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ART. XXXV.—*The Queen's Chamber in the Great Pyramid;*
by Prof. H. L. SMITH. With Plate II.

THE so-called "Queen's Chamber" in the Great Pyramid of Jizeeh has not hitherto received as much attention as has been bestowed upon the other portions of that most remarkable structure; and yet, as we shall presently show, it is of prime importance in deciding upon the truth or falsity of certain hypotheses which have been proposed as to the objects of causation of this wonderful building. For the present, without expressing any opinion ourselves, we shall present certain facts, leaving our readers to make their own inferences. We may, however, premise that the most commonly received hypothesis is, that the great pyramid, and all the other pyramids in Egypt, are gigantic tombs, built for the safe preservation of an enclosed sarcophagus and the kingly body once placed therein.

So far as all the other pyramids are concerned, we will consider this hypothesis admitted, but there are a few who believe that the great pyramid was built for the preservation of certain grand metrological standards, and for the purpose of conveying to man in these latter days some remarkable astronomical truths and high mathematical (geometrical) relations; and they call upon us to admit, either that this knowledge was given by inspiration, or that a high degree of civilization existed, and a knowledge of geometry, arithmetic and astronomy, not merely up to, but in some respects surpassing, that of the present day, was possessed by man, 4000 years ago. Either horn of this dilemma is sharp enough; and it is, as bearing upon

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the necessity of accepting one of them, that we present, as briefly as possible, the result of our investigations in the "Queen's Chamber." For the benefit of those who may not at once recall the structure of the great pyramid, we refer to fig. 2, plate II, which is a sectional view, in the plane of the meridian, looking eastward.

1, is the entrance passage, inclined at an angle of about $26^{\circ} 27'$; this passage, at a distance of some 300 feet, enters into the subterranean chamber 11, which is an unfinished room, situated about 106 feet vertically below the base of the pyramid, and it is cut in the limestone rock upon which the pyramid is built. The entrance passage is rectangular, about 41 inches in breadth and 47 inches in height, lined with polished limestone of a much finer quality than that constituting the mass of the pyramid; and the joints are exquisitely fine. About 30 feet within the passage, at 2, in fig. 2, there is on either side a *thin, fine, exquisitely-ruled line*, cut by a master hand, exactly vertical by severest tests. Whatever else these lines may symbolize, we find them to be so situated that, to an observer placed midway between them, in the axis of the passage, the walls at the entrance would be tangents to the diurnal path of a pole star (α Draconis), at a distance of $3^{\circ} 42'$ from the pole; within limits of the thickness of the casement. At 3, the ascending passage commences, at an inclination of $26^{\circ} 18'$ very nearly. Originally, this ascending passage was closed by a triangular stone, inserted in the roof of the descending passage at 3, and effectually concealing it. This stone was accidentally dislodged, during the progress of the forced entrance made into the pyramid by the Khaliph Al Mamoon, some 3000 years after the erection of the pyramid, and by its fall revealed the ascending passage, still closed, however, by a granite block about 8 feet long, which, apparently, had been caused to slide down the passage, previous to the insertion of the triangular stone; it still remains in place, and the ascent is now made by passing around it, in what is known as Al Mamoon's hole. A careful examination of the floor directly under the ascending passage shows the insertion of a different, much harder stone, with joints diagonal to the passage, all the others being at right angles; perhaps intended to indicate to any one really searching for it, that just above was the place to look for the upper passage.

Ascending for about $128\frac{1}{2}$ feet, the grand gallery is entered at 4. This gallery has a length of 157 feet, nearly, and a height of 28 feet; the walls have seven overlappings, and a stone bench, or ramp, along either side; the distance between the ramps being 3 feet 6 inches, and above them 6 feet 10 inches. Originally the floor of the gallery was continuous, but a portion

removed near the commencement of the ascent shows the horizontal passage into the Queen's Chamber, 9. This neat room is lined with white limestone, exquisitely polished; though now covered with dirt, and saline incrustations in some places nearly or quite one inch in thickness; the joints of the stone are cemented, and of microscopic fineness, often within the thickness of a hair. Unlike the other chambers of the pyramid, this has a gable-roof; and the floor is rough; the floor is 205 inches by 227 inches, the height to the apex of the roof is 244·2 inches, and under the eaves 182 inches, nearly. A very nicely constructed niche, about 15 feet high, and some 40 inches deep, is cut into the eastern wall; it is eccentrically placed, its axis being 25 inches south of the central line of the east wall. The workmanship of this niche is very excellent. This is the room and niche to which we now invite attention, and hitherto there has been scarcely an attempt at an explanation; none whatever upon the tombic hypothesis.

Passing up the grand gallery, we enter the little ante-chamber 6, a very curiously constructed room, partially lined with polished granite; from this we enter the so-called "King's Chamber," 8, upon the floor of which stands the coffer, or sarcophagus, as some suppose, 7. This noble room, probably the finest ever built, is lined with polished granite, in five courses all of the same height, 47 inches, except the lower course, which by a rise of 5 inches of the polished granite floor, inside the room, is only 42 inches in height; the object of this appears to be quite clear, but it is no part of our present purpose to discuss the features of this room. The chamber is approximately 34 feet 4 inches long, 17 feet 2 inches in breadth, and 19 feet 2 inches in height. Nine beams of polished granite cross the room from north to south, forming the ceiling, (one of these stones would weigh about 60 tons.) The stone coffer is rectangular, of polished granite, but now shamefully broken and abused; it is entirely plain, and indeed, no hieroglyphs are anywhere found in the great pyramid. For a discussion of the significance of the coffer, and the dimension of the King's Chamber, we must refer to the works of Prof. Piazza Smyth, John Taylor, Mr. St. Vincent Day, William Petrie and others. Above the King's Chamber are five small chambers, entered by a forced passage; probably constructed to relieve the enormous pressure of the superincumbent mass above the ceiling of the King's Chamber. In these "chambers of construction" were found some rough quarry marks, (not incised,) expressing the names of kings of the fourth dynasty, Shofu and Nu-Shofu. The date of erection of the great pyramid is astronomically determined by computing the time when α Draconis was the pole star, with a distance of $3^{\circ} 42'$ from the

pole; and it is especially as bearing on this question, that our own investigations in the Queen's Chamber become valuable. Upon this supposition, the pyramid was built, in round numbers, 2170 years B. C. The base of the pyramid is a square covering some thirteen acres, and the sides of this square are very exactly, two in the plane of the meridian, and two in that of the prime vertical.

The standard of length, which appears to have been adopted in the pyramid, is 25,025 British inches, which happens to be the one-ten millionth of the polar radius of the earth, and, so far, a much more convenient unit than the French meter, which is the same proportional part of a particular quadrant. The height of the pyramid, as determined by the most careful discussion of all the measures hitherto made, is given by Prof. Piazzzi Smyth at 5813 pyramid inches, = 5819 British inches; and the length of base as 9131·05 pyr. inches, = 9140 British inches, nearly. These measures we adopt.

Supposing now the triangle XNS, fig. 2, to be so constructed that $2CX:4NS::1:3\cdot14159$, &c., i. e., the circle struck with the height of the pyramid as radius, to have a perimeter equal to the perimeter of the square base of the pyramid, then, upon this supposition, the angles XNS, and XSN, can be computed, and they are found to be $51^{\circ} 51' 14'' \cdot 3$. And again, peculiar to a triangle with the proportions we have supposed, are three other angles, viz.

26° 18' 10'' *	Upper culmination of pole star.
30	Latitude.
33 41 50	Lower culmination.

And also, derived from above, $41^{\circ} 59' 48'' \cdot 7$, angle of arris. With these angles demanded, we enter the Queen's Chamber; let us see what we shall find there.

We remark that Prof. Piazzzi Smyth had no conception of the amazing importance of the Queen's Chamber when he made his measurements; and, as he had no theory with regard to it, whatever we deduce from these measurements comes, so far, unbiassed; at the same time, there are certain portions which we could have wished he had more carefully measured; but when he states that other portions *have been* measured with care, we feel bound to adhere very closely to his figures.

Having an idea from the roughness of the floor, that at some time a lining might have existed, he searched for evidence of it along the vertical walls; he assures us that he "had to follow the walls upward from their very bottom, to the level of the first overlappings of the niche in the eastern wall, and to the

* See "Life and Work at the Great Pyramid," by Prof. C. Piazzzi Smyth, vol. iii, p. 41, et seq.

top of the northern doorway, before any horizontal cuttings were met with"; this is the height of the line K'L, fig. 1, and was measured by him as 66·7 Br. inches, or 66·634 pyr. inches nearly; does this mean anything?

We turn to a table of natural tangents, and lo! 6668 natural tangent, $33^{\circ} 41' 50''$. Farther up, Prof. Smyth measured the height of the line KS', fig. 1, as 127 inches, mean of two measures, one being 127·2 inches; he says, however, that having no ladder to stand upon, the "measurements of the upper portions are rude in the extreme;"* but from table of tangents we have 1·2732 equal to natural tangent, $51^{\circ} 51' 14''$. Taking, however, 127· given as the actual measure, makes but a difference of some 4' in the angle, a difference which, considering that the floor just under the niche, and also the back of the niche itself, up to this height, has been more or less broken, and removed, in the search for treasure, or hidden passage, is, we think, quite as near as could be expected. Passing now two stages higher of the niche, we find two measures of this, one 184·8 Br. in., the other 186·4 Br. in. We feel quite justified, then, in this difference of 1·6 in., in adopting 185·218 Br. in., or 185·033 pyr. in., as the true height, especially as this particular number is again tested hereafter by an entirely different process, and proved correct. The line N'B, fig. 1, marks this height, and BK is equal to 57·713 pyr. in., but 57·713 is the natural tangent of $29^{\circ} 59' 24''$. The true latitude of the pyramid is, according to the laborious measurements of Prof. Smyth, $29^{\circ} 58' 51''$, and by M. Nouet, in the celebrated national French work, $29^{\circ} 59' 6''$.

The pyramid, then, is not in lat. 30° N. exactly; but the ancient builders, in their endeavor to make it so, have pushed it to the extreme N.E. boundary of the hill, and have used the chippings of the stones, an immense mass, to build out the hill on this side, and to strengthen it. We have now three of the angles required, with sufficient exactness, and the others follow from them; but we have, as yet, scarcely made a beginning. We consider now the geometrical construction of the room, assuming as data, for east wall,

Height,	185·033 pyr. in., as above.
Breadth,	205·00 " "
Height of gable,	244·2 " "

205 in. and 244 Br. in., are given by Prof. Smyth as the most probable approximations.† Our numbers are evidently not quite exact, as the construction will show, yet they are very close. We begin then with a rectangle, fig. 1, N'BDH; N'B, equal to 205 in., and BD, 185·033 in. (all hereafter pyr. measure, i. e., 25 pyr. in. = 25·025 Br. in.); bisect N'B in O, and HD in A, join

* "Life and Work, &c.," vol. ii, p. 66.

† Ibid.

AO and prolong it to E, making AE equal to 244.2 in. Join EB and EN'; the angles EBO and EN'O are each equal to 30° [I].*

Join ED; it will cut N'B in C, and OC will equal 25 inches. Join HO; produce it to meet EB in F, and from F drop a perpendicular upon OB; it will meet OB in C; prolong FC, to meet HD in B' it is the axis of the niche. Our equations [II] and [III] give results which differ not quite .2 in. from 25 inches, but, as this number has been adopted by Prof. Smyth, and as a very slight variation in our assumed elements would give it exactly, we assume $OC = 25$ in. Set off DL equal to 66.68 in., and DK equal to 127.32 inches, as indicated above; draw K'L and S'K parallel to each other and to HD. Join EL; the triangle EBL is isocles; the angles at the base are each 30° ; the same is true of the triangle EN'K' [IV]. The triangle EK'L is equilateral; the angles at E, K', and L being each 60° , $\therefore EK' = K'L = EL = 205$ in.

Join N'L and BK'; they bisect each other in the point M, in the line EA; and the angles MN'O, MBO, MK'N, and MLN, are each equal to 30° [V]; $\therefore K'B = LN' = 2BL = 2N'K'$, and $BL = MB = BE = EM = EN' = N'M = N'K' = K'M$. With M as a center, and radius = ME, describe the arc N'EB; N'M will represent the polar axis, and E the zenith, at the latitude of the pyramid. Draw S'MX' parallel to S'K; it will represent the horizon. Through the intersection S, of EL and N'B, draw MSF'; it will bisect EB in F', and since the angle F'MX' is equal to 60° , MF' will represent the equator at the latitude of the pyramid; also MF' will equal OB [VI].

From the point F', drop the perpendicular F'H' upon HD; it will bisect OB in C'', and pass through the point P, where MB and EL bisect each other [VII]. K'B cuts S'K, in I and IK = 100 inches [VIII], (and I is one inch from the corner of the niche, x.) Join ID, and prolong it to meet AE in γ ; join γ H; then will H γ D represent a meridional section of the great pyramid, and the angles γ HD and γ DH will each be equal to $51^\circ 51' 14''$ [IX].

Join L γ , and prolong it to meet HN' produced, in γ' ; join $\gamma'M$; the angle $\gamma'MS''$ will equal $33^\circ 42'$ nearly [X], upper culmination of a pole star, which at this distance from the pole in 2170 B. C. was found by Sir John Herschel to have been α Draconis.

Through J, intersection of K'L, and axis of niche, draw DJ, meeting HN' in δ ; join δM ; the angle $\delta MS''$ will equal $26^\circ 18'$ nearly [XI], direction of lower culmination of α Draconis.

Set off CO', equal to 12.5 inches join K'O', intersecting EA in V. Join HV, and DV; the angles HVA; and AVD will each be equal to $33^\circ 42'$ nearly [XII].

* "The numbers in brackets refer to the equations at the end of this article.

Produce HV, it will meet H''C'' produced in C', in the arc EC'B [xiii]; join MC'; the angle C'MF' will represent $4^{\circ} 20'$ north declination [xiv].

But the declination north, of η Tauri, the lucida of the Pleiades is computed by the well-known astronomer Brunnöw, to have been, in the year 2170 B.C., $4^{\circ} 13' 46''$.

His computation shows further, that its right ascension at this time was $0^{\circ} 57' 26''$, or that it was nearly in the equinoctial point; and moreover, that at this time, when η Tauri and the equinoctial point were on the meridian *above*, at C' and E', α Draconis was within 20^m of culmination at δ , *below* the pole; and hence the reason why the entrance passage was made to point to the lower rather than the upper culmination, the date of the pyramid erection being commencement of the grand Pleiades year.

We will now proceed to construct the niche, having already the axis and the point O'. We assume also that the line C'H'' marks out one side of the niche, though upon calculation it differs from it .12 inches, with our assumed data for height and breadth of the east wall. We set off A'H', equal to 61.4 inches, as we find upon trial that 61.3, measured by Piazzi Smyth, is somewhat too small, yet this number, with slight change of breadth of the room, would do as well. The line S'D' will cut the corner of the niche very nearly in N'' [xv].

Join N''K; it will cut C'H'' in D', corner of the niche; by the computation D'G = 31.86 inches [xvi], but as the line C'D'' does not quite agree with the side of the niche (with our present data), we take 31.84 inches as the true height. From G, draw GX parallel to γ G'; it will meet KS' in X, a corner of the niche.

If the point G was in the line C'H'', we would have 52.5 in. for the breadth of this part of the niche; this would be perhaps too great a strain on Piazzi Smyth's measure. 52.3 Br. inches. If, however, we suppose the corner of the niche, G, to be .1 in. nearer the axis, this would give his measure exactly; thus, for the half breadth; $CB - (C''B + .1)$ or $77.5 \text{ in.} - 51.35 \text{ in.} = 26.15 \text{ inches}$. We next compute XX'', remembering, however, that the line GX'' is .1 inch toward the axis; it is found to be 47.63 [xvii]; adding to this .1 inch and also 51.25 in., the value of KX, is found to be 98.98 in., or practically 99 inches. $\therefore$ IX = one inch. The corner of the niche X is therefore 3.5 inches from the middle of the room, which gives us 21.5 inches for the half breadth of the niche at this place.

The height of the fourth compartment is determined by the line T, T', drawn parallel to N'B, through the point P, and is found to be 28.13 inches, and at the same time the height of the upper compartment is determined to be 29.58 inches (xviii). The breadth of the fourth compartment, 32.2 inches, is deter-

mined by joining the points O and H'. The line OH' cuts TT' in W', a corner of the niche, bisecting the angle at W' (xix). The breadth of the top compartment is already determined as 25 pyramid inches, or the "sacred cubit."

We now place these determinations alongside the measurements of Prof. Piazza Smyth for comparison.

	HEIGHTS.		BREADTHS.	
	C. P. S. Br. in.	H. L. S. Pyr. in.	C. P. S. Br. in.	H. L. S. Pyr. in.
1st Compartment,	66·7	66·68	61·3	61·4
2d "	31·6	31·84	52·3	52·3
3d "	28·7	28·8	43·3*	43·
4th "	29·4	28·13	34·3*	32·2
5th "	29·4	29·58	25·3*	25·

The agreement is quite within the errors of measurement; but as these computed values depend on the assumed values of height and breadth of the east wall, they are subject to slight correction when these are better known. With these computed values we will determine the angles made by drawing the diagonals of the niche, O'R, W'W, IG', and ZH', and we obtain

The angle at O'' (xx),	26° 21' upper culmination.
The angle at W' (xxi),	33° 30' lower culmination.
The angle at X (xxii),	38° 8' 46" exactly,

being the half angle at the summit of the pyramid, and giving, for the angle at the base, 51° 51' 14". And lastly, the angle at Z (xxiii), 30°, latitude of the pyramid.

The discovery of these angles, which are also substantially given by using Prof. Smyth's numbers, was first made by me, and announced to Prof. Smyth before any one had suspected this design of the niche. It is most fortunate that the measures of Prof. Smyth, recorded in vol. ii of his "Life and Work at the Great Pyramid," were all made, so far as the Queen's Chamber is concerned, independent of any theory.

We have not by any means finished with this peculiar chamber. If we suppose now a diagonal drawn from E (toward the spectator), and touching the west wall at the height of the line LK', we obtain once more the angle 51° 51' very nearly (xxiv), and we may also notice, that the length of the room is a mean proportional between the height of the niche and the breadth of the room. The ratio of height to breadth of the east wall is nearly 9:10, and the diagonal HB gives therefore HBD, the arris angle, 42° nearly. ∴ BHD=48° [xxv].

We have alluded to the π theory of the great pyramid; let us see what the Queen's Chamber gives us here, observing

* Estimated—not measured by Prof. Smyth, but determined by considering the offsets all equal, i. e., 4·5 inches.

simply that divisions of 5^s and 10^s characterize the pyramid ratios—

$$185\cdot033 \times 10 \times \pi = 5812\cdot978 \text{ pyr. inches.}$$

But 5813 in. is the most recent and accurate determination of the height of the great pyramid adopted by Prof. Smyth after diligent comparison; it is equal to 5819 Br. inches nearly.

At a height of about 40 inches above the floor, in the first compartment of the niche, is a bank or shelf L/L'', and while the main portion of the niche is 40 in. deep, this is about 100 in. in depth; the line δ JD gives its height approximately; it was not measured carefully by Prof. Smyth.

Taking the height at $39\cdot71$ inches, we have, for the height of the niche above this shelf, $145\cdot32$ inches.

$$\text{Now } 145\cdot32 \times 10 \times \pi = 4565\cdot5,$$

but this is half the base length of the pyramid, according to the most accurate determinations, viz: 9131 pyr. in. or 9140 Br. in.

We have then, in these two portions of the niche,

$$185\cdot03 : 145\cdot3 :: \text{height of pyramid} : \text{half base of do.}$$

If now we extend the line L/L'' to meet γ H and γ D, and take this for the base of the pyramid, the line K/L will represent the level of the King's Chamber, given by Prof. Smyth as 1688 inches above the base; and by computation 1691·6 inches above the base, on the hypothesis that the area of a section of the pyramid at this height is half the area of the base as first announced by us, the vertex of the pyramid being represented by γ . The computed quantity for the height of the base of the pyramid above the Nile level, is 1769 in., giving us the *rough* floor as representing the level of the Nile at the date of erection; this height was estimated by Mr. Perring at 1767 in.

It may be noted further that γ A = 130·5 inches, and $130\cdot5 \times 10 \times 7 = 9135$, which is the length nearly for one side of base of the pyramid.

It is much to be regretted that *absolutely correct* measurements are not at hand to test how closely the ancient builders could work to what seems to have been the design, at least in part. Upon the whole, the results show that they were more accurate, and capable of doing better work, than many modern scientific men. Let us see then what modern men *have* done, measuring the same things, and it is lamentable to notice discrepancies like these for the Queen's Chamber.

	C. Piazza Smyth.	H. Vyse.	E. Lane.	M. Jomard.
	Br. in.	Br. in.	Br. in.	Br. in.
Length,	226·7	225	228	228·13
Breadth,	205·8	204	204	205·72
Height to roof,	182·4	177	162	162·01

	C. Piazzi Smyth.	H. Vyse.	E. Lane.	M. Jomard.
	Br. in.	Br. in.	Br. in.	Br. in.
Height to angle of roof,	244·4	243	246	258·41
“ of niche,	185·8	183	----	----
Width of lower part,	61·3	61	----	----
“ of top,	25·3 ?	23·5 ?	----	----
Distance of axis from } center of wall,	25·3 ?	26· ?	----	----

In view of these discrepancies, shall we demand superhuman accuracy on the part of the pyramid builders?

Notwithstanding the remarkable proportions, and evident geometrical relations, from a rectangle of the height of the niche, yet, in practice, the builders appear to have made the walls of the north and south sides of the chamber almost 2·5 inches lower than the niche. Was this careless workmanship, or was it designed? Let us see. We adopt 182·62 in. as the true height of the north and south walls, which is very near Prof. Smyth's mean—

182·62 in. $\times 100 \div 3 \cdot 14159 = 5813$ in. = height of the pyramid, and calling the 182·62 so many cubits—

182·62 in. $\times 25 = 4565 \cdot 5$ in. = half base length of the pyramid ;

and again, $\frac{182 \cdot 62 \times 10}{185 \cdot 033} = 9 \cdot 869587728 + = \pi^2$.

Moreover, $182 \cdot 62 \times 2 = 365 \cdot 24 =$ days in a year. If we take now 5813 in. and 9131 in., representing height and base of the pyramid, we get the following proportions :

Arc equal to radius = $3438' : 9131' :: 10^\circ : 26^\circ 34'$
 “ “ “ “ $3438' : 5813' \times 2 :: 10^\circ : 33^\circ 48'$.

The following are the results of the very accurate measurements of Prof. Smyth for the internal dimensions of the coffer :

Mean length,	77·93 inches.
“ breadth,	26·73 “
“ depth,	34·34 “

The last number is, almost, the right sine of 90° , or 3437·75, or the arc equal to radius. If now we multiply 3437·75 by the natural sine of $51^\circ 51' 14''$ i. e., ·7864, we get 27·03 for breadth of coffer, which is quite within some of the individual measures. We cannot expect any similar result for the length, if the cubical dimensions were designed to be a given quantity. Finally, we have, breadth of coffer : depth of do. :: height of pyr. : half base of do.

$5813 : 4565 :: 34 \cdot 34 : 26 \cdot 9$.

We have thus the π ratio in the coffer, and exactly, if we might substitute for Prof. Smyth's numbers 34·3775 and 27.

Thus, $\frac{27 \times 4}{34 \cdot 3775} = 3 \cdot 14159.$

Are these all coincidences? And was the niche built as a mere freak of ignorant, and debased idolaters?

Since the preceding was written, Prof. Smyth calls our attention to the pavement of the ante-chamber, which is not of one material, but partly limestone, partly granite, with accurate joints. He has not yet stated his own results to us, but doubtless he has already the following, and probably more.

The pavement is a rectangle, and lengthwise, is situated exactly north and south; AD being the length of the northern, shorter, and limestone portion; DB that of the southern, longer, and granite portion; AA' the breadth, and AB the whole length; we find

$$\frac{DB}{AD} = \pi, \quad \frac{2AD}{AA'} = \pi, \quad \therefore AA' \times DB = 2AD^2.$$

Theory.	Measured by C. Piazzzi Smith.	Diff.
AA' = $(1 + \pi) \times 10 = 41 \cdot 4159$ in.	41.45 in.	— .034 in.
AD = $(\pi + \pi^2) \times 5 = 65 \cdot 0559$ "	64.5 "	— .56 "
DB = $(\pi^2 + \pi^3) \times 5 = 204 \cdot 379$ "	203.6 "	— .62 "

AD and DB were determined by Prof. Smyth by measuring from the edge of the great step (now much worn) to the successive joints of the pavement, and cannot be as correct as the measurement of AA', which is the distance between the polished granite walls, comparatively in good condition.

Equations.

- I. OB:R::EO:tan. EBO. $102 \cdot 5:1::59 \cdot 17:57727$; nat. tang. 30° nearly.
- II. AE:AD::EO:OC. $244 \cdot 2:102 \cdot 5::59 \cdot 17:24 \cdot 83$; diff. .17.
- III. BO:OE::BC:CF. AO:HA=BO::CF:CO. $\therefore$ AO:EO::BC:CO, $185 \cdot 033:59 \cdot 17::77 \cdot 5:24 \cdot 8$.
- IV. Sin. 30° :EO::R:BE, $.5:59 \cdot 17::1:118 \cdot 34 = BE = BL$; angle EBL= $90^\circ + 30^\circ = 120^\circ$. $\therefore$ BEL= $30^\circ = BLE$.
- V. K'L:BL::R:tan. BK'L. $205:118 \cdot 353::1:5773$; nat. tang. 30° .
- VI. MO=EO, OS is common, and ang. O= 90° $\therefore$ ES=MS and ESM= 120° , OSM= $60^\circ = SM'X'$. EOB and BF'M are similar, and EB=BM; $\therefore$ F'M=BO and EO=BF'.
- VII. R:F'M::sin. 30° :MX', $1:102 \cdot 5::5:51 \cdot 25 = \frac{1}{4} N'B = \frac{1}{4} BO$. BM and EL bisect each other in P. In the triangle EBM, F'H, parallel to the base, bisects BE; $\therefore$ it will bisect BM, i. e., will pass through P.
- VIII. LB:K'L::BK:IK, $118 \cdot 353:205::57 \cdot 713:99 \cdot 98$; diff. .02.
- IX. IK=100=Rad. DK=127.32=IA. IK:R::IA:tang. IDA, $100:1::127 \cdot 32:1 \cdot 2732$; nat. tang. $51^\circ 51' 14''$.

- X. In the triangle γAD , $R : AD :: \text{tang. } \gamma DA : \gamma A$, $1 : 102.5 :: 1.2732 : 130.5$. $\gamma N = \gamma A - DL = 130.5 - 66.68 = 63.82 = \frac{1}{2} KD$. $\gamma'K' = 2 \gamma N = 127.64$, $S''K' = MN = (MA - AN) = (AE - EM - AN) = (244.2 - 118.353 - 66.68) = 59.167 = EO$. $\gamma'K' - S''K' = \gamma'S'' = 127.64 - 59.167 = 68.473$. $MS'' : R :: \gamma'S'' : \text{tang. } \gamma'MS''$, $102.5 : 1 :: 68.47 : 66.8$; nat. tang. $33^\circ 45'$; diff. $3'$.
- XI. $AM = AE - EM = 125.85$, $B'D : B'J :: DH : H\delta$, $77.5 : 66.68 :: 205 : 176.3$. $S''\delta = H\delta - S'H (=AM) = 176.3 - 125.85 = 50.45$. $S''M : R :: S''\delta : \text{tang. } \delta MS''$, $102.5 : R :: 50.45 : .492+$; nat. tang. $26^\circ 13'$; diff. $5'$.
- XII. $K'VN''$ and VOO' are similar, and V' divides ON in the ratio $KN'' : OO'$; $ON = AO - AN = 185.03 - 66.68 = 118.35$. $OO' = 37.5$. $K'N'' = 102.5$. $140 : 102.5 :: 118.35 : 86.6 = NV$. $AV = NV + AN = 153.28 (= \frac{3}{4} HD)$. $AV : R :: AH : \text{tang. } HVA$. $153.28 : 1 :: 102.5 : .668$; nat. tang. $33^\circ 45'$.
- XIII. Join HC' . The arc $C'B = 34^\circ 20'$ (xiv) $\therefore$ Chord $C'B = 2R$. $\text{Sin. } 34^\circ 20' \div 2 = 69.86$, $C'C'' = \sqrt{C'B^2 - BC''^2} = 47.4+$ $\therefore C'H'' = 232.43$. $C'H'' : R :: HH'' : \text{tang. } HC'H''$, $232.43 : 1 :: 153.75 : .661$ nearly; tang. $33^\circ 28'$. This so nearly equals the angle HVA , that we consider HVC' a straight line.
- XIV. $MC' : R :: MX'' : \cos. C'MX''$, $118.35 : 1 :: 51.25 : .433$; nat. cos. $64^\circ 20'$. $64^\circ 20' - 60^\circ = 4^\circ 20'$.
- XV. $S''H = EA - EM = 125.85$, $DH : HS'' :: A'D : A'N''$, $205 : 125.85 :: 108.2 : 66.42$; diff. $.26$.
- XVI. $N'L : KL :: N'G : GD'$, $108.2 : 60.64 :: 56.85 : 31.86$; diff. $.02$.
- XVII. Tan. $GXX'' : GX'' :: R : XX''$, $1.2732 : 60.64 :: 1 : 47.63$; but, as the line GX'' is .12 in. nearer center of room than $C'H''$, $X''K = 51.37$; adding XX'' gives $XK = 99$ in. $\therefore IX =$ one inch, and from center of room to $X = 3.5$ in. $OC - 3.5 = 21.5 =$ half breadth.
- XVIII. $IK : KB :: PT : TB$, $100 : 57.713 :: 51.25 : 29.58 = BT$; $KB - BT = KT = 28.13$.
- XIX. $AO : AH' :: OV' : V'W'$, $185.03 : 55.7 :: 29.58 : 8.9$. $OC - V'W' = W'U$, $25 - 8.9 = 16.1 =$ half breadth. $H'O$ bisects angle at W' , nearly; this angle as shown in (xxi) is $33^\circ 30'$. $AO : R :: AH' : \text{tang. } AOH'$, $185.03 : 1 :: 55.7 : .301$; nat. tang. $16^\circ 45'$.
- XX. Dropping a perpendicular from O'' upon SK , we have a right-angled triangle, of which the vertical side $= 57.713$, the horizontal $= 28.6$. $\frac{57.71}{28.6} = 2.018$; nat. tang. $63^\circ 39' =$ angle at base $\therefore$ ang. at $O'' = 26^\circ 21' =$ lower culmination nearly.
- XXI. In like manner $\frac{56.93}{37.6} = 1.51 =$ nat. tang. $56^\circ 30' =$ ang. at W $\therefore$ ang. at $W' = 33^\circ 30' =$ upper culmination.
- XXII. $\frac{60.64}{47.65} = 1.273 =$ nat. tang. $51^\circ 51' 14'' =$ ang. at G , $\therefore$ ang. at $X = \frac{1}{2}$ summit ang. $= 38^\circ 8' 46''$.

XXIII. $\frac{98.52}{56.85} = 1.732 = \text{nat. tang. } 60 = \text{ang. at H'} \therefore \text{ang. at Z} = 30^\circ = \text{latitude.}$

XXIV. $177.52 : 1 :: 226.4 : 1.275$; nat. tang. $51^\circ 55'$. $185.03 : 205 :: 205 : 227.1$; diff. $.7$.

XXV. $185.03 : 205 :: 1 : 1.1078$; nat. tang. $47^\circ 55'$; diff. $5'$.

Fig. 1.

Fig. 2.