. 3. Curves showing the linear rotational velocities observed during the years 1901-02 and 1903

may be seen from the following consideration. Let the error of the position angle be Δp , so that a point on the solar limb whose computed distance from the equator, measured in the direction of the position angle, is Π , is actually at a heliographic latitude $\Pi + \Delta p$. If, now, we pass to the opposite side of the pole to a point whose computed distance from the equator, again measured in the direction of the position angle, is $180^\circ - \Pi$, such a point would actually be in the heliographic latitude $\Pi - \Delta p$. Hence

our first point would be nearer the pole and the second nearer the equator than is shown by computation, and the arithmetic mean of both must therefore agree with the actual latitude. If, then, we arrange, as has indeed been done in these observations, the measurements so that equal numbers of points are observed on opposite sides of the poles, the mean values of the heliographic latitudes are practically uninfluenced by a systematic error of the position angle. The question, however, whether such an error actually exists, is readily answered by the observations. We have only to separate the

Fig. 4 showing the values $v \cdot \sec \beta$ in the two periods 1901-2
and 1903

observations into two groups in such a manner that the heliographic latitudes *increase* with the position angle in one group, and *decrease* in the other. The error would thus show itself by the fact that the same values of the velocity would not correspond to exactly the same latitudes in the two groups; or, to put it in other words, that corresponding latitudes should show slightly different values of the velocities. I have made such investigation for the whole time from 1901 to 1903, with the following result:—

TABLE II.

Heliographic Latitude.	Group I. km.	Group II. km.
82.5°	0.21	0.22
75.0	0.34	0.41
65.0	0.65	0.64
55.0	0.92	0.91
45.0	1.18	1.17
35.0	1.43	1.42
25.0	1.64	1.63
15.0	1.77	1.79
5.0	1.89	1.89

With the exception of latitude 75° , the differences between the two groups are far within the limits of their probable errors. This shows, as was anticipated, that practically no error of the position angle was present.

We have now to deal with the question, How are these remarkable differences between the two groups to be interpreted? Is their cause indeed to be sought for in the sun itself? We are, of course, not yet in a position to attempt an answer to this question, but there is a probability of the solar origin of this phenomenon, for which I should like to give some reasons. First, we have to take into consideration that the observations have been made throughout by the same observer, with the same entirely unaltered instrument, and at the same place,—that means, under the same atmospheric conditions. While it must be granted that deficiencies in the instrument and in the method of observation, or the personal perception of the line-displacements, which may be different for different observers, but most of all atmospheric conditions, which shall be discussed later—that all these circumstances may vitiate the results, there seems, however, to be no reason for the assumption that these influences should have altered so decidedly from one year to the next.

Secondly, that a real change of the rotation must have taken place from the one group to the other seems to be indicated by a significant peculiarity of the *angular* velocities which have been derived from the linear velocities v contained in Table I. By multiplying the values of v by $\sec. \beta$, β being the heliographic latitude, we obtain numbers which obviously must be proportional to the angular velocities. This computation having been made, the values were plotted down in fig. 4, again for both groups separately. At first glance we recognise the retardation of the angular velocity from the equator towards the poles, but it will be remarked that the amount of this retardation during the second period is considerably smaller than during the first. The most significant fact, however, appears to be this: if in both groups the values of the curve for corresponding latitudes are subtracted from the equatorial velocity, these differences can be made to agree perfectly if the values for 1901–2 are multiplied by the factor 0.4. This result seemed to me so remarkable that I decided to test its correctness by a special investigation, including also the results of Professor DUNÉR's observations.

First of all I endeavoured to find an empirical formula which should represent the angular velocities in every group in a satisfactory manner. After various attempts at representing these velocities in the usual way by the sine and cosine functions of the heliographic latitude, the method had to be abandoned, as my observations could be represented only by extremely complicated expressions of such a form. Accidentally, however, I arrived at a formula very different from those hitherto used, which satisfies the observations in all three groups with a high degree of accuracy, and which has the further advantage of being extremely simple. This formula can be expressed in the following way—

$$\xi = a - bc^\beta,$$

where ξ is the angular velocity in latitude β , and a , b and c are constants. When

computing the constants of this entirely empirical formula, I found that c had the same value for all the three groups, but that a and b showed considerable differences. The value of c was found to be 1.01447 if β is expressed in degrees, and 2.2784 if expressed in units of the sun's radius. The constants a and b in each of the three groups are given in the following table, which also contains the comparison between the observed values of v and those computed by means of the preceding formula.

TABLE III.

GROUP	a	b
1887-9 (Dunér)	2.349	0.354
1901-2 (Halm)	2.292	0.370
1903 do.	2.066	0.148

GROUP 1887-9.

β	v (comp.)	v (observed)	obs. - comp.
0.4°	1.99 km.	1.98 km.	-0.01
15.0	1.85	1.85	0.00
30.0	1.56	1.58	+0.02
45.0	1.19	1.19	0.00
60.0	0.76	0.74	-0.02
74.8	0.34	0.34	0.00

GROUP 1901-2.

β	v (comp.)	v (observed)	obs. - comp.
1.6°	1.908	1.908	0.000
5.2	1.881	1.894	+0.013
8.3	1.851	1.871	+0.020
12.8	1.797	1.802	+0.005
18.0	1.720	1.720	0.000
23.8	1.617	1.594	-0.023
30.6	1.474	1.488	+0.014
38.7	1.281	1.265	-0.016
47.3	1.055	1.061	+0.006
55.8	0.822	0.840	+0.018
65.1	0.566	0.560	-0.006
75.5	0.299	0.307	+0.008

GROUP 1903.

β	v (comp.)	v (observed)	obs. - comp.
3.7°	1.906	1.898	-0.008
9.1	1.874	1.883	+0.009
15.5	1.813	1.831	+0.018
21.1	1.741	1.753	+0.012
27.7	1.633	1.631	-0.002
34.0	1.513	1.512	-0.001
40.3	1.374	1.365	-0.009
47.1	1.209	1.207	-0.008
54.7	1.008	1.014	+0.006
62.3	0.792	0.797	+0.005
69.2	0.592	0.564	-0.028
76.0	0.393	0.408	+0.015
83.0	0.192	0.187	-0.005

The probable value of a difference "obs. - comp." is in all the groups approximately the same, viz., ± 0.010 km., whereas the probable errors of an observed single value of v are respectively ± 0.013 , 0.013 , 0.018 . The formula therefore represents the observations in all the groups with sufficient accuracy. The constancy of c is a somewhat significant feature. If it should be confirmed by future observations, it would almost certainly point to the conclusion that the formula is not purely empirical, but has some physical meaning. A decision on this question would be highly important, for if the rotation of the surface layers be actually governed by a law which has its analytical expression in the formula here adopted, we may conclude that *surface* phenomena are the cause of these peculiarities. I shall not dwell, however, on this at present, but shall now draw attention to the result of these computations with regard to the constants α and b of our equation. Here we find a satisfactory agreement between the values for 1887-9 and 1901-2. These are also the two groups which have exactly the same position in the sun-spot cycle, both embracing a time of minimum activity. On the other hand, as already pointed out, the year 1903 was characterised by vigorous displays of spots and solar eruptions. Simultaneously with this activity we notice an enormous change in the values of the constants of our equation. The "retardation" of the higher latitudes appears now to be reduced to less than half its former amount. This is a novel and, I am sure, unexpected result. So far, it is true, it can only be said to represent a coincidence in time, but I trust the discussion has made it sufficiently clear that the discrepancies observed, since they can in no way be ascribed to observational errors, must, in all probability, be interpreted as a new and very peculiar feature of the still mysterious mechanism of the sun. Care has been taken throughout not only to investigate all possible errors which may arise from deficiencies of the instrument, and to eliminate such errors from the reduced values of the observations, but the measurements were also arranged in such a way that the possible remainders of such errors which could not be elicited in the reductions should have a minimum effect on the results.

The possible bearing of an investigation of this kind on important questions of solar physics makes it very desirable that the observations should not only be continued at this Observatory, but that other observatories should also take part in them. Next to the question of possible alterations in the amount of solar heat, an answer to which may now be expected from the ingenious researches of Professor LANGLEY and his staff, the problem of solar rotation should command the greatest attention from the part of solar physicists, for no other seems to me so well adapted to give us information on the mechanism of the solar forces. That the periodic play of these forces should in some manner be conducive to changes of the distribution of the moments of rotation is a logical conclusion on which I trust astronomers are unanimous. It is well to remark in this connection that the idea has already received theoretical consideration in an interesting paper by Mr EMDEN.

The construction of the apparatus required for these observations is well within the

reach of our large astrophysical observatories, and there are several beautiful heliometers which might be resuscitated for this purpose from their present unprofitable state of repose. Nor is the present the only investigation which can be taken up with an instrument of the kind here described. The construction of the apparatus, combined with its high dispersive power, makes it possible to separate by a mere glance the telluric lines from the solar ones, and at the same time to determine by a single observation the wave-length of a line with an accuracy of about ± 0.05 tenth-metres. The instrument appears therefore to be particularly adapted for an investigation of the telluric spectrum in accordance with the method first suggested by CORNU.

As regards the accuracy of the determination of the rotational velocities, I am confident that, under more favourable atmospheric conditions, the annual output of observations can be considerably increased, and thereby the probable error of the annual means correspondingly lessened. It does not seem to me impossible that in this way changes of only one to two hundredths of a kilometre could be traced with certainty.

I must not conclude this paper, however, without drawing attention to an inconvenience encountered in this spectroscopic determination of the rotational velocity. With a hazy atmosphere, a not inconsiderable quantity of scattered daylight, reflected from the particles of the aqueous vapour contained in the air, is thrown upon the slit of the spectroscope. As a certain percentage of this light will have emanated from the interior of the sun's disc, the solar lines of this "day" spectrum appear less displaced than the lines of the two solar limbs. By a superposition of the two spectra, the intensity of the lines will therefore be lessened on the one side and increased on the other, the effect always being to bring the lines nearer to their normal position. The velocities obtained under such conditions must therefore be too small. I have often had occasion to observe this phenomenon when, after a measurement made under favourable atmospheric conditions, the sky was suddenly overspread with a veil of haze, a meteorological feature not at all infrequent in the climate of Edinburgh. When the observation was then repeated, the measured displacement was invariably found less than that obtained before. Observations under misty conditions of the sky were therefore carefully avoided. A reliable scale for the transparency of the atmosphere was supplied by the optical appearance of the dark band separating the spectra of the two limbs in the viewing telescope. Whenever this space became so bright as to show traces of the absorption lines of the solar spectrum, the observation was broken off. All the measurements used in this discussion were made at moments when the band mentioned was uniformly dark, so that the spectrum of the diffused daylight cannot have seriously influenced the measured displacements.

TABLE IV.

Single Values of Linear Rotational Velocities of the Sun observed during the Years 1901-1903.

Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.
1901.		3.4	1.69	0.5	2.05					32.1	1.54				
Aug. 13.		5.5	1.75	16.9	2.09	Sept. 10.		Feb. 17.		40.7	1.32	Apr. 25.			
24.2	1.41	14.9	1.65			62.9	0.77	67.8	0.31	49.2	0.99	18.0	1.61		
38.1	1.23	2.3	1.75	Aug. 28.		52.2	1.04	55.0	0.63			27.3	1.73		
50.4	0.84	31.4	1.31	10.0	1.89	41.4	1.20			Mar. 24.					
64.1	0.55	40.4	1.13	18.6	1.92	30.8	1.59	Feb. 18.		62.0	0.64	Apr. 28.			
77.0	0.22	49.3	0.83	27.4	1.83	19.7	1.87	35.7	1.39			27.9	1.72		
79.2	0.12	58.4	0.66	36.3	1.59	8.7	1.92	27.1	1.35	Mar. 25.		17.1	1.66		
64.6	0.71	67.2	0.35	43.5	1.33			18.9	1.46	70.6	0.44	6.4	1.84		
48.5	0.97	75.9	0.32	52.4	1.07	Sept. 11.		10.3	1.71	59.8	0.75	5.0	1.74		
32.9	1.17	5.5	1.77	51.0	0.72	15.9	1.99	1.8	1.67	48.8	1.10	16.1	1.47		
17.0	1.49	10.5	1.83	1.1	1.90	24.4	1.77	6.8	1.74	37.8	1.28	27.5	1.51		
						33.3	1.67	14.8	1.75	26.4	1.61				
Aug. 15.		Aug. 22.		Aug. 30.		42.4	1.37	23.0	1.67	15.5	1.90	Apr. 30.			
23.0	1.52	0.8	2.07	19.3	1.70	51.3	1.00	31.5	1.52	4.2	1.96	18.1	1.78		
		8.2	1.87	2.5	1.88	60.0	0.87	39.8	1.14	7.3	1.85				
Aug. 16.		17.0	1.84	8.3	1.89	68.6	0.56					May 29.			
8.5	2.03	25.5	1.83					Mar. 7.		Mar. 29.		0.6	1.92		
5.6	1.77	34.3	1.46	Aug. 31.				46.0	0.96	21.1	1.58	14.9	1.87		
19.8	1.76	43.2	1.19	1.9	2.17	Oct. 26.		54.5	0.78			25.5	1.41		
33.3	1.45	52.2	1.03	1.1	1.95	6.9	1.98	63.0	0.51	Apr. 1.		36.5	1.28		
47.6	1.17	61.1	0.75			1.8	2.11	71.4	0.35	6.7	1.89	47.0	1.07		
60.7	0.67	69.7	0.42	Sept. 1.		10.3	1.84			Mar. 15.		57.6	0.82		
74.0	0.34	5.4	2.06	6.5	1.90	18.9	1.49	Mar. 16.		76.2	0.23	68.7	0.64		
		6.6	2.11	19.3	1.97			65.9	0.55	9.2	1.90				
Aug. 19.		Aug. 24.		10.3	2.03	Nov. 5.		54.7	0.88	1.8	1.89	June 27.			
11.5	1.70	3.7	2.06	1.8	2.13	39.6	1.30	43.3	1.25	26.7	1.47	52.7	0.69		
2.5	1.85	5.0	2.11	7.0	2.09	48.2	0.99	32.3	1.43	35.5	1.22	63.0	0.43		
16.7	1.60	3.4	1.96	15.9	1.98	56.3	0.86	13.9	1.90	43.9	1.04	73.8	0.22		
30.8	1.23					64.6	0.63	2.9	2.05	51.5	0.84	44.2	1.02		
44.5	1.20			Sept. 2.		73.2	0.39					34.7	1.30		
58.3	0.57	11.1	1.82	50.6	1.03	1.2	2.00			Apr. 7.		25.2	1.53		
71.8	0.16	20.5	1.67	59.2	0.76	9.0	1.73	Mar. 16.		50.1	0.98	15.5	1.76		
77.9	0.33	29.1	1.54	68.0	0.32	16.8	1.67	21.0	1.64						
65.0	0.51	34.9	1.20	2.2	1.94			9.2	1.90	Apr. 10.		June 30.			
51.0	1.08	44.2	1.08	8.1	1.88	1902.		1.8	1.89	52.6	1.04	10.4	1.79		
37.1	1.28			1.0	2.17	Feb. 12.		26.7	1.47	61.5	0.86	1.0	1.88		
23.6	1.43	Aug. 25.		9.5	1.92	13.0	1.72	35.5	1.22	70.6	0.67	8.8	1.75		
9.4	1.48	78.6	0.11	18.3	1.83	5.3	1.89	43.9	1.04	78.5	0.16	18.0	1.59		
4.5	1.75	68.8	0.52	27.0	1.59	2.7	2.01	51.5	0.84			27.3	1.48		
		58.1	0.89			10.5	1.73			Apr. 16.		36.8	1.13		
Aug. 20.		47.4	1.19	Sept. 4.		18.9	1.68	Mar. 20.		79.5	0.24	46.2	0.93		
1.0	2.03	36.7	1.41	28.8	1.53			9.5	1.77	71.1	0.46	55.8	0.63		
2.2	1.80	25.8	1.64	17.4	1.71	Feb. 13.		6.8	1.84	75.9	0.13				
11.4	1.59	14.6	1.87	5.7	2.03	7.9	2.14	18.1	1.76	65.5	0.61	July 5.			
25.2	1.39	3.8	1.98	5.9	2.21					54.9	0.85	62.3	0.70		
39.0	1.08	7.2	2.05			Feb. 15.		Mar. 21.		44.3	1.14	51.8	0.98		
52.8	0.84	7.9	2.04	Sept. 5.		3.3	1.85	17.6	1.66	33.6	1.46	41.5	1.23		
66.5	0.32	26.7	1.84	0.8	2.02	14.6	1.79			21.8	1.55	31.1	1.41		
4.5	1.93	17.8	1.86	Sept. 7.		26.1	1.46	Mar. 22.		10.8	1.63	20.6	1.66		
5.5	1.80					37.2	1.07	6.5	1.85	0.4	1.83	8.0	1.73		
21.1	1.52	Aug. 27.		7.7	1.85	48.9	0.98	0.2	1.77			0.3	1.85		
		6.6	2.00	41.1	1.34	60.7	0.77	7.6	1.90	Apr. 18.		10.8	2.00		
Aug. 21.		8.1	1.85	48.9	1.19	71.4	0.42	16.0	1.83	5.1	1.85	21.5	1.76		
2.3	1.79							24.3	1.66	16.0	1.80	32.3	1.39		

TABLE IV.—continued.

Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.	Lat.	Vel.
1902.		Sept. 8.		Nov. 1.		59.7 0.81 48.6 1.03		May 29.		51.2 0.92 42.3 1.13 32.9 1.47 23.5 1.56 24.5 1.68		51.2 0.92 42.3 1.13 32.9 1.47 23.5 1.56 24.5 1.68		70.0 0.55 59.1 0.71 47.2 1.20 35.9 1.45 25.1 1.67	
July 5 contd.		79.1 0.39		80.0 0.41		May 23.		72.8 0.49 83.3 0.24 85.9 -0.14 75.8 0.33 65.4 0.68 54.8 0.82 44.2 1.13 34.0 1.56 23.5 1.73 13.2 1.78 2.9 1.84 7.7 1.73				Aug. 27.		Oct. 21.	
44.2 1.02		74.2 0.13		72.0 0.53		3.9 1.84 7.1 1.86 17.8 1.80 28.5 1.65 38.7 1.29 49.0 1.17 59.7 0.77 70.9 0.53 81.3 0.26 74.8 0.48 64.3 0.75 53.7 1.04 43.1 1.35 32.5 1.46						1.9 1.81		18.1 1.90 6.4 1.94 5.5 1.83 17.4 1.84	
54.4 0.83		53.5 0.89				Nov. 4.									
64.9 0.58		42.9 1.14		59.7 0.84		17.8 1.80									
65.3 0.56		30.4 1.40		51.0 1.05		28.5 1.65									
75.5 0.29		19.7 1.65		42.1 1.32		38.7 1.29									
		8.9 1.80		33.4 1.54		49.0 1.17									
Aug. 21.				25.7 1.64		59.7 0.77									
49.8 1.00		Sept. 9.		21.2 1.61		70.9 0.53									
		18.6 1.72				81.3 0.26									
Aug. 23.		8.0 1.86		Nov. 10.		74.8 0.48									
59.8 0.67				77.7 0.29		64.3 0.75									
50.2 0.92		Sept. 13.		1903.		53.7 1.04									
41.5 1.16		2.9 1.81		Feb. 25.		43.1 1.35									
33.0 1.23		11.0 1.86		24.0 1.65		32.5 1.46									
23.8 1.49				38.8 1.30		May 26.									
14.7 1.73		Sept. 17.		Feb. 27.		46.6 1.25									
		12.2 1.90		35.4 1.52		36.3 1.41									
Aug. 25.		1.4 2.01		17.2 1.87		25.9 1.69									
5.9 1.71		9.4 1.85		9.4 1.94		15.0 1.77									
2.5 1.54						3.9 1.91									
10.2 1.75		Sept. 23.		Mar. 15.		6.8 1.83									
17.7 1.66		9.9 1.80		14.5 1.83		17.3 1.93									
		1.0 1.95		2.3 2.02		27.9 1.74									
Aug. 26.		6.9 1.89		8.9 1.99		38.4 1.54									
35.6 1.41		15.1 1.77		20.5 2.00		48.9 1.20									
24.9 1.65		21.1 1.71		31.8 1.68		59.6 0.99									
11.2 1.65		29.8 1.52		52.8 1.19		69.9 0.63									
0.7 1.73				63.1 0.87		May 27.									
22.4 1.69		Oct. 4.		Apr. 17.		64.8 0.89									
3.6 1.78		36.7 1.50		7.2 1.85		75.3 0.56									
37.1 1.44		45.5 1.17		18.3 1.78		86.3 0.19									
		54.5 1.08		29.3 1.65		83.1 0.14									
Aug. 27.		62.4 0.71		Apr. 19.		72.9 0.33									
6.4 1.72		70.8 0.33		16.3 1.79		61.5 0.79									
				28.3 1.60		51.0 1.02									
Aug. 28.		Oct. 19.		Apr. 24.		40.5 1.35									
8.2 1.92		2.1 1.70		9.9 2.13		28.8 1.61									
17.2 1.85		4.5 1.89		20.8 2.01		18.4 1.68									
25.7 1.66		13.3 1.76		31.6 1.70		7.7 1.84									
34.8 1.44		21.9 1.64		41.9 1.34		2.8 1.91									
43.6 1.31		30.7 1.50		53.0 1.03		13.1 1.93									
52.8 0.96		39.4 1.17		64.0 0.74		May 28.									
61.5 0.69		48.3 1.02		74.3 0.42		4.4 1.93									
70.5 0.40				83.8 0.16		15.1 1.86									
78.7 0.15		Oct. 30.		81.7 0.24		25.9 1.66									
78.1 0.37		62.1 0.57		71.5 0.53		36.1 1.46									
		70.4 0.29				47.3 1.26									
Aug. 29.		78.3 0.13				57.6 0.95									
74.4 0.41		79.0 0.48				68.1 0.53									