XL.—*On the Mechanical Action of Heat.* By WILLIAM JOHN MACQUORN RANKINE,
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SECTION VI.—A REVIEW OF THE FUNDAMENTAL PRINCIPLES OF THE MECHANICAL
THEORY OF HEAT; WITH REMARKS ON THE THERMIC PHENOMENA OF CURRENTS
OF ELASTIC FLUIDS, AS ILLUSTRATING THOSE PRINCIPLES.

(Article 46.) I have been induced to write this Section, in continuation of a paper on the Mechanical Action of Heat, by the publication (in the *Philosophical Magazine* for December 1852, Supplementary Number) of a series of experiments by Mr JOULE and Professor WILLIAM THOMSON, on the Thermal Effects experienced by Air in rushing through small Apertures. Although those authors express an intention to continue the experiments on a large scale, so as to obtain more precise results; yet the results already obtained are sufficient to constitute the first step towards the experimental determination of that most important function in the theory of the mechanical action of heat, which has received the name of CARNOT'S Function.

By the theoretical investigations of Messrs CLAUSIUS and THOMSON,—which are based simply on the fact of the convertibility of heat and mechanical power, the determination of their relative value by Mr JOULE, and the properties of the function called *temperature*, without any definite supposition as to the nature of heat,—CARNOT'S function is left wholly indeterminate.

By the investigations contained in the previous sections of this paper, and in a paper on the Centrifugal Theory of Elasticity,—in which the supposition is made, that heat consists in the revolutions of what are called Molecular Vortices, so that the elasticity arising from heat is in fact centrifugal force,—a *form* is assigned to CARNOT'S function; but its numerical values are left to be ascertained by experiment.

The recent experiments of Messrs JOULE and THOMSON serve (so far as the degree of precision of their results permits) at once to determine numerical values of CARNOT'S function for use in practice, and to test the accuracy with which the phenomena of heat are represented by the consequences of the hypothesis of molecular vortices, from which the investigation in this paper sets out.

SUB-SECTION 1.—*Properties of Expansive Heat.*

(47.) To shew more clearly the nature of the questions, towards the decision of which these experiments are a step, I shall now briefly review the fundamental principles of the theory of heat, and the reasoning on which they are based; and the object of this being illustration rather than research, I shall use algebraical symbols no farther than is absolutely necessary to brevity and clearness, and shall follow an order of investigation, which, though the same in its results with that pursued in the previous sections of this paper, is different in arrangement.

By a mind which admits as an axiom, that, in the present order of things, physical power cannot be annihilated, nor produced out of nothing, the law of the mutual convertibility of heat and motive power must be viewed as a necessary corollary from this axiom, and Mr JOULE's experiments, as the means of determining the relative numerical value of those two forms of power. By a mind which does not admit the necessity of the axiom, these experiments must be viewed also as the proof of the law.

This law was virtually, though not expressly, admitted by those who introduced the term Latent Heat into scientific language; for when divested of ideas connected with the hypothesis of a subtle fluid of caloric, and regarded simply as the expression of a fact, this term denotes heat which has disappeared during the appearance of expansive power in a mass of matter, and which may be made to reappear by the expenditure of an equal amount of compressive power.

(48.) Without for the present framing any mechanical hypothesis as to the nature of heat, let us conceive that unity of weight of any substance, occupying the bulk V under the pressure P , and possessing the absolute quantity of thermometric heat whose mechanical equivalent is Q , undergoes the indefinitely small increase of volume dV ; and let us investigate how much heat becomes latent, or is converted into expansive power, during this process; the thermometric heat being maintained constant, so that the heat which disappears must be supplied from some external source.

During the expansion dV , the body, by its elastic pressure P , exerts the mechanical power $P dV$. Part of this power is produced by molecular attractions and repulsions; and although this part may be modified by the influence of heat upon the distribution of the particles of the body, it is not the direct effect of heat. The remainder must be considered as directly caused by the heat possessed by the body, of which the pressure P is a function; and to this portion of the power developed, the heat which disappears during the expansion must be equivalent.

To determine the portion of the mechanical power $P dV$ which is the effect of heat, let the total heat of the body, Q , be now supposed to vary by an indefinitely

increase of total heat dQ , while the volume remains constant, is expressed as follows:—

$$\frac{dS}{dQ} dQ = \left\{ Q \int \frac{d^2 P}{dQ^2} dV + \phi'(Q) \right\} dQ \quad (71.)$$

the heat Q being treated as a constant in the integration.

If we now investigate the entire quantity of heat, both sensible and latent, which is consumed by a body during a simultaneous small change of total heat dQ and volume dV , we find the following results:—

Sensible heat (which retains its condition)	$= dQ$
Latent heat, or heat which disappears in overcoming molecular action	$\frac{dS}{dQ} dQ + \frac{dS}{dV} dV$
Latent heat equivalent to the visible mechanical effect	$P dV$
The amount being	

$$\left. \begin{aligned} dQ + dS + P dV &= \left(1 + \frac{dS}{dQ} \right) dQ + \left(\frac{dS}{dV} + P \right) dV = \\ &\left(1 + Q \int \frac{d^2 P}{dQ^2} dV + \phi'(Q) \right) dQ + Q \cdot \frac{dP}{dQ} \cdot dV \end{aligned} \right\} \quad (72.)$$

This formula expresses completely the relations between heat, molecular action, and expansion, in all those cases in which the expansive power developed, $P dV$, is entirely communicated to the bodies enclosing the substance which expands.

(49.) The following coefficients are contained in, or deducible from it.

The ratio of the specific heat at constant volume to the real specific heat:—

$$\frac{K_v}{K} = 1 + \frac{dS}{dQ} = 1 + Q \int \frac{d^2 P}{dQ^2} \cdot dV + \phi'(Q) \quad (73.)$$

The coefficient of latent heat of expansion at constant heat:—

$$\frac{dS}{dV} + P = Q \frac{dP}{dQ} \quad (74.)$$

The ratio of the specific heat at constant pressure to the real specific heat is found as follows. To have the pressure constant, we must have

$$\frac{dP}{dQ} dQ + \frac{dP}{dV} dV = 0; \quad \text{or} \quad \frac{dV}{dQ} = - \frac{\frac{dP}{dQ}}{\frac{dP}{dV}}$$

consequently the ratio in question is

$$\left. \begin{aligned} \frac{K_p}{K} &= 1 + \frac{dS}{dQ} - \left(\frac{dS}{dV} + P \right) \frac{\frac{dQ}{dP}}{\frac{dV}{dP}} = 1 + Q \int \frac{d^2 P}{dQ^2} \cdot dV + \phi'(Q) \\ &\quad - Q \frac{\left(\frac{dP}{dQ} \right)^2}{\frac{dP}{dV}} \end{aligned} \right\} \quad (75.)$$

(50.) In order to investigate the laws according to which heat is converted into mechanical power, in a machine working by the expansion of an elastic body, it will be convenient to use a function

$$F = \int \frac{dP}{dQ} dV \quad (Q = \text{const.})$$

of such a nature that the difference between two of its values, corresponding to different volumes of the body at the same total heat, represents the ratio of the heat converted into power by expansion between those volumes, to the given constant total heat. I shall call this function a *heat-potential*.

Introducing this function into Equation 72, we find, for the total heat consumed by a body during the increase of total heat dQ , and the expansion dV ,

$$dQ + d.S + P dV = \left(1 + \phi' \cdot (Q)\right) dQ + Q d.F \quad . \quad . \quad . \quad (76.)$$

$$\left(\text{observing that } d.F = \frac{dF}{dQ} dQ + \frac{dF}{dV} dV = \left(\int \frac{d^2P}{dQ^2} dV\right) dQ + \frac{dP}{dQ} dV\right)$$

Let us now suppose that the body changes its volume without either losing or gaining heat by conduction. This condition is expressed by the equation

$$0 = (1 + \phi' \cdot (Q)) dQ + Q d.F$$

from which we deduce the following,

$$-d.F = \frac{1 + \phi' \cdot (Q)}{Q} \cdot dQ \quad . \quad . \quad . \quad (77.)$$

which expresses the following theorem:—

When the quantity of heat in a body is varied by variation of volume only, the variation of the heat-potential depends on the heat only, and is independent of the volume.

In order that a machine working by the expansive power of heat may produce its greatest effect, all the heat communicated from external bodies should be employed in producing expansive power, and none in producing variations of the quantity of heat in the body; for heat employed for the latter purpose would be wasted, so far as the production of visible motion is concerned. To effect this, the body must receive heat by conduction, and convert it into expansive power, while containing a certain constant quantity of heat Q_1 ; give out by conduction heat produced by compression, while containing a smaller constant quantity of heat Q_2 ; and change between those two quantities of thermometric heat by means of changes of volume only, without conduction. For this purpose a cycle of operations must be performed similar to that described by CARNOT; as follows:—

(I.) Let F_A be the initial value of the heat-potential; let the body expand at the constant heat Q_1 , till the heat-potential becomes F_B . Then the heat received and converted into expansive power is

$$H_1 = Q_1 (F_B - F_A)$$

(II.) Let the body further expand without receiving or emitting heat, till the quantity of heat in it falls to Q_2 ; the heat-potential varying according to Equation 77, and becoming at length F_c . The heat converted into expansive power in this operation is

$$Q_1 - Q_2$$

(III.) Let the body be compressed, at the constant heat Q_2 , till the heat-potential becomes F_d ; a quantity differing from the initial heat-potential F_a by as much as F_c differs from F_b . In this operation the following amount of power is reconverted into heat, and given out by conduction:—

$$H_2 = Q_2 (F_c - F_d)$$

(IV.) Let the body be further compressed, till the heat-potential returns to F_a , its original value. Then, by the power expended in this compression alone, without the aid of conduction, the total heat of the body will be restored to its original amount, exactly reversing the operation II.

At the end of this cycle of operations, the following quantity of heat will have been converted into mechanical power:—

$$H_1 - H_2 = Q_1 (F_b - F_a) - Q_2 (F_c - F_d)$$

but it is obvious that the difference between the heat-potentials is the same in the first and third operations; therefore, the useful effect is simply

$$\left. \begin{aligned} H_1 - H_2 &= (Q_1 - Q_2) (F_b - F_a) \\ \text{while the whole heat expended is,} \\ H_1 &= Q_1 (F_b - F_a) \end{aligned} \right\} \quad (78.)$$

Hence, *the ratio of the heat converted into mechanical effect, in an expansive machine working to the greatest advantage, to the whole heat expended, is the same with that which the difference between the quantities of heat possessed by the expansive body during the operations of receiving and emitting heat, respectively, bears to the quantity of heat possessed by it during the operation of receiving heat; and is independent of the nature and condition of the body.*

This theorem is thus expressed symbolically,—

$$\frac{H_1 - H_2}{H_1} = \frac{\text{Effect}}{\text{Heat Expended}} = \frac{Q_1 - Q_2}{Q_1} \quad (79.)$$

(51.) When a body expands without meeting with resistance, so that all its expansive power is expended in giving velocity to its own particles, and when that velocity is ultimately extinguished by friction, then a quantity of heat equivalent to the expansive power is reproduced.

The heat consumed is expressed by taking away the term representing the expansive power, $P dV$, from the expression 72, the remainder of which consists merely of the variation of actual heat, and the heat expended in overcoming molecular attraction, viz. :—

$$dQ + dS = \left(1 + \frac{dS}{dQ}\right) dQ + \frac{dS}{dV} dV = \left(1 + Q \int \frac{d^2 P}{dQ^2} dV + \phi'(Q)\right) dQ \\ + \left(Q \frac{dP}{dQ} - P\right) dV.$$

This expression is a complete differential, and may be written thus:—

$$d(Q + S) = d \left\{ Q + \phi(Q) + \left(Q \frac{dP}{dQ} - 1 \right) \int P dV \right\} \quad . \quad . \quad (80.)$$

(Q being treated as a constant in performing the integration $\int P dV$).

Its integral, $Q + S$, the sum of the heat of the body, and of the potential of its molecular actions, is the same quantity which I have denoted by the symbol Ψ in the 10th article of a paper on the Centrifugal Theory of Elasticity, and whose differences are there stated to represent the total amount of power which must be exercised on a body, whether in the form of expansive or compressive power, or in that of heat, to make it pass from one volume and temperature to another. This integral corresponds also to the function treated of by Professor WILLIAM THOMSON in the fifth part of his paper on the Dynamical Theory of Heat, under the name of “Total Mechanical Energy.”

(52.) We have now obtained a system of formulæ, expressing all the relations between heat and expansive power, analogous to those deduced from a consideration of the properties of temperature, by Messrs CLAUSIUS and THOMSON, and from the Hypothesis of Molecular Vortices in the previous sections of this paper; but, in the present section, both the theorems and the investigations are distinguished from former researches by this circumstance;—that they are independent, not only of any hypothesis respecting the constitution of matter, but of the properties, and even of the existence, of such a function as Temperature; being, in fact, simply the necessary consequences of the following

DEFINITION OF EXPANSIVE HEAT.

Let the term EXPANSIVE HEAT be used to denote a kind of Physical Energy convertible with, and measurable by, equivalent quantities of Mechanical Power, and augmenting the Expansive Elasticity of matter, in which it is present.

(52 A.) It is further to be remarked, that the theorems and formulæ in the preceding articles of this section are applicable, not only to heat and expansive power, but to any two directly convertible forms of physical energy, one of which is actual, and the other potential. They are, in fact, the principles of the conversion of energy in the abstract, when interpreted according to the following definitions of the symbols.

Let Q denote the quantity of a form of actual physical energy present in a given body;

V, a measurable state, condition, or mode of existence of the body, whose tendency to increase is represented by

P, a force, depending on the condition V, the energy Q, and permanent properties of the body; so that

$P dV$ is the increment of a form of potential energy, corresponding to a small increment dV of the condition V.

Let dS be the quantity whereby the increment of potential energy $P dV$ falls short of the quantity of actual energy of the form Q, which is converted into the potential form, by the change of condition dV .

Then, as in Equation 69

$$\frac{dS}{dV} = Q \frac{dP}{dQ} - P$$

an equation from which all those in the previous articles are deducible, and which comprehends the whole theory of the mutual conversion of the actual form of energy Q, and the potential form $\int P dV$, whatsoever those forms may be, when no other form of energy interferes. The application of these principles to any form or any number of forms of actual and potential energy, is the subject of a paper read to the Philosophical Society of Glasgow, on the 5th January 1853, and published in the Philosophical Magazine for February 1853.

SUB-SECTION 2.—*Properties of Temperature.*

(53.) Still abstaining from the assumption of any mechanical hypothesis, let us proceed a step beyond the investigation of the foregoing articles, and introduce the consideration of *temperature*; that is to say, of an arbitrary function increasing with heat, and having the following properties.

Definition of Equal Temperatures.

Two portions of matter are said to have equal temperatures, when neither tends to communicate heat to the other.

Corollary.

All bodies absolutely destitute of heat have equal temperatures.

The ratio of the real specific heats of two substances, is that of the quantities of heat which equal weights of them possess at the same temperature.

Theorem.

The ratio of the real specific heats of any pair of substances, is the same at all temperatures.

For, suppose equal weights of a pair of homogeneous substances to be in contact, containing heat in such proportions as to be in equilibrio. Then, let additional

portions of each substance, of equal weight, and destitute of heat, be added to the original masses ; so that the quantities of heat in unity of weight may be diminished in each substance, but may continue to be in the same ratio. Then, if the equality of temperature do not continue, portions of heat which were in equilibrium must have lost that equilibrium, merely by being transferred to other particles of a pair of homogeneous substances, which is absurd. Therefore, the temperatures continue equal.

It follows, that the quantity of heat in unity of weight of a substance at a given temperature, may be expressed by the product of a quantity depending on the nature of the substance, and independent of the temperature, multiplied by a function of the temperature, which is the same for all substances.

Let τ denote the temperature of a body according to the scale adopted ; κ , the position, on the same scale, of the temperature corresponding to absolute privation of heat ; $\mathfrak{k}$, a quantity depending on the nature of the substance, and independent of temperature. Then the quantity of heat in unity of weight may be expressed as follows:—

$$Q = \mathbb{K}(\psi, \tau - \psi, K) \quad . \quad . \quad . \quad . \quad . \quad . \quad (81.)$$

(54.) If we introduce this notation into the formula (79) which expresses the proportion of the total heat expended, which is converted into useful power by an expansive machine working to the best advantage, the quantity $\mathfrak{k}$, peculiar to the substance employed, disappears, and we obtain CARNOT'S THEOREM, as modified by Messrs CLAUSIUS and THOMSON, viz.,—that this ratio is a function solely of the temperatures at which heat is received and emitted respectively, and is independent of the nature of the substance; or symbolically,

$$\frac{\text{Effect}}{\text{Heat Expended}} = \frac{Q_1 - Q_2}{Q_1} = \frac{\psi \cdot \tau_1 - \psi \cdot \tau_2}{\psi \cdot \tau_1 - \psi \cdot K} \quad (82.)$$

(55.) Let us now apply the same notation to the formula (67) for the latent heat of a small expansion dV at constant heat, viz:—

$$Q \frac{dP}{dQ} \cdot dV$$

we have evidently

$$\frac{dP}{dQ} = \frac{1}{\frac{dQ}{d\tau}} \cdot \frac{dP}{d\tau} = \frac{1}{k \psi' \cdot \tau} \cdot \frac{dP}{d\tau}$$

and consequently, the heat which disappears by the expansion dV is

$$Q \frac{dP}{dQ} \cdot dV = \frac{\psi \cdot \tau - \psi \cdot \kappa}{\psi' \cdot \tau} \cdot \frac{dP}{d\tau} \cdot dV \quad . \quad . \quad . \quad . \quad (83.)$$

from which formula the specific quantity $\mathfrak{k}$ has disappeared.

Now, in the notation of Professor THOMSON we have

$$\frac{\psi \cdot \tau - \psi \cdot \kappa}{\psi' \cdot \tau} = \frac{J}{\mu}$$

where J is JOULE's equivalent, and μ a function of the temperature, the same for all substances, to be determined empirically ; and consequently,

$$\begin{aligned} \text{hyp. log. } (\psi \cdot \tau - \psi \cdot \kappa) &= \frac{1}{J} \int^{\tau} \mu d\tau \\ \text{or, } \psi \cdot \tau - \psi \cdot \kappa &= \epsilon \frac{1}{J} \int^{\tau} \mu d\tau \\ \text{and } Q = \kappa (\psi \cdot \tau - \psi \cdot \kappa) &= \kappa \cdot \epsilon \frac{1}{J} \int^{\tau} \mu d\tau \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{hyp. log. } (\psi \cdot \tau - \psi \cdot \kappa) &= \frac{1}{J} \int^{\tau} \mu d\tau \\ \text{or, } \psi \cdot \tau - \psi \cdot \kappa &= \epsilon \frac{1}{J} \int^{\tau} \mu d\tau \\ \text{and } Q = \kappa (\psi \cdot \tau - \psi \cdot \kappa) &= \kappa \cdot \epsilon \frac{1}{J} \int^{\tau} \mu d\tau \end{aligned}} \right\} \quad (84.)$$

These expressions will be recognised by those who have studied Professor THOMSON's papers on the Dynamical Theory of Heat. By introducing the value given above of the quantity of heat in unity of weight, into the formulæ of the preceding articles of this section, they are at once transformed to those of Professor THOMSON, and in particular, the formulæ 79 and 82 become the following:—

$$\frac{\text{* Effect of Machine}}{\text{Heat Expended}} = \frac{\epsilon \frac{1}{J} \int^{\tau_1} \mu d\tau - \epsilon \frac{1}{J} \int^{\tau_2} \mu d\tau}{\epsilon \frac{1}{J} \int^{\tau_1} \mu d\tau} = 1 - \epsilon \frac{1}{J} \int^{\tau_2} \mu d\tau \quad (85.)$$

SUB-SECTION 3.—*On the Hypothesis of Molecular Vortices.*

(56.) The use of a Mechanical Hypothesis in the Theory of Heat, as in other branches of physics, is to render it a branch of Mechanics, the only complete physical science ; and to deduce its principles from the laws of Force and Motion, which are better understood than those of any other phenomena.

The results of the investigations in the preceding part of this section are consistent alike with all conceivable hypotheses which ascribe the phenomena of heat to invisible motions amongst the particles of bodies.

Those investigations, however, leave undetermined the relation between temperature and quantity of heat, except in so far as they shew that it must follow the same law of variation in all substances.

By adopting a definite hypothesis, we are conducted to a definite relation between temperature and quantity of heat ; which, being introduced into the formulæ, leads to specific results respecting the phenomena of the mutual transformation of heat and visible mechanical power ; and those results, being compared with experiment, furnish a test of the soundness of the hypothesis.

Thus the hypothesis of Molecular Vortices, which forms the basis of the investigations in the first five sections of this paper, and in a paper on the Centrifugal Theory of Elasticity, leads to the conclusion, that, if temperature be mea-

* It is to be observed, that in Professor THOMSON's notation, heat is supposed to be measured by an arbitrary unit, whose ratio to a unit of mechanical power is denoted by J ; while in this paper, the same unit is employed in expressing quantities of heat and of mechanical power.

sured by the expansion of a perfect gas, the total quantity of heat in a body is simply proportional to the elevation of its temperature above the temperature of absolute privation of heat; or, in the notation of the preceding article,

$$\begin{aligned} \psi \cdot \tau &= \tau, \quad \psi' \cdot \tau = 1, \text{ and} \\ Q &= \kappa (\tau - \kappa) \end{aligned} \quad (86.)$$

κ being the real specific heat of the body.

If this value be substituted for the quantity of heat Q , in all the formulæ, from 67 to 80 inclusive, which are founded simply on the definition of expansive heat, it reproduces all the formulæ which, in this and the other paper referred to, have been deduced directly from the hypothesis. In the sequel I shall apply one of these formulæ to the calculation, from the experiments of Professor THOMSON and Mr JOULE on the heating of currents of air by friction, of approximate values of the absolute temperature corresponding to total privation of heat, that the mutual consistency of those values may serve as a test of the soundness of the hypothesis, and the accuracy of the formulæ deduced from it.

(57.) Before proceeding further, it may be desirable to point out how far this hypothesis agrees with, and how far it differs from, that proposed by Mr HERAPATH and Mr WATERSTON, which supposes bodies to consist of extremely small and perfectly elastic particles, which fly about in all directions with a velocity whose half-square is the mechanical equivalent of the heat possessed by unity of weight, and are prevented from dispersing by their collisions with each other and with the particles of surrounding bodies. Let v be the velocity of motion, then

$$\frac{v^2}{2g} = Q$$

represents the heat possessed by unity of weight, expressed in terms of the force of gravity.

The expansive pressure due to such motions is found by conceiving a hard, perfectly elastic plane of the area unity to be opposed to the collision of the particles, and calculating the pressure which would be required to maintain its position against them. If all the particles were to strike and rebound from such a plane at right angles, the pressure would be represented thus:

$$\frac{v^2}{g} \cdot \frac{1}{V}$$

where V is the volume which contains so many particles as amount to unity of weight. But the particles are supposed to fly in equal numbers in all directions. Then if θ denote the angle of incidence on the plane

$$\frac{\sin \theta d\theta}{\int_0^{\frac{\pi}{2}} \sin \theta d\theta} = \sin \theta d\theta$$

represents the proportion of the whole particles which fly in those directions which make the angle θ with the normal to the plane. Of this proportion, again,

the fraction $\cos \theta$ only strikes the plane; while the force of the blow also is less than that of a normal blow in the ratio $\cos \theta : 1$. Hence the mean force of collision is

$$\int_0^{\frac{\pi}{2}} \cos^2 \theta \sin \theta d\theta = \frac{1}{3}$$

of the force of a perpendicular collision; so that the expansive pressure is represented by

$$\frac{1}{3} \cdot \frac{v^2}{g} \cdot \frac{1}{V} = \frac{2}{3} \cdot \frac{Q}{V} = P$$

Hence, according to this hypothesis, we should have for a perfect gas

$$P V = \frac{2}{3} Q$$

or the product of the pressure and volume of a mass of a perfect gas equal to two-thirds of the mechanical equivalent of its total heat.

It is known, however, that the product of the pressure and volume of a mass of sensibly perfect gas is only about four-tenths of the equivalent of its total heat. The hypothesis, therefore, requires modification.

By supposing the particles to attract each other, or to be of appreciable bulk compared with the distances between them, the ratio in question is diminished; but either of these suppositions is inconsistent with the perfectly gaseous condition.

It appears to me, that, besides this difficulty connected with the gaseous condition, there exists also great difficulty in conceiving how the hypothesis can be applied to the solid condition, in which the particles preserve definite arrangements. The limited amount of time and attention, however, which I have hitherto bestowed on this hypothesis, is not sufficient to entitle me to pronounce whether these difficulties admit of a solution.

(58.) The idea of ascribing expansive elasticity to the centrifugal force of vortices or eddies in elastic atmospheres surrounding nuclei of atoms, originated with Sir HUMPHRY DAVY. The peculiarity of the view of the hypothesis taken in this paper consists in the function ascribed to the nuclei or central physical points of the atoms, which, besides retaining the atmospheres round them by their attraction, are supposed, by their actions on each other, to constitute the medium which transmits radiant heat and light; so that heat is radiant or thermometric, according as it affects the nuclei or their atmospheres.

In this form the hypothesis of Molecular Vortices is not a mere special supposition, to elucidate the theory of expansive heat, but becomes connected with the theory of the elasticity of matter in all conditions, from solid to gaseous, and with that of the transmission of radiations.

I have already investigated mathematically the consequences of this hypothesis by two different processes, which are necessarily somewhat complicated.

Equilibrium of heat and pressure between portions of two different perfect gases in contact requires that the pressures independent of heat, and the pressures caused by heat, shall separately be in equilibrio. Let the suffixes a and b be used to distinguish quantities relative to two different substances in the perfectly gaseous condition. Then the first condition of equilibrium is expressed as follows:—

$$\left(\frac{h}{V}\right) (a) = \left(\frac{h}{V}\right) (b) \quad . \quad . \quad . \quad . \quad . \quad . \quad (91.)$$

that is to say, *the densities of two perfect gases in equilibrio are inversely proportional to the coefficients of elasticity of their atomic atmospheres.*

The second condition is expressed as follows:—

$$\left(\frac{N Q}{V}\right) (a) = \left(\frac{N Q}{V}\right) (b)$$

which, being taken in connection with the first condition, gives

$$\left(\frac{N}{h} Q\right) (a) = \left(\frac{N}{h} Q\right) (b) \quad . \quad . \quad . \quad . \quad . \quad . \quad (92.)$$

Now by Equation 90, we have

$$\frac{N}{h} Q = \frac{P V}{h} - 1$$

Hence the condition of equilibrium of heat between two perfect gases is

$$\left(\frac{P V}{h}\right) (a) = \left(\frac{P V}{h}\right) (b) \quad . \quad . \quad . \quad . \quad . \quad . \quad (93.)$$

consequently, *temperature may be measured by the product of the pressure and volume of a perfect gas, divided by a coefficient, which is proportional to the volume of the gas at a standard pressure and temperature.*

Temperatures thus measured are reckoned from the point known as the *zero of gaseous tension*, or *absolute zero of a perfect gas thermometer*, $274^{\circ} \cdot 6$ centigrade below the temperature of melting ice.

Let V_0 denote the volume of unity of weight of a perfect gas, at a standard pressure P_0 , and absolute temperature τ_0 ; then any other absolute temperature has the following value:—

$$\tau = \tau_0 \frac{P V}{P_0 V_0} = \frac{\tau_0}{P_0 V_0} (N Q + h) \quad . \quad . \quad . \quad . \quad . \quad . \quad (94.)$$

while the absolute temperature of total privation of heat is

$$\kappa = \tau_0 \frac{h}{P_0 V_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (94 A.)$$

Hence it appears that quantity of heat in unity of weight bears the following relation to temperature,—

$$Q = \frac{1}{N} (P V - h) = \frac{P_0 V_0}{N \tau_0} (\tau - \kappa) \quad . \quad . \quad . \quad . \quad . \quad . \quad (95.)$$

in which, if we substitute the symbol of real specific heat,

$$\mathfrak{K} = \frac{P_0 V_0}{N \tau_0} \quad (96.)$$

we obtain the formula already given (86) for the relation between heat and temperature.*

(59.) The introduction of the value given above of the quantity of heat in terms of temperature, into the formula 67, gives for the latent heat of a small expansion dV at constant temperature

$$(\tau - \kappa) \frac{dP}{d\tau} \cdot dV \quad (97.)$$

The formulæ 79 and 82, for the proportion of heat rendered available by an expansive engine working to the greatest advantage, becomes

$$\frac{\tau_1 - \tau_2}{\tau_1 - K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98.)$$

or the ratio of the difference between the temperatures of receiving and emitting heat, to the elevation of the former temperature above that of total privation of heat. This is the law already arrived at by a different process in Section V. of this paper.

When the same substitution is made in Equation 80, which represents the total energy, whether as heat or as compressive power, which must be applied to unity of weight of a substance to produce given changes of heat and volume, the following result is obtained:—

$$\begin{aligned}
d \cdot \Psi &= d \cdot Q + d \cdot S = \left\{ \mathbb{K} + f'(\tau) + (\tau - \kappa) \int \frac{d^2 P}{d\tau_2} dV \right\} d\tau \\
&\quad + \left\{ (\tau - \kappa) \frac{dP}{d\tau} - P \right\} dV \\
&= d \cdot \left\{ \mathbb{K} \tau + f(\tau) + \left((\tau - \kappa) \frac{d}{d\tau} - 1 \right) \cdot \int P dV \right\} \quad . \quad . \quad . \quad (99.)
\end{aligned}$$

As it cannot be simplified, it is unnecessary here to recapitulate the investigation, which leads to the conclusion that the functions $f(\tau)$ and $f'(\tau)$ have the following values :—

$$f(\tau) = \kappa \text{ N} \left(\kappa \text{ hyp. log. } \tau + \frac{\kappa^2}{\tau} \right) ; f'(\tau) = \kappa \text{ N} \left(\frac{\kappa}{\tau} - \frac{\kappa^2}{\tau^2} \right) \quad . \quad . \quad (99 \text{ A.})$$

We have thus reproduced Equation 26 of the paper formerly referred to, on the Centrifugal Theory of Elasticity.

The coefficient of the variation of temperature in the first form of Equation 99 is the specific heat of the substance at constant volume. Denoting this by K_v , the formula becomes

$$d \cdot \Psi = K_v \cdot d\tau + \left\{ (\tau - \kappa) \frac{dP}{d\tau} - P \right\} dV \quad . \quad . \quad . \quad (100.)$$

* See Appendix, Note A.

SUB-SECTION 4. *Thermic Phenomena of Currents of Elastic Fluids.*

(60.) When a gas previously compressed is allowed to escape through small apertures, as in the experiments of Mr JOULE and Professor THOMSON, and has its velocity destroyed entirely by the mutual friction of its particles, without impediment from any other substance, and without conduction of heat to or from any other substance; then its condition is expressed by making

$$d\Psi = 0$$

that is to say,

$$-d\tau = \frac{1}{K_v} \cdot \left\{ \tau \cdot \left(\frac{dP}{d\tau} - \frac{P}{\tau} \right) - \kappa \frac{dP}{d\tau} \right\} dV \quad . \quad . \quad . \quad (101.)$$

If we assume (as is really the case in the experiments) that the specific heat of the gas at constant volume does not sensibly vary within the limits of the experiments as to temperature and volume, so that K_v is sensibly constant, and also that the variation of temperature is very small as compared with the absolute temperatures, then we have the following approximate integral:—

$$-\Delta\tau = \frac{1}{K_v} \cdot \left\{ \tau \int_{V_1}^{V_2} \left(\frac{dP}{d\tau} - \frac{P}{\tau} \right) dV - \kappa \int_{V_1}^{V_2} \frac{dP}{d\tau} dV \right\} \quad . \quad . \quad (102.)$$

which represents the cooling effect of an expansion from the volume V_1 to the volume V_2 .

If it were possible to obtain any substance in the state of perfect gas to be used in experiments of this kind, the first integral in the above expression would disappear, because, for a perfect gas,

$$\frac{dP}{d\tau} = \frac{P}{\tau}$$

and as the other term is negative, the result would be a slight heating effect. As no gas, however, is perfect, and as $\frac{dP}{d\tau}$ always exceeds $\frac{P}{\tau}$, the mode of reducing the experimental data is to calculate the value of the first term, which represents the effect of cohesion, from the known properties of the gas, to subtract from it the actual cooling, and from the remainder to compute values of κ , the temperature of absolute privation of heat, according to the following formula:—

$$\kappa = \frac{\frac{1}{K_v} \tau \int_{V_1}^{V_2} \left(\frac{dP}{d\tau} - \frac{P}{\tau} \right) dV - (-\Delta\tau)}{\frac{1}{K_v} \int_{V_1}^{V_2} \frac{dP}{d\tau} dV} \quad . \quad . \quad . \quad (103.)$$

When the gas is nearly perfect, as in the case of atmospheric air, it is unnecessary to take into consideration its deviation from the perfect condition in computing the integral in the denominator; whose approximate value is found to be

$$\frac{P_0 V_0}{K_v \tau_0} \cdot \text{hyp. log } \frac{V_2}{V_1} = N \cdot \text{hyp. log } \frac{P_1}{P_2} \text{ nearly } (\tau \text{ being nearly constant}),$$

and K_v nearly = k .

The value of the integral in the numerator is found as follows:—

The Centrifugal Theory of Elasticity indicates that the pressure of an imperfect gas may be represented by the following formula:—

$$P = P_0 \frac{V_0}{V} \left\{ \frac{\tau}{\tau_0} + A_0 - \frac{A_1}{\tau} - \frac{A_2}{\tau^2} - \&c. \right\} \quad (104.)$$

where V_0 is the volume in the perfectly gaseous state, at a standard pressure P_0 , and absolute temperature τ_0 , and A_0 , A_1 , &c., are a series of functions of the density, to be determined empirically. From this formula it is easily seen that

$$\frac{dP}{d\tau} - \frac{P}{\tau} = P_0 \frac{V_0}{V} \left\{ -\frac{A_0}{\tau} + \frac{2A_1}{\tau^2} + \&c. \right\} \quad (105.)$$

so that the first term in the numerator of the expression (103) has the following value:—

$$\frac{\tau}{K_v} \int_{v_1}^{v_2} \left(\frac{dP}{d\tau} - \frac{P}{\tau} \right) dV = \frac{P_0 V_0}{K_v} \left\{ -\int_{v_1}^{v_2} \frac{A_0}{V} dV + \frac{2}{\tau} \int_{v_1}^{v_2} \frac{A_1}{V} dV + \&c. \right\} \quad (106.)$$

in which $\frac{P_0 V_0}{K_v} = N \tau_0$ nearly.

In order to represent correctly the result of M. REGNAULT'S experiments on the elasticity and expansion of gases, it was found sufficient to use, in the formula for the pressure (104), the first three terms; and the functions of the density which occur in these terms, as determined empirically from the experiments, were found to have the following values, in which the unit of volume is the theoretical volume of unity of weight of air under the pressure of one atmosphere, at the temperature of melting ice,* and the values of the constants are given for the centigrade scale.

$$\frac{A_0}{V} = b \left(\frac{1}{V} \right)^{\frac{3}{2}}; \quad \frac{A_1}{V} = a \left(\frac{1}{V} \right)^{\frac{3}{2}} \quad (107.)$$

Com. log $b = \bar{3} \cdot 8181545$; Com. log $a = 0 \cdot 3176168$.

Hence it appears that the integrals in the formula (106) have the following values:—

$$\int_{v_1}^{v_2} \frac{A_0}{V} dV = 2b \cdot \Delta \cdot \left(\frac{1}{V} \right)^{\frac{1}{2}}; \quad \frac{2}{\tau} \int_{v_1}^{v_2} \frac{A_1}{V} dV = \frac{10}{3} \cdot \frac{a}{\tau_0} \cdot \frac{\tau_0}{\tau} \cdot \Delta \cdot \left(\frac{1}{V} \right)^{\frac{1}{2}} \quad (107 A.)$$

in which the common logarithms of the constants are

* This unit of volume is greater than the actual volume of air, under the circumstances described, in the ratio of 1·00085 to 1.

$$\text{Com. log } 2b = \bar{2}.1101845 ; \log \frac{10}{3} \cdot \frac{\alpha}{\tau_0} = \bar{2}.4017950 ;$$

and these values suit any scale of temperatures.

In calculating, for use in these formulæ, the densities $\frac{1}{V}$ from the observed pressures, it is sufficiently near the truth, in the case of air, to use the approximate equation

$$\frac{1}{V} = \frac{\tau_0}{\tau} \cdot P \text{ (in atmospheres).}$$

The common logarithm of τ_0 , the absolute temperature of melting ice, for the centigrade scale, is $\bar{2}.4387005$.

The constant N for atmospheric air is 0.4 nearly; therefore

$$\text{Com. log } (N \times \text{hyp. log } 10) = \bar{1}.9642757.$$

The following, therefore, is the approximate value of the formula (103), to be used (with the numerical constants already given) in reducing the experiments of Mr JOULE and Professor THOMSON on atmospheric air, so as to obtain approximate values of the absolute temperature of total privation of heat:—

$$\kappa = \left\{ N \tau_0 \left(\frac{10}{3} \frac{\alpha}{\tau_0} \cdot \left(\frac{\tau_0}{\tau} \right)^{\frac{1}{6}} \Delta \cdot (P^{\frac{1}{6}}) - 2b \left(\frac{\tau_0}{\tau} \right)^{\frac{1}{2}} \Delta \cdot (P^{\frac{1}{2}}) \right) - (-\Delta \tau) \right\} \\ \div N \text{ hyp. log } 10 \times \Delta \cdot \text{com. log } \frac{1}{P} \quad . \quad . \quad . \quad . \quad . \quad (108.)$$

In using this formula, the mean absolute temperature should be taken as the value of τ .

The following table shews the values of the quantity κ , computed from ten mean experimental data, taken respectively from the first ten series of experiments described in the recent paper of Messrs JOULE and THOMSON, in the supplementary number of the *Philosophical Magazine* for December 1852. The temperatures in the table, for the sake of convenience, are reduced to the centigrade scale, because that scale has been used throughout the previous sections of this paper.

The final pressure in each case was that of the atmosphere.

SERIES AND NUMBER OF EXPERIMENTS.	Quantity of Air used in cubic inches per second.	Initial Temperature. — Centigrade.	Mean Absolute Temperature τ .	Initial Pressure in Atmospheres.	Actual Cooling. — Centigrade.	Calculated Cooling by Cohesion. — Centigrade.	Difference.	Approximate Values of α , the Absolute Temperature of Total Privation of Heat. — Centigrade.
Series I. } Mean of 3 Experiments, }	Not noted.	16.11	290.4	3.61	0.76	1.70	0.94	1.83
Series II. } Mean of 5 Experiments, }	Not noted.	71.11	345.6	2.64	0.28	0.70	0.42	1.08
Series III. } Mean of 7 Experiments, }	6.4	77.11	351.5	4.28	0.37	1.25	0.88	1.51
Series IV. } Mean of 3 Experiments, }	5.6	17.39	281.7	3.04	0.56	1.49	0.93	2.09
Series V. } Mean of 2 Experiments, }	8.4	3.89	278.0	4.11	1.00	2.18	1.18	2.087
Series VI. } Mean of 4 Experiments, }	11.2	5.21	279.4	4.40	0.92	2.31	1.39	2.345
Series VII. } Mean of 5 Experiments, }	1.4	17.87	292.3	1.894	0.23	0.66	0.43	1.683
Series VIII. } Mean of 5 Experiments, }	2.8	18.04	292.5	2.41	0.31	0.93	0.62	1.762
Series IX. } Mean of 4 Experiments, }	5.6	18.21	292.6	2.92	0.43	1.385	0.955	2.228
Series X. } Mean of 3 Experiments, }	11.2	18.37	292.6	4.11	0.72	1.93	1.21	2.14

If the value of κ is really $2^{\circ}\cdot 1$ centigrade, as computed above, the calculated maximum theoretical duty in Section V. is too small by about one one-hundred-and-ninetieth part of its amount,—a quantity of no practical importance in such calculations.

(61.) It may be anticipated, that when Mr JOULE and Professor THOMSON shall have performed experiments on the thermic phenomena exhibited by air in more copious currents, and by gases of more definite composition, and more simple laws of elasticity, much more precise results will be obtained.

When a gas deviating considerably from the perfectly gaseous condition, or a vapour near the point of saturation, is employed, it will no longer be sufficiently accurate to treat the specific heat at constant volume as a constant quantity, nor the cooling effect as very small. It will therefore be necessary to employ, for the reduction of the experiments, the integral form of equation (99); that is to say,

$$\begin{aligned} 0 = \Delta \Psi &= \Delta \left\{ \kappa \tau + \kappa N \kappa \left(\text{hyp. log } \tau + \frac{\kappa}{\tau} \right) + \left((\tau - \kappa) \frac{d}{d\tau} - 1 \right) \int P dV \right\} \\ &= \kappa (\tau_2 - \tau_1) + \Delta \int \left(\tau \frac{dP}{d\tau} - P \right) dV \\ &\quad - \kappa \left\{ \Delta \int \frac{dP}{d\tau} dV - \kappa N \left(\Delta \cdot \frac{\kappa}{\tau} + \Delta \text{ hyp. log } \tau \right) \right\} \quad . \quad . \quad (110.) \end{aligned}$$

(62.) Preliminary to the application of this equation, it is necessary to determine the mechanical value of the real specific heat κ . Supposing the law which connects the pressure, density, and temperature of the gas to be known, it is sufficient for this purpose to have an accurate experimental determination, either of the apparent specific heat at constant pressure for a given temperature, or the velocity of sound in the gas under given circumstances.

First, let us suppose that the apparent specific heat at constant pressure is known.

The value of this coefficient (Centrifugal Theory of Elasticity, art. 12) is

$$K_P = \kappa + (\tau - \kappa) \left\{ \frac{P_0 V_0}{\tau_0} \frac{\kappa}{\tau^2} + \int \frac{d^2 P}{d\tau^2} dV + \frac{\left(\frac{dP}{d\tau} \right)^2}{-\frac{dP}{dV}} \right\} \quad . \quad . \quad . \quad (111.)$$

In order that the lower limit of the integral may correspond with the condition of perfect gas, it is convenient to transform it into one in terms of the density. Let D be the weight of unity of volume, then

$$\int \frac{d^2 P}{d\tau^2} dV = - \int_0^D \frac{1}{D^2} \cdot \frac{d^2 P}{d\tau^2} dD \quad . \quad . \quad . \quad (111 A.)$$

If, then, we have the pressure of the gas under consideration expressed by the the following approximate formula:—

$$P = \frac{P_0 V_0}{V} \left\{ \frac{\tau}{\tau_0} + A_0 - \frac{A_1}{\tau} \right\}$$

The following will be the values of the functions of the pressure which enter into the above equation:—

$$\left. \begin{aligned} \frac{dP}{d\tau} &= \frac{P_0 V_0}{V} \left\{ \frac{1}{\tau_0} + \frac{A_1}{\tau^2} \right\}; \quad \frac{d^2 P}{d\tau^2} = -2 \frac{P_0 V_0}{V} \cdot \frac{A_1}{\tau^3} = -2 P_0 V_0 \frac{A_1 D}{\tau^3} \\ \int \frac{d^2 P}{d\tau^2} dV &= - \int_0^D \frac{1}{D^2} \cdot \frac{d^2 P}{d\tau^2} dD = + 2 \frac{P_0 V_0}{\tau^3} \cdot \int_0^D \frac{A_1}{D} dD; \\ - \frac{dP}{dV} &= D^2 \frac{dP}{dD} = P_0 V_0 D^2 \left\{ \frac{\tau}{\tau_0} + \frac{d \cdot A_0 D}{dD} - \frac{1}{\tau} \cdot \frac{d \cdot A_1 D}{dD} \right\} \\ \therefore \frac{\left(\frac{dP}{d\tau} \right)^2}{- \frac{dP}{dV}} &= P_0 V_0 \cdot \frac{\left(\frac{1}{\tau_0} + \frac{A_1}{\tau^2} \right)^2}{\frac{\tau}{\tau_0} + \frac{d \cdot A_0 D}{dD} - \frac{1}{\tau} \cdot \frac{d \cdot A_1 D}{dD}} \end{aligned} \right\} \quad (111 B.)$$

To illustrate the application of these formulæ, let us calculate the difference between the real specific heat, and the apparent specific heat, at constant pressure, of carbonic acid gas, at the temperature of melting ice, and at the density which, if the gas were perfect, would correspond to a pressure of one atmosphere at the temperature of melting ice. Let this density be denoted by D_0 , and its reciprocal by V_0 . As the constants have been deduced from M. REGNAULT'S experiments, the calculations will be made in French measures and for the latitude of Paris.

The actual density of carbonic acid at 0° centigrade, and under one atmosphere of pressure, exceeds the theoretical density, in the perfectly gaseous state, in the ratio of 1.0065 to 1 nearly. Hence the height of a homogeneous atmosphere of actual carbonic acid at 0° centigrade being 5225.5 metres, the corresponding height in the state of perfect gas is $P_0 V_0 = 5259.5$,, and $\frac{P_0 V_0}{\tau} = 19.53$ metres per centigrade degree = 62.84 feet.

The functions which express the influence of density on the deviation of carbonic acid gas from the perfectly gaseous state, have the following values:—

$$\left. \begin{aligned} A_0 &= b \cdot \frac{D}{D_0}; \quad A_1 = a \cdot \frac{D}{D_0}; \quad \text{when} \\ \text{Com. log } b &= \bar{3}.1083932; \quad \text{Com. log } a = 0.3344538 \\ b &= 0.00128349 \quad \quad \quad a = 2.16; \\ \int_0^D \frac{A_1}{D} dD &= \int_0^{\frac{D}{D_0}} A_1 \frac{D_0}{D} \cdot \frac{d \cdot D}{D_0} = a \cdot \frac{D}{D_0}; \quad \frac{d}{dD} \cdot A_0 D \\ &= 2b \cdot \frac{D}{D_0}; \quad \frac{d}{dD} \cdot A_1 D = 2a \frac{D}{D_0} \end{aligned} \right\} \quad (111 C.)$$

For the purposes of a first approximation, we may assume that the value of κ

$$\Delta \int \left(\tau \frac{dP}{d\tau} - P \right) dV = -\Delta \int \frac{1}{D^2} \left(\tau \frac{dP}{d\tau} - P \right) dD = -P_0 V_0 \Delta \left\{ \frac{2}{\tau} \int_0^D \frac{A_1}{D} dD - \int \frac{A_0}{D} dD \right\}$$

$$\int \frac{dP}{d\tau} dV = - \int \frac{1}{D^2} \cdot \frac{dP}{d\tau} dD = -P_0 V_0 \Delta \left(\frac{1}{\tau_0} \text{hyp. log } D + \frac{1}{\tau^2} \int \frac{A_1}{D} dD \right) \quad (113.)$$

For carbonic acid, the first of these formulæ becomes simply

$$\left. \begin{aligned} &+ P_0 V_0 \left\{ \frac{2a}{D_0} \left(\frac{D_1}{\tau_1} - \frac{D_2}{\tau_2} \right) - \frac{b}{D_0} (D_1 - D_2) \right\} \\ \text{and the second,} \\ &+ P_0 V_0 \left\{ \frac{1}{\tau_0} \text{hyp. log } \frac{D_1}{D_2} - \frac{a}{D_0} \left(\frac{D_1}{\tau_1^2} - \frac{D_2}{\tau_2^2} \right) \right\} \end{aligned} \right\} \quad (113 A.)$$

GLASGOW, 27th December 1852.

APPENDIX.

NOTE A (to Article 58). Since this section was read, the theoretical views relative to the relation between heat and temperature contained in it and the previous sections of this paper, have received a strong confirmation by the publication by M. REGNAULT of the fact, that he has found the specific heat of air to be sensibly constant at all temperatures from -30° centigrade to $+225^\circ$, and at all pressures from one to ten atmospheres (*Comptes Rendus*, 18th April 1853); so that equal lengths on the scale of the air thermometer represent equal quantities of heat.

NOTE B (to Article 62). Until very recently, there existed no exact experimental determination of the specific heat of any gas. The specific heat of air at constant pressure, as compared with that of water, was calculated theoretically in the previous part of this paper, from JOULE's equivalent and the velocity of sound, and found to be 0.24. This value has since been confirmed very closely by Mr JOULE's experiments, whose mean result was 0.23, and still more exactly by M. REGNAULT's experiments, already referred to, which give the value 0.2379. The following table shews the results of the application of the formulæ of this paper to the specific heats of five different gases at constant pressure, selected from M. REGNAULT's table (*Comptes Rendus*, 18th April), as being those in which the velocity of sound can be computed, and has been determined experimentally. The table shews also a comparison of the calculated and observed velocities of sound. This table appeared originally, in French measures, in the *Philosophical Magazine* for June 1853: the metres are here reduced to feet. K_P , K_V , and K_W , are expressed in feet of fall per centigrade degree. K_W (JOULE's equivalent) = 1389.6.

GASES.	EXPERIMENTAL DATA.			THEORETICAL RESULTS.			VELOCITY OF SOUND AT 0° CENT.		
	PV at 0° C. REGNAULT.	$K_P \div K_W$ REGN.	K_P	$K_P - K_V$	K_V	$K_P \div K_V$	By Theory.	By Obser- vation.	Observers.
	Feet.		Feet.	Feet.	Feet.		Ft. per sec.	Ft. per sec.	
Air, . . {	26214.4	0.2379	330.6	96.0	234.6	1.4094	1090.4	1090.5 1090.1	BRAVAIS & MARTINS. MOLL & VAN BEEK.
Oxygen, . .	23710.4	0.2182	303.2	86.8	216.4	1.4014	1036.4	1042.3	DULONG.
Hydrogen, .	378819.0	3.4046	4731.0	1388.0	3343.0	1.4150	4153.3	4165.1	DULONG.
Carbonic oxide,	27097.8	0.2479	344.5	99.25	245.25	1.4047	1106.8	1107.0	DULONG.
Carbonic acid,	17144.7	0.2169	300.7	64.2	236.5	1.2714	837.55	858.28	DULONG.

The real specific heat of carbonic acid gas is 235.5 feet of fall per centigrade degree. That of the other gases does not differ from the apparent specific heat at constant volume by an amount appreciable in practice.