THE RESOLUTION OF THE $K\alpha$ DOUBLET IN THE POWDER METHOD OF X-RAY CRYSTAL ANALYSIS.*

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IN the powder method of X-ray crystal analysis, suitably oriented crystals large enough to give separately distinguishable spots on the photographic film reflect the two wave-lengths of the $K\alpha$ doublet— $K\alpha_1$ and $K\alpha_2$ —at slightly different angles, so that these spots are also doublets.

Using a copper anti-cathode, W. Gerlach¹ reported that the separation of such doublets was only about half of the value which he expected from the known values of the wave-lengths. As I mentioned in a recent paper,² the positions of double spots in my photographs taken with a molybdenum anti-cathode were in rough agreement with the equations

$$\lambda_1 = 2d \sin \theta_1 \quad (1)$$

$$\lambda_2 = 2d \sin \theta_2 \quad (2)$$

wherein $2\theta_1$ and $2\theta_2$ are the angles which lines joining the centre of the sample to the centres of the spots make with a line joining the centre of the source with the centre of the sample.

In a recent note, F. Kirchner³ has explained the small values for spot separations obtained by Gerlach as theoretically justified, and Gerlach⁴ has recalculated the values of wave-length difference, $K\alpha_2 - K\alpha_1$, from his measured separations, getting very good agreement with experimental and theoretical results for copper and nickel radiators.

This leaves my results apparently out of line with both theory and experiment, but it is quite easy to show that there is no real contradiction, both results agreeing with a complete theory of the phenomenon.

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¹ W. Gerlach, *Phys. ZS.*, **22**, 557-559 (1921).

² L. W. McKeehan, *FRANK. INST. JOURN.*, **193**, 231-242 (1922).

³ F. Kirchner, *Phys. ZS.*, **23**, 114 (1922).

⁴ W. Gerlach, *Phys. ZS.*, **23**, 114-120 (1922).

In Fig. 1 let S be a plane source of any finite extent giving X-rays of two wave-lengths, λ_1 and λ_2 ($\lambda_2 > \lambda_1$). Let C be a group of crystal lattice-planes with primitive interval d . There will be two reflected beams, as shown, where equations (1) and (2) determine the values of θ_1 and θ_2 .

If the crystal is of unlimited area each point of the source will

FIG. 1.

give a complete circular cone of reflected rays with its apex at the ordinary mirror image of the chosen point in the plane C , and cutting the film MN in a curved line. With long crystals of tungsten, such traces have been obtained about 2 cm. long and showing a slight convexity on the less deflected side, as they should, considering the actual shape of MN , which was a circular cylinder centered at P . An enlarged reproduction of such traces is given in Fig. 2 (a). The double character of these long traces is evident, but they are not well enough defined for accurate measurement of the separation.

Smaller crystals give patterns on the film which show clearly the distribution of intensity among the points of the source which,

FIG. 2.

(a)

(b)

(c)

(d)

Actual width of fields is 2.0 cm.

by pin-hole photographs was found to be a brilliant annulus with a relatively non-emitting centre. The two pairs of parallel projections indicated in Fig. 1 give rise in the film to two very nearly similar elliptic bands, the ratio of the major to the minor diameter of each ellipse being about 8 and the major diameter being about 0.6 cm., the same as the diameter of the circular source. A pair of such ellipses is shown in Fig. 2 (b).

These ellipses are not sharply defined except near the ends of their minor axes, because of the width of the crystal. The wider the crystal the longer is the trace of the cone of reflected rays which were emitted at a point on the source, and the "image" of the source is built up by the superposition of these parallel lines of limited length exactly as a real image in any optical system is built up by the superposition of the diffraction patterns due to the points of the source. An additional cause of blurring is the thickness of the crystal perpendicular to the reflecting planes, but this was not serious with the thin sheets of metal which I found most convenient for other reasons.⁵

Inspection of Fig. 1 shows that the arc length R_1R_2 between the centres of the two elliptical spots is given by

$$R_1R_2 = (SP + PR)(\theta_2 - \theta_1) .$$

In the actual apparatus SP and PR were nearly equal, so that to a first approximation

$$R_1R_2 = 2PR(\theta_2 - \theta_1) .$$

Crystals too small to intercept both of the reflectable beams in their entirety will give rise to defective ellipses, one or both being cut off by lines due to the boundaries of the crystal, so that the ellipses may be cut off by lines in any position with respect to the axes of the ellipse. An example of such defective ellipses is given in Fig. 2 (c). It is clear that with the disposition shown in Fig. 1 it would be possible to have one ellipse complete and the other reduced to a narrow arc on the side nearest to the unaffected spot. If the beams do not overlap on C one spot of the pair may be wholly absent.

Carrying the reduction in size of the reflecting surface still farther, a very small area at P , in the region common to the two beams, will reflect only rays from O_1 and O_2 , to Q_1 and Q_2 , respectively, so that the two ellipses are now represented simply

⁵ L. W. McKeehan, *Loc. cit.*

by two chords, that is, by two very narrow parallel lines on the film. This is the case obtained by Gerlach, and occurs in my experiments with metals where special heat treatment has permitted the formation of very small crystals free from strain. A region showing several such pairs of lines is shown in Fig. 2 (*d*). An area outside the region common to the two beams cannot give rise to two lines of this sort, so that single lines are also of frequent occurrence.

The arc length Q_1Q_2 between the lines due to reflection from the same tiny crystal is given by

$$Q_1Q_2 = PR(\theta_2 - \theta_1),$$

half the approximate value found above for R_1R_2 . If differences in wave-length are to be determined this may be put in the rigorous form

$$\lambda_2 - \lambda_1 = \frac{\lambda_2 + \lambda_1}{2} \cot \frac{\theta_2 - \theta_1}{2} \tan \frac{\theta_2 - \theta_1}{2}$$

or for small values of $(\theta_2 - \theta_1)$

$$\lambda_2 - \lambda_1 = \frac{\lambda_2 + \lambda_1}{2} \cdot \frac{Q_1Q_2}{PR} \cot \frac{\theta_2 + \theta_1}{2}.$$

In the case of molybdenum, Duane⁶ gives

$$\lambda_2 = K\alpha_2 = (0.71212 \pm 8) \times 10^{-8} \text{ cm.}$$

$$\lambda_1 = K\alpha_1 = (0.70783 \pm 7) \times 10^{-8} \text{ cm.}$$

so that

$$\lambda_2 - \lambda_1 = (4.29 \pm 11) \times 10^{-11} \text{ cm.}$$

My measurements upon a single photograph where many sharply defined pairs of lines occurred gave

$$\lambda_2 - \lambda_1 = (4.36 \pm 5) \times 10^{-11} \text{ cm.}$$

in satisfactory agreement with Duane's value. Gerlach's values computed in this way agree equally well with previously published values of comparable accuracy for copper and nickel.

SUMMARY.

Double spots due to reflection of two nearly equal wave-lengths by the same crystal may be of two kinds.

(1) Fine parallel lines are due to reflection of rays from different parts of the source by a very small crystal. The separa-

⁶ W. Duane, *Nat. Res. Council. Bull.*, 1, 383-408 (1920).

tion of such lines depends only upon the difference in wave-length, the mean wave-length, and the mean of the observed angles of reflection. They are excellent objects under the microscope and give an accurate method for determining small differences in wave-length.

(2) Projected images of the whole source are due to reflection of rays from each point of the source at different points in the crystal. The separation of such images depends upon the difference in wave-length, the mean wave-length, the mean of the observed angles of reflection and upon the ratio of the whole path-length of the rays to the path-length after reflection. In the usual arrangement spots of the second kind are about twice as far apart as spots of the first kind.

Duane's value for the wave-length difference of the $K\alpha$ doublet of molybdenum has been checked by measurements on spots of the first kind.

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Facts About Asbestos. (*Engineering World*, June, 1922, p. 371.)—The art of weaving the mineral fibre in asbestos, which is ordinarily indestructible, was rediscovered at a comparatively late period of civilization. Woven asbestos was used in the ancient pyre to preserve the royal ashes. Charlemagne is said to have had a table-cloth made of asbestos and to have cleaned it by throwing it into the fire, which consumed the dirt, thus illustrating in a spectacular manner one of the most valuable properties of this material.

The fibre of the best grade of asbestos is beautiful and silky and has great flexibility, elasticity and tensile strength. It can be spun into thread so fine as to run 225 yards to the ounce, and as it is incombustible as well as a non-conductor of heat and electricity and resists the action of most ordinary acids, its field of use is large. The possible applications of asbestos are far from fully appreciated, not only by the general public but by manufacturers who are in search of material for special uses to which asbestos may well be applied. Perhaps it is most generally used to make fireproof cloth for theatre curtains. It has been used also for making firemen's clothing. Everywhere in cold countries it is extensively employed for covering furnaces, boilers and pipes to prevent loss of heat. Asbestos is a good insulator.

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