THE CONDITIONS GOVERNING THE RISE OF TEMPERATURE OF ELECTRIC RAIL- WAY MOTORS IN SERVICE.

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In a former paper read before the INSTITUTE (TRANS. AM. INST. E. E., Vol. XIX., p. 129), I described a method for determining the motor-capacity and energy required to make any schedule speed for a run of any distance, with electric motors. Since that time I have extended the method to include the pre-determination of the rise of temperature of the motor when those constants of the motor are known, upon which the rise of temperature depends.

In that investigation the time required for each of the three

FIG. 1

periods of the "type-run," shown in Fig. 1, was first determined; this was an algebraical problem, the solution of which involved

the initial acceleration (a), the coasting retardation (b), the retardation after braking (c), the length of the run and the average speed from start to stop. These equations are:

$$x/T = 2 A_0/a - K_1 (A_0 - A)^{1/2}$$

$$y/T = K_2 (A_0 - A)^{1/2}$$

$$z/T = 2 A_0/c - K_3 (A_0 - A)^{1/2}$$

in which K_1, K_2, K_3 and A_0 are simple functions of (a), (b) and (c) only; x, y and z are respectively the times of initial acceleration, coasting and braking, and T the total time, as shown in Fig. 1. The only quantity occurring in these equations other than the three accelerations, (a), (b) and (c), then, is

$$A = V/T = 0.682 L/T^2$$

T being the time and V the average speed from start to stop; the form of the equation shows that all runs having the same values of (a), (b), (c) and A will be made in the same proportionate time for the different periods, *i.e.*, the same fraction of the total time will be required for accelerating, coasting and braking. In other words, all runs having the same value of the accelerations and of $A = V/T$ are represented by the same figure, with merely a change in scale for the different lengths of run, the ordinates and abscissas being proportional to $L^{1/2}$.

This quantity, A , reduces the variables from three— V, L and T , to two— A and L . A is a convenient quantity to use as independent variable for the curve-sheets; it simplifies the presentation, since all runs having the same values of A , (a), (b) and (c) require the same energy per ton-mile, independent of the length of run or the average velocity, and require a motor-capacity directly proportional to the square root of the length of run—that is, directly proportional to the time from start to stop.

Moreover, all values of A occurring in practice can be included between the values of 0.15 and 0.35—a relatively small variation whereas variations in V, L and T cover a much wider range.

It is to be remembered that the previous investigation, as well as this one, assumes a straight, level track, constant train friction and constant braking force. These assumptions are necessary to any general treatment, since curvature and gradients are arbitrary. I have, however, in part, developed a method for making the necessary allowances for curves, grades and variable train friction, in the form of corrections to the values found for the corresponding simple case.

These equations involve four quantities, of which two—(b) and (c)—are assumed to be constants and two—(a) and A —

the variables. The solution of the equations is best given by curve-sheets, with x/T , y/T and z/T as ordinates, and A as abscissas; a separate curve is required for each value of (a) . CS 1 is plotted with

$$(b) = .15 \text{ mph/sec.} = 13.6 \text{ lb. per ton}$$

$$\text{and } (c) = 2.0 \text{ mph/sec.} = 182 \text{ lb. per ton;}$$

(c) is the total retardation after braking, including train friction. This curve-sheet is similar to CS 2 of the first paper, save that there the values, $(b) = .2$ and $(c) = 3.0$ mph/sec., were used.

Fig. 1 shows the type case, with three constant accelerations;

FIG. 2

Fig. 2 the actual case, in which acceleration on the motor-curve is added to the constant initial acceleration; Fig. 2 then involves the characteristics of the motor used. CS 5 of the first paper gave the average speed- and torque-curves of a number of tramway motors in common use; I have since added other motor-curves, and in CS 2 show average motor-curves, which represent with practical accuracy the torque and speed relations of any General Electric or Westinghouse motor, throughout the range of ordinary operation. The extreme difference between the speed of any motor of either company and the speed on this curve-sheet for a given torque is greatest at the high speeds, but within the range of ordinary operation this variation is less than 5 per cent.

It is necessary to assume the same motor performance in order that the variation in energy consumption and motor-capacity required by the different schedules, when made with various initial and motor accelerations, may be shown independently of variations in characteristics of the motor. If one assumes other motor-curves, the methods outlined here can be applied by making certain corrections; but, as a matter of fact, the small variations in the shape of the motor-curve are of comparatively slight consequence in the final results.

The efficiency of different motors varies through a much wider range than do the speed and torque; but the efficiency is used only in connecting the input and output, and not in determining heat losses, which are the subject of a separate consideration; hence considerable differences in efficiency curves are of minor consequence.

Here, as in the first paper, the datum point for these motor curves is the load determined by the "hour-rating,"—that is, the load the motor will carry for one hour with a temperature rise of 75% C. when tested on the stand, with covers off; this value is assumed to be the commutation-limit of the motor-capacity and all the results of the investigation are expressed in terms of the input donated by the datum point; therefore it only remains to determine the conditions of a run which will give the limiting temperature elevation when the motor is used at this particular input. The hour-rating for the maximum input is not unduly conservative; it leaves a margin for greater power on curves, grades and for emergencies.

CS 3 and CS 4 give energy and motor-capacity in terms of through-acceleration A ; they are the same as CS 13 and CS 14 of the first paper, except that here the abscissas are through acceleration with the motor-curve instead of the corresponding through acceleration with the type-curve; that is, the quantity A_m of the first paper is used as abscissa instead of the A of that paper, but is designated A ; and that the motor-capacity is given for a run of one mile. This change simplifies the use of the curves, in that it makes it unnecessary to add corrections to the value of A calculated from the data given for the run. CS 3 then gives directly the energy per ton-mile for any run and for initial accelerations from 0.6 to 3.0 mph/sec., and for a train friction of 13.6 lbs. per ton, equivalent to $(b) = .15$, a retardation after braking equivalent to 2 mph/sec., and all for an acceleration on the motor-curve to a speed of 50 per cent. greater than the speed with all resistance out.

CS 4 gives the horse-power per ton for the same conditions; in the first paper, motor-capacity was given in kilowatt input; here it is reduced to horse-power output. These curves give the motor-capacity per ton required for any run expressed in terms of the maximum power input at the end of the initial acceleration.

An example will best explain their use. Suppose,

Schedule speed = 30 mph.

Distance = 7000 feet.

Time of stop = 25 sec.

Then

Time from start to start = 159 sec.

“ “ “ “ stop - $T = 134$ “;

Average velocity = $30 \times 159 / 134 = 35.6$ mph

Through acceleration, $A = 35.6 / 134 = .266$

Suppose further that the speed, energy and motor-capacity are to be determined for

(a) = 1.25 mph/sec.,

and a velocity on the motor-curve of 150.

Then,

Relative maximum velocity on type-curve = 160 (CS 5).

(CS 5 is plotted from the velocity-time curves for the various initial accelerations.)

$$x/T = .306 \quad (\text{CS } 1)$$

Hence,

$$ax/T = .306 \times 1.25 = .382$$

and

$$ax = .382 \times 134 = 51.2 \text{ mph.}$$

This is the maximum velocity on the type-curve.

Hence,

$$\text{Maximum velocity on motor-curve} = 150/160 \times 51.2 = 48 \text{ mph}$$

and

$$\text{Velocity at resistance out} = 100/150 \times 48 = 32 \text{ mph}$$

hence,

$$\text{Velocity coefficient} = .32$$

(Velocities are uniformly expressed in terms of the velocity at maximum power, as 100: the actual velocities are then equal to β 100, where β is a coefficient determined as indicated here, or in the manner explained in the first paper.)

$$\text{Time of initial acceleration} = 32/1.25 = 25.6 \text{ sec.}$$

Usually it will not be necessary to determine the times required by the various parts of the run, but this can be done as follows:

$$x/T = .306 \quad (\text{CS } 1)$$

$$y/T = .545 \quad (\text{CS } 1)$$

$$z/T = .149 \quad (\text{CS } 1)$$

$$1.0000$$

Hence

$$x/T = 41 \text{ sec.}$$

$$y/T = 73 \text{ "}$$

$$z/T = 20 \text{ "}$$

$$T = 134 \text{ sec.}$$

Referring to Fig. 2:

$$\text{Time of initial acceleration} \dots\dots\dots (\text{OP}) = 25.6 \text{ sec.}$$

$$\text{Time of acceleration on motor-curve} \dots\dots\dots (\text{PR})$$

$$= \beta \times 112 = .32 \times 112, (\text{CS } 6) \dots\dots\dots = 35.8 \text{ "}$$

(CS 6 is taken from the separate velocity-time curves.)

$$\text{Time of total acceleration} \dots\dots\dots (\text{OR}) = 61.4 \text{ sec.}$$

$$\text{Time of total acceleration on type-run} \dots\dots\dots (\text{OD}) = 41 \text{ "}$$

$$\text{Difference} \dots\dots\dots (\text{DR}) = 20.4 \text{ sec.}$$

$$\text{Time coasting on type-run} \dots\dots\dots (\text{DE}) = 73. \text{ "}$$

$$\text{Difference, time coasting on motor-run} \dots\dots\dots (\text{RE}) = 52.6 \text{ sec.}$$

The various times are then,

Initial acceleration.....	(OP)	= 25.6 sec.
Motor acceleration.....	(PR)	= 35.8 "
Coasting.....	(RE)	= 52.6 "
Braking.....	(EC)	= 20. "
		<hr/>
Total.....	(OC)	= 134.9 "

It is noteworthy that this total, made up from readings taken directly or indirectly from half a dozen curve-sheets, should check exactly; of course such accuracy is partly accidental, but it shows how well graphical work of this character can be done.

The distances passed over for the various portions of the run can be calculated in a similar manner from other curve-sheets, which are not given here.

Further, from CS 4.

$$\text{Motor-Capacity} = 10.8 \left(\frac{7000}{5280} \right)^{\frac{1}{2}} = 12.5 \text{ hp/ton.}$$

$$\text{Energy per ton-mile} = 92 \text{ wh.} \quad (\text{CS } 3).$$

The results of the assumption of any other initial acceleration are obtained in the same manner.

These calculations show that the energy consumption and motor-capacity can be determined when the "through acceleration," the initial acceleration and the amount of use of the motor-curve are given. As I show below, the initial and motor accelerations are fixed by considerations of heating; hence the conditions are entirely determinate.

The input and output of the motor when accelerating to any maximum velocity at any rate can both be obtained by integrating the appropriate power and velocity curves. A comparison of these figures shows that the efficiency for constant maximum velocity is practically independent of the initial acceleration. Both the input and output vary, but their ratio varies slightly; for instance, for a maximum velocity of 125, the input of the motor varies from 804 to 653 wh. when the initial acceleration is varied from .6 to 3.0 mph/sec.; for the same range the output varies from 517 to 415; hence the efficiency varies only from 64.3 to 63.5 per cent., a total variation of only .8 per cent. for a variation from .6 to 3.0 mph/sec. in the initial acceleration. Sixty-four per cent. may be considered the efficiency for a maximum velocity of 125, regardless of the initial acceleration. This is for the motor shown on CS 2, of which the efficiency at rated load is 85 per cent. For a maximum velocity of 175, the efficiency varies from 71 to 73 per cent.

CS 7 gives the efficiency of this motor equipment in terms of the maximum velocity. The values of efficiency from this curve-sheet can be used with small error to determine the input from the output.

A deduction from the fact that the efficiency is independent of the initial acceleration is that the average heat-loss is independent of the initial acceleration, which is also deduced directly. It is true that in the losses, friction loss is included, but the average friction-loss from zero to a fixed maximum velocity will not vary with the initial acceleration; hence the difference,

representing the heat-loss, must be constant to the same degree of approximation that the efficiency is constant.

The formula following gives the work done at the car-axle per ton-mile for various schedules, in terms of the through-acceleration and the proportional part of the total time used in braking. This formula and the efficiency from CS 7 permit the easy determination of the energy-input per ton-mile.

The formula for work per ton at the car-axle is deduced by calculating the work of friction for the entire distance, and the kinetic energy at braking; by the substitution of

$$L = .682 AT^2$$

the through-acceleration is included, and hence the schedule. The formula is:

Work per ton-mile at car-axle = $180b + 90 \left(c - b \right) \frac{c}{A} \left(\frac{z}{T} \right)$ wh.

The first term, $180b$, gives the work of friction per ton-mile; the other term, the energy wasted in brakes per ton-mile. z/T is a function of the three accelerations, (a) , (b) and (c) , as well as of the through-acceleration. Hence the values of z/T to be used in this formula must be obtained from curve-sheets plotted for the particular values of (b) and (c) that are substituted in the formula. CS 1 is calculated for $(b) = .15$ and $(c) = 2$; substituting these values in the above formula, it becomes,

$$\text{Work} = 27 + 333/A \times \left(\frac{z}{T} \right)^2 \text{ wh.}$$

To apply this formula, find first the value of the through-acceleration on the type-curve corresponding to the through-acceleration A on the motor-curve, given by the data of the problem. CS 8 gives this relation for a maximum velocity of 150. Having this value of A^1 , the through-acceleration on the

type-curve, select from CS 1 the value of z/T corresponding to that value of A^1 and of the initial acceleration in question. For example,

$$\begin{aligned}\text{For } A &= .266 \\ \text{and velocity} &= 150\end{aligned}$$

CS 8 shows that A^1 equals .275. The value of z/T for this value of A^1 and for an acceleration of 1.5 mph/sec., from CS 1 is .15, substituting in the above formula the work per ton-mile at the car-axle is found to be equal to 56 wh.; the efficiency, from CS 7 to 70 per cent.; hence the energy-input is 80 wh. CS 3 shows for the same conditions; *i.e.*, for $A = .266$ and $(a) = 1.5$, and energy-input equal to 83 wh., an agreement sufficiently close.

In the first paper, I assumed that an electric tramway motor in service will carry, on an average, a heat loss of 3 per cent. of its "hour-load," with a temperature elevation of 75° C. This is a fair average figure, but individual motors differ greatly in their heating characteristics; hence I have elaborated the method so that it may be used for motors having any copper-loss and any core-loss, for any schedule with any initial acceleration.

For this purpose I determine the ratio of average loss to loss at rated load for the copper-loss and core-loss separately, and for the several initial accelerations; knowing then the ratio of average loss during acceleration to loss at rated load, the average loss for any particular motor is found by multiplying the proper value of this ratio by the loss at rated load, separately for copper- and core-loss; the results added will give the total average loss during acceleration. These ratios of average to maximum loss were determined for velocities of 125, 150 and 175 on the motor-curve; that is, for maximum velocities 25, 50 and 75 per cent. greater than the velocity with resistance out. In describing this method further, it is to be understood that this procedure was followed in all cases; in other words, three separate sets of curves were carried through for the three maximum velocities, for the heat losses, for ratio of distribution, for motor-capacity, for energy-curves, and for the various other intermediate curves which were used as steps to the final results.

The rise of temperature of the parts of a motor, as determined by tests, depends upon the ratio of the distribution of the total heat-loss between the armature and field; to apply such test results correctly this ratio must then be known for the various conditions; hence the ratio of distribution was calculated for

the several initial accelerations and maximum velocities; the average loss during acceleration and its distribution having been determined in this manner, the average loss during the run will be known when the ratio of time of acceleration to time of run, for the various initial accelerations, maximum velocities and for all values of the through acceleration is known.

Having then the average loss during the run and its distribution between armature and field, the rise of temperature of a motor, used under the conditions of service fixed by any particular ratio of distribution, can be determined by comparing this calculated average loss with the rise of temperature per watt lost, for the same ratio of distribution, as determined by tests in service.

These steps required the preparation of nearly a hundred curve-sheets and some four hundred curves; the matter is too voluminous to present here in detail; I shall therefore give only the results and call attention to a few of the principal stops.

The ratio of average to maximum copper-loss is definite and was determined in the usual manner by plotting values for the current and integrating.

The ratio of average to maximum core-loss depends on the variation of core-loss with speed and current; to fix this variation, the individual core-loss curves of a number of motors were plotted, and a curve determined showing the average variation of the core-loss. CS 9 shows the curve of the average variation of core-loss used in this investigation; the values from this curve were plotted on the various velocity-time sheets and integrated; this gives the core-loss during acceleration on the motor curve. In addition to this, core-loss curves of several

motors, differing widely, were selected, and the ratios of average to maximum for different operating conditions were determined. These ratios were found to differ comparatively little for the different curves, much less than the actual values of the core-losses differ. The average core-loss during the period of acceleration on resistance was taken at 42 per cent. of the maximum core-loss. This value was obtained from a number of separate core-loss curves.

The average percentage heat-loss during acceleration to the same maximum velocity for any motor is practically independent of the initial acceleration; the ratio of average to maximum copper-loss increases with the acceleration, while the ratio for the core-loss decreases.

The variation of the loss with initial acceleration in two typical cases is given in the tables following:

TABLE I.

Average heat-loss, for velocity of 150.

Copper loss at rated load = 3%

Core " " " " = 3%

Initial Acceleration.	Average Loss.
1 mph/sec.	3.88%
1.5 "	3.91
2. "	3.91
2.5 "	3.91

TABLE II.

Average heat loss, for velocity of 150.

Copper loss at rated load, = 7%

Core " " " " = 1%

Initial Acceleration.	Average Loss.
1.0 mph/sec.	4.77%
1.5 "	4.92
2.0 "	5.00
2.5 "	5.03

For velocities greater than 150, the variation is greater; for lower velocities it is less. The values for $(a) = 1.5$ are used.

Although this approximation is satisfactory for the purpose, yet if greater accuracy is desired, it is possible to plot the average loss for the various initial accelerations separately; but the complication is very much increased.

CS 10 gives the average percentage heat loss during acceleration for any combination of copper- and core-losses within the range covered, for a velocity of 150 on the motor-curve. The ordinates are average loss, the abscissas core-loss, and the inclined lines copper-loss; for instance, for a 4.0 per cent. copper loss and 2.5 per cent. core-loss, the intersection of the vertical at 2.5 with the inclined line marked 4, gives the average, loss of 4.12 per cent., and this will be the average loss occurring in a motor accelerating by series parallel, and then on the motor-curve to a velocity 50 per cent. greater than the velocity when resistance is cut out, the motor having a 4 per cent. copper- and 2.5 per cent. core-loss; the capacity being determined by the product of the torque during initial acceleration and the velocity with resistance out, that is the maximum power input.

The ratio of distribution is practically independent of the initial acceleration for a given maximum velocity, and for the range of initial accelerations used, *i.e.*, from $(a) = 1$ to $(a) = 3$. While this is not strictly correct, yet the difference between the values calculated accurately and the values found by assuming an average ratio of mean to maximum copper and mean to maximum core-loss, does not exceed 5 per cent. for the range covered; since this ratio is used only to fix the appropriate value of the radiation-constant (of which the determination is in itself necessarily rough) the degree of accuracy is sufficient.

CS 11 gives, for a velocity of 150, the ratio of distribution as ordinates, with core-loss as abscissas, and with inclined lines for the various copper losses; for instance, with copper-loss of 4.0 per cent. and core-loss of 2.5 per cent., the ratio of distribution is 2.44. It is assumed that the copper-loss is half in armature and half in field.

These curves then determine the average loss during acceleration and its distribution; to get the average loss during the run the ratio of time of acceleration to time of run must be known.

Referring to Fig. 3,

$$RH/OR = AD/OD$$

i.e.,

$$(100 + at_2)/Y_t = (t_1 + t_2)/x$$

and

$$\begin{aligned} (100 + at_2)/Y_t \times x/T &= (t_1 + t_2)/T \\ &= \text{Time of acceleration divided by total time.} \end{aligned}$$

x/T is taken from CS 1 for the various values of A and (a) , thus the ratio of the times is determined for all conditions; the corrections to the type-curve must be allowed for.

For example,

For (a) = 1.25 mph/sec.

and A = .2 "

$$\text{Average loss} = 1.48\% \quad (\text{CS } 12)$$

and therefore

$$1.63 \times 1.48\% = 2.41\%$$

is the average loss or the second motor for the same run. Similarly for other combinations of copper- and core-loss.

These curve-sheets determine the average heat-loss of any motor for any run, and its ratio of distribution; it remains then to connect these calculated values with results of test made under comparable conditions of service, giving the temperature rise for various losses, *i.e.*, with the radiation coefficient of the motor under consideration.

Many tests of this nature have been made by the General Electric Company on its experimental track at Schenectady, by running trains back and forth over the track until permanent conditions of temperature have been attained, and at the same time, measuring the losses in the motors.

CS 13 gives such test results for a motor in general use; the ordinates give the average loss in percentage of input that the motor will carry with a rise of temperature of 75° C. measured by resistance under the conditions of service determined by the ratios of distribution denoted by the abscissas; these curves are calculated from test results which give the rise of temperature per watt lost for field and armature separately; the curves are given for tests made with covers on the motors and covers removed. Thus CS 13 shows that for a ratio of distribution of 2, the armature of motor No. 13 will carry, with covers off, a load giving a loss of 4.82 per cent., whereas the fields will stand only 3.1 per cent.; but if this motor be used with covers on, then at the same ratio the armature will stand a load giving a 2.05 per cent., and the fields at 1.88 per cent. loss, very nearly the same, as should be.

These test results are used as follows:

Motor No, 13 has

Copper-loss = 6.05%

Core- " = 2.10%

Then

Average loss during acceleration

$$\text{to velocity 150} = 5.06\% \quad (\text{CS } 10)$$

$$\text{Ratio of distribution of losses} = 1.77 \quad (\text{CS } 11)$$

At this ratio of distribution, the permissible loss, with covers on, is

Armature, 1.98% (CS 13)

Field, 1.80% (CS 13)

Hence the field controls, and the motor can carry only a load giving a loss of 1.8 per cent.; the armature will then run much cooler than 75°; in fact, at $1.8/1.98 \times 75^\circ = 68^\circ \text{ C}$.

CS 12 is plotted for an average loss of 4.12 per cent., hence to apply it to motor No. 13, the ordinates should be increased in the ratio of $5.06/4.12 = 1.23$: if this were done and a horizontal section of CS 12 were taken with the 1.8 per cent. line for ordinate, the initial accelerations would be determined, for each through-acceleration, that would result in a loss of just 1.8 per cent. and consequently in the exact temperature rise permitted;

but the same numerical results will be obtained by taking a cross-section of CS 9 at the ordinate $1.8\%/1.23 = 1.46\%$.

The dotted line marked "Motor No. 13" shows this cross-section. If the results of measurement by resistance, with covers off, be taken, the permissible loss for the field is 2.9 per cent., by CS 13; for this condition the cross-section line should be drawn on CS 12 at $2.9/1.33 = 1.35$ per cent.

The values of (*a*), the initial acceleration for measurements by resistance with covers off, as determined by the cross-section line at 2.35 per cent. (not shown on CS 12), are plotted on CS 14 in terms of *A* as abscissa; the corresponding values of energy per ton-mile and of horse-power per ton for velocity of 150 are taken from CS 3 and 4. Thus, CS 14 gives the data for this motor under any conditions of service, for a maximum velocity of 150. The comparison has been made on the tacit assumption that the test results of CS 13 represent average losses during running time; if, as is usual, the test values represent average losses during total elapsed time, they must be increased in the ratio of total elapsed time to running time and this increased value used for the cross-section of CS 12.

This is all for a rise of temperature of $75^{\circ}\text{C}.$; if the permissible rise be 60° , then the permissible loss will be only

$$60/75 \times 1.8\% = 1.44\%,$$

and 12 must be cross-sectioned at

$$1.44/1.23 = 1.17\%,$$

to determine the proper accelerations for the various through-accelerations.

CS 12 then gives the rise of temperature of any motor for any through acceleration, with any initial acceleration, after its cross-section line has once been fixed; for example, for this motor the 75° rise is given by the accelerations having for ordinates 1.46 per cent.; suppose that for the through-acceleration $A = .2$, an initial acceleration of 1.0 be used; then the average loss will be 1.93 per cent., and since temperatures are proportional to losses, the temperature of the hottest part will be $1.93/1.46 \times 75^{\circ} = 99^{\circ}\text{C}.$ If an initial acceleration of 1.75 mph/sec. be used, the rise of temperature will be $1.00/1.46 \times 75^{\circ} = 51^{\circ}\text{C}.$

That is, for any motor and any value of *A*, the ordinates of CS 12 will give the rise of temperature for any initial acceleration; each initial acceleration corresponds to a definite rise of temperature for each through-acceleration; a vertical cross-section of CS 12 at any value of *A* shows the relation of rise of temperature and initial acceleration for that through-

acceleration. The dotted vertical line on CS 12 gives the temperature scale for motor No. 13, for $A = .25$ and $(a) = 1$, the rise will be 134° ; for $A = .15$ and $(a) = 2$, the rise will be 40.5° . The temperature scale can thus be drawn on CS 12 for any motor, knowing its heating constants.

In order to avoid confusion, it is essential to recall that each change of initial acceleration means a proportional change in motor-capacity, the motor-capacity being directly proportional to the product of initial torque and the rated velocity. It is thus tacitly assumed that the engineer has a complete range of motors of all capacities, all with the same heating constants; but the results are given in horse-power per ton and hence they give merely a determination of the load that any given motor will carry, under the various conditions.

Returning to the example given on page 6, where the initial acceleration was assumed to be 1.25 mph/sec.; if the heating constants of the motor to be used are known, the initial acceleration will be fixed; assume the motor for this run to have these constants:

$$\begin{aligned} I^2 R\text{-loss} &= 3.6 \% \\ \text{Core-loss} &= 2.75\% \end{aligned}$$

Then

$$\text{Ratio of distribution} = 2.8 \quad (\text{CS 11})$$

$$\text{Average loss during acceleration} = 4.06\% \quad (\text{CS 10})$$

Assume,

$$\text{Radiation at ratio of 2.8} = 1.6\%$$

Then a horizontal section of CS 12 having as ordinate

$$4.12/4.06 \times 1.6\% = 1.62\% \text{ gives,}$$

for $A = .266$ — the through acceleration under consideration, —

$$\text{Initial acceleration} = 1.5 \text{ mph/sec.}$$

and

$$\text{Energy} = 83 \text{ wh} \quad (\text{CS 3})$$

$$\text{Capacity} = 1.16 \left(\frac{7000}{5280} \right)^{\frac{1}{2}} = 13.3 \text{ hp/ton.} \quad (\text{CS 4})$$

This example corresponds to an actual case; the engineers of the project have decided to use an initial acceleration of 1.5 mph/sec., and expect to obtain an energy consumption of 87 wh. per ton-mile; these figures agree closely with those above.

CS 14 gives what may be called the "characteristic performance" of Motor No. 13, used for any schedule as determined by resistance measurements with covers off. It is obvious that curve-sheets similar to this can readily be plotted

for maximum velocities of 125 and 175. A comparison of such curve sheets for the different maximum velocities will, in general, show slight variations in motor-capacity for different initial accelerations, for a given schedule. In some cases the minimum motor-capacity is attained with an average maximum velocity, say, from 140 to 150; in other cases the motor-capacity diminishes as the maximum velocity is increased, up to the limit used in this investigation. No general conclusions can be drawn for the particular relative maximum velocity to be used; this depends principally upon the heating constants of the motor, including the radiation. It is clear, however, that by the aid of curves such as CS 12 for the various maximum velocities, the minimum motor-capacity can be found for any particular motor; all that is necessary is to draw the cross-section lines at the proper

points, thus fixing the value of the initial acceleration, and then to select the values of energy and horsepower from the different curve-sheets. In the case of Motor No. 13, CS 14 for velocity of 150 gives practically the minimum motor-capacity; there is a diminution for greater maximum velocities, but it is comparatively unimportant.

In all cases, the particular initial acceleration leading to the minimum motor-capacity should be adopted, even though this initial acceleration does not give minimum energy. The curves give the energy required per ton of total weight; hence the energy required per car is proportional to these figures multiplied by the relative weight of the car. The difference in the energy per ton for any schedule, is comparatively slight for the different maximum velocities, and in all cases the energy required per car is a minimum for the minimum motor-capacity with which the schedule can be made; the increase in the weight of equipment much more than balancing the diminished energy per ton obtained by using higher initial acceleration and higher motor-capacity. CS 15 gives the ratio of weight of equipment of different capacities; it is substantially the same as CS 15 of the former paper. The energy per ton multiplied by the ordinates of this curve-sheet gives relative figures showing the energy per car.

By this method, the two quantities that have, up to this point, been matters of arbitrary choice, are determined, *i.e.*, the initial acceleration and the relative maximum velocity on the motor-curve. Having then a motor with known heating constants, all the conditions which should govern its use for any particular schedule are fixed, and consequently the difference in the results attained by the use of motors with different heating constants is also determined.

I have illustrated the application of this method in comparing the availability of motors with different heating constants for a particular service. It is clear that it can also be used to obtain the data for different schedules. As an example of this use, suppose the data for various schedule speeds for a run of 7,000 feet, with twenty-five second stops, are to be determined.

Assume the motor represented by CS 14; then Table III. will show the performance of this motor for the various schedule speeds. In this Table, columns 1, 2 and 3 are calculated from the length and time of stop; columns 4, 5 and 6 are taken from CS 14; column 7 gives the ratio of the numbers of column 6, taking 7.4 as 100; column 8 is taken from CS 15; column 9 gives the relative total energy required per car; it is obtained by dividing the product of the figures of columns 5 and 8 for each schedule speed, by the corresponding product for a schedule speed of 25 mph. As the weight ratios of CS 15 are for a motor-car of the same size, but with different motor equipments, the figures of column 9 give the relative energy re-

quirements per car-seat or per passenger for the different schedule speeds. Column 7 of this table may be taken as a sort of rough indication of the relative cost of the equipment for the different schedule speeds, and column 9 of the relative operating costs. They illustrate forcibly the great increase required both in motor equipment and in energy consumption for comparatively small increases in schedule speed.

TABLE III.

Schedule Speed	Average Speed.	Through Acceleration.	Initial Acceleration.	Wh/Ton.	Hp/Ton.	Relative Motor-Capacity.	Relative Weights of Equipment.	Relative Total Energy.
1	2	3	4	5	6	7	8	9
25.	28.8	.174	.77	63	7.4	100	1.49	100
27.5	32.1	.216	.93	77	9.4	127	1.65	137
30.	35.6	.266	1.15	99	12.3	166	1.89	199
32.5	39.1	.320	1.42	127	15.9	215	2.37	320
35.	42.9	.386	1.84	180	24.0	324	2.92	560

The curve-sheets show initial accelerations as high as 3 mph/sec., and through-accelerations up to $A = .4$; these values are beyond the range of practice, both on account of the excessive motor-capacity demanded and the close approach to the slipping point of the wheels, with the usual percentage of total weight on drivers. The boundary lines of the curve-sheets can fairly be taken as

$$A = .1 \text{ to } A = .3$$

and

$$(a) = .8 \text{ to } (a) = 2.$$

This indicates the comparatively narrow limits within which all schedules are brought by the use of the quantity A .

It is obvious that the methods outlined here can be applied to alternating current motors by the use of appropriate speed-torque curves in place of those of CS 2.

The method which has been developed takes account of all the factors governing the rise of temperature of motors under con-

ditions of service. While certain assumptions have of necessity been made—as, for instance, a constant value for train-friction—it is perfectly feasible to re-calculate the curve-sheets with other values of train-friction; a set of curve-sheets similar to CS 12, including as many maximum velocities as may seem desirable, can be prepared and used as standard curves from which the rise of temperature of any motor can be determined very simply, for any conditions of service.

While the results obtained by this method are of necessity an approximation because of the number of assumptions that it is necessary to make in developing the method, yet numerous applications of the curve-sheets to practical examples have shown that the approximation is close, and that the curves furnish a useful means for determining the performance of any motor in service.