

IX.—*On Some Relations between Magnetism and Twist in Iron and Nickel.* Part I.

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In a former paper* I described certain experiments on the relations of magnetism and twist in iron and nickel, the chief results of which it may be well to give briefly here. When an iron or nickel wire is under the influence of longitudinal and circular magnetisations, it twists in a direction which is definitely related to the direction of the magnetising forces. This effect in iron was discovered by WIEDEMANN,† and for convenience I shall call it the Wiedemann Effect. It was pointed out by CLERK MAXWELL that the Wiedemann effect might be explained as a consequence of the earlier discovery made by JOULE, that iron lengthens in the direction of magnetisation, and contracts at right angles thereto.‡ Led by a consideration of BARRETT's discovery§ of the shortening of nickel wire in the direction of magnetisation, I determined to test nickel in the same way in which WIEDEMANN had tested iron. It was quite obvious that, if MAXWELL's explanation of the Wiedemann effect were the true one, nickel wire should, *ceteris paribus*, twist in a sense opposite to that in which iron twists. The experiment when made completely fulfilled the expectation. Thus, when an iron wire, with one end fixed, is traversed by an electric current in the direction in which it is at the same time longitudinally magnetised, the wire is twisted so that the free end rotates right-handedly with reference to the traversing current, or the longitudinal magnetisation. In nickel, on the contrary, the corresponding rotation is left-handed. This was the chief conclusion arrived at in my earlier paper; and a little consideration will show how very readily the Wiedemann effect, whether in iron or in nickel, is explained in terms of the simpler strains studied by JOULE and BARRETT.

There were, however, other results of interest touched upon, especially with regard to the influence of tension, which seemed to call for further investigation; but it was not till the spring of 1887 that I was able to return again to the subject. In that year I was fortunate in obtaining the assistance of Mr NAGAOKA, a graduating student of physics in the Imperial University of Japan, who undertook a very thorough examination of the influence of tension on the Wiedemann effect. His results, taken in conjunction with Mr BIDWELL's recent elaborate measurements of the changes of length of iron and nickel in varying magnetic fields, go far to establish the sufficiency of MAXWELL's explanation, as will be seen further on.

* "On Superposed Magnetisms in Iron and Nickel," *Trans. Roy. Soc. Edin.*, vol. xxxii. p. 193, 1883.

† WIEDEMANN'S *Galvanismus*, Bd. ii. § 491 (1st edit.).

‡ STURGEON'S *Annals of Electricity*, vol. viii.; also *Phil. Mag.*, 1847.

§ See *Nature*, vol. xxvi., 1882.

The experiments were conducted in exactly the same manner as described in my earlier paper. The wire hung vertically within a magnetising solenoid, and to its lower and free end was attached a dipper, which dipped into a pool of mercury and made the necessary contact for the current along the wire. The tension in the wire was varied by putting on and taking off ordinary lead weights. The twist was measured by means of a mirror which was fixed to the free end of the wire, and reflected a spot of light in the customary manner upon a scale.

When the wire was being traversed by a steady current so as to be circularly magnetised, the longitudinal magnetic field in which the wire was placed was repeatedly reversed until a steady difference of readings on the scale was obtained. The wire was, in other words, subjected to a cyclic straining, which gave a total to-and-fro twisting of the wire. For any one condition, some six or eight distinct successive readings were noted down, so that a good mean for the total cyclic twisting of the wire was obtained. Half the amount of this total twist was taken as the twist corresponding to the particular combination of magnetising forces.

With a given current along the wire, the current in the magnetising helix was taken through an ascending series of values. The highest field so obtained usually lay between 25 and 35 (C.G.S.) electromagnetic units. In the case of the iron wires, this greatest field was considerably higher than the field for which the maximum twist was obtained. In the case of the nickel wires, there was no such maximum twist observed up to the fields employed.* This point of maximum twist is one which offers distinct facilities for the discussion of the problem in hand, namely, the effect of tension on the phenomenon of twist. Does change of tension cause a change in the *position* of this maximum twist, and does it cause a change in its amount?

Some 120 distinct curves were obtained for different iron wires, showing the relation between twist and field for different currents along each wire and for different tensions. If we compare the curves for any one steady current along any one wire, we shall be able to study the direct influence of tension on the Wiedemann effect. The first conclusion is, that there is no evident relation between the tension and the position of the maximum; or, more accurately, that the field, which for given current along the wire corresponds to the maximum twist, is in no way affected by change of tension through a considerable range. On the other hand, as was also clearly shown in my earlier paper, it is abundantly evident that the position of the maximum twist depends on the strength of the current along the wire—the stronger this current, the higher the field needed to produce the maximum twist.

The effect, however, of tension upon the *amount* of twist is very marked. Thus, if we take any iron wire, and subject it at different tensions to the same combination of circular and longitudinal magnetic stresses, we shall find that the twist due to this com-

* In more recent experiments, conducted on a somewhat different plan, I have been able to obtain a maximum twist in nickel for intermediate values of field; but much higher fields must be used than were available in Mr NAGAOKA's experiments.

θ = angular displacement, measured in radians, of any cross-section of the wire relatively to the cross-section 1 c.m. distant from it.

TABLE I.—*Iron Wires.*

No. 1. $r = \cdot 02$.			No. 2 (<i>continued</i>).			No. 2 (<i>continued</i>).		
	T	$10^{\ast} \theta$		T	$10^{\ast} \theta$		T	$10^{\ast} \theta$
C = 1·14 H = 10·5 H' = 11·4	196 594 992 1388 1784 2181 2577 2975	4·60 4·25 3·35 2·55 2·35 1·60 1·55 1·15		543 698 853 1087 1164 1312 1630 1785 2093 2256 2463 2618 2773 1087 77	2·81 2·5 2·25 1·97 1·88 1·90 1·47 1·34 1·19 1·10 .90 .84 .81 2·06 3·50		1630 1785 1940 2093 2256 2463 2618 2773 1087 77	1·97 1·75 1·69 1·56 1·56 1·22 1·16 1·03 2·69 3·84
C = 1·73 H = 14·0 H' = 17·3	196 594 992 1388 1784 2181 2577	5·00 4·35 4·00 3·25 3·00 2·35 1·90		77 232 388 543 698 853 1087 1164 1312	3·94 3·78 3·69 3·34 3·00 2·88 2·69 2·44 2·19			
No. 2. $r = \cdot 032$.			C = 1·52 H = 11·8 H' = 9·5			No. 3. $r = \cdot 0415$.		
	T	$10^{\ast} \theta$					T	$10^{\ast} \theta$
C = .95 H = 10·4 H' = 5·9	77 232 388	3·75 3·44 3·14				C = 1·08 H = 7·1 H' = 5·2	48 243 439 633 864 1058 1251 1413 1661	2·96 2·43 2·02 1·76 1·28 1·18 1·04 .92 .67

TABLE I.—*Iron Wires*—continued.

No. 3 (<i>continued</i>).			No. 4 (<i>continued</i>).			No. 5 (<i>continued</i>).		
	T	$10^4\theta$		T	$10^4\theta$		T	$10^4\theta$
	2050	·58		814	1·70	C = 2·47	23	2·58
	2241	·53		1097	1·60	H = 11·3	403	2·13
	2505	·53		1345	1·22	H' = 8·37	784	1·78
				1625	1·06		1164	1·39
C = 1·6	48	3·16		1876	·96		1551	1·08
H = 8·9	243	2·80					1931	·93
H' = 7·7	439	2·39	C = 2·07	31	2·43	No. 6. $r = \cdot 072$.		
	633	2·17	H = 12·0	284	2·33			
	864	1·69	H' = 8·25	561	2·15		T	$10^4\theta$
	1058	1·52		814	1·91			
	1251	1·33		1097	1·70	C = 2·28	20	1·79
	1413	1·20		1345	1·46	H = 10	280	1·59
	1661	1·01		1625	1·30	H' = 6·33	536	1·39
	2050	·77		1876	1·12		794	1·12
	2241	·65	No. 5. $r = \cdot 059$.			C = 3·04	15	1·88
	2505	·68				H = 11·6	270	1·68
	2894	·53		T	$10^4\theta$	H' = 8·44	527	1·46
No. 4. $r = \cdot 0502$.			C = 1·52	23	2·32		784	1·22
	T	$10^4\theta$	H = 8·7	41	1·78			
			H' = 5·15	591	1·39			
C = 1·5	31	2·21		797	1·36			
H = 12·3	284	2·13		981	1·17			
H' = 6	561	1·91		1180	1·02			

A careful study of these numbers seems to lead to the following conclusions:—

1. Other things being the same, the twist is greater in the thinner wire. It should be noted here that there is considerable difficulty in deciding as to the meaning of “other things being the same.” The best mode is clearly, not to take the current through the wire as a guide, nor yet the current density, but something which may be regarded as giving an approximate estimate of the magnetic effect of the current. I have therefore taken the quantity H' , which measures the direct magnetic force at the circumference of the wire due to an axial current of the magnitude used. The method is certainly open to criticism; but in our absolute ignorance of the magnetic distribution in an iron wire due to a current passing along it, any other approximation could hardly be so justifiable. It will be shown later, that, on the simplest supposition possible, a calculation based on MAXWELL'S explanation of the Wiedemann effect leads to the result that the twist produced in a thin *tube* under the influence of given longitudinal and circular magnetising forces is inversely as the radius. It will readily be granted, when all the conditions are taken into account, that the experimental result just given is in fair accordance with the result deduced from theory.

2. The longitudinal field necessary to produce the maximum twist is greater for the greater current along the wire. This result also may be shown to be in harmony with MAXWELL's explanation.

3. For a given combination of magnetising forces, the twist diminishes steadily as the tension increases. This relation also holds for all the other combinations of magnetising forces used in the experiments. The conclusion is in remarkable agreement with the results obtained by Mr BIDWELL in his elaborate investigations into the changes of length of iron in magnetic fields. Amongst other results, he found that under increasing loads the elongation of iron wire due to moderate magnetising forces decreases.* Here again MAXWELL's explanation of the Wiedemann effect, in terms of the simpler Joule effect, accords well with the facts—both being powerfully influenced by tension, and that in the same direction.

At first sight, it might be supposed that the maximum twist shown to exist in these experiments was exactly the same phenomenon as the maximum elongation of iron, obtained by Mr BIDWELL in his experiments. But in trying to connect the two peculiarities, we encounter discrepancies which are hard to explain. Thus, the field which corresponds to the maximum elongation is much higher than the field which corresponds to the maximum twist. The latter, of course, depends on the strength of the current along the wire; but in no case (out of nearly 200 distinct experiments with different wires) has the magnetic field corresponding to the maximum twist been higher than 25 electromagnetic units—usually considerably lower, as in the table given above. According to Mr BIDWELL's experiments, the field producing maximum elongation in a wire 1·2 mm. in diameter, varied from 45 for low loads to about 20 for high loads. This very striking effect of increased tension upon the strength of field required to produce the maximum elongation has, further, no analogue in the experiments on the maximum twist. Again, as bearing on this point, it may be mentioned that, although it is possible to obtain a maximum twist in nickel, there is no evidence of a maximum contraction. Hence the existence of the maximum twist does not imply the existence of maximum elongation or contraction, but clearly depends on other considerations. As I hope to show later, however, these considerations seem to be necessarily involved in the complete statement of the MAXWELL explanation.

We shall now pass to the discussion of the results for the nickel wires. Here we are not able to make use of such a well-marked singular point as a maximum, since no maximum was obtained with the magnetic fields employed. The main purpose of the present inquiry is, however, to consider the effect of tension on twist, other things being the same. It suffices, therefore, to fix upon some one value of the longitudinal field, which is common to all the experiments. In the following tables, only those twists are given which correspond to the field 28·5. In the great majority of cases this was in reality one of the fields employed; and in the comparatively few instances where it was not so, it was an easy matter to obtain by a simple interpolation an accurate enough value for

* See *Proc. Roy. Soc.*, vol. xl. p. 262 (1886).

the required twist. Only three different specimens of nickel wire were at our disposal, and of these the thinnest one had no special claims to purity. The symbols, r , c , H , H' , T , θ , have the same meanings as before, except that H has no reference whatever to a maximum twist.

r = radius of wire in centimetres.

C = current along the wire in ampères.

H = longitudinal magnetic field.

$H' = C/5r$ = circular magnetic field at the circumference of the wire.

T = tension in kgs. weight per sq. cm.

θ = measure in radians of the twist per centimetre length of the wire.

TABLE II.—*Nickel Wires.*

No. 1. $r = \cdot 0253$.			No. 2. $r = \cdot 0422$.			No. 3. $r = \cdot 05$.		
	T	$10^4\theta$		T	$10^4\theta$		T	$10^4\theta$
$C = 1\cdot 52$	160	5.21	$C = 1\cdot 52$	57.4	1.54	$C = 1\cdot 52$	41	1.86
$H = 28\cdot 5$	657	3.67	$H = 28\cdot 5$	236	1.37	$H = 28\cdot 5$	168	2.31
$H' = 12\cdot 0$	1155	3.01	$H' = 7\cdot 2$	415	1.25	$H' = 6\cdot 1$	296	2.56
	1652	2.43		594	1.19		423	2.64
	2150	1.82		773	1.07		550	2.47
							736	2.35
							1107	1.86
$C = 2\cdot 28$	160	6.32	$C = 2\cdot 28$	57.4	1.51	$C = 2\cdot 31$	41	3.01
$H = 28\cdot 5$	657	3.87	$H = 28\cdot 5$	236	1.45	$H = 28\cdot 5$	168	3.35
$H' = 18\cdot 0$	1145	2.41	$H' = 10\cdot 8$	415	1.45	$H' = 9\cdot 2$	296	3.38
	1652	2.22		594	1.38		423	3.28
	2150	1.73					550	3.15
	2687	1.85					736	2.67
							1107	2.10

It will be noticed that the thinnest specimen behaves very like iron—that is, increasing tension is accompanied by diminishing twist, and that very markedly. A very similar effect is produced in the case of the intermediate specimen, with the single difference that the effect is not so pronounced. With the thickest, and what is probably the purest specimen, however, the effect of tension on the twist is quite peculiar. At first, as the tension is increased, the twist increases until the tension attains a value of 300 or 400 kgs. weight per sq. cm. After this, as the tension is further increased, a pretty rapid decrease of twist sets in. It may be mentioned that this maximum twist for some intermediate value of tension was obtained in my earlier experiments* with very thin nickel wire. It existed, however, only in one series of experiments, and disappeared when the current along the wire was doubled. In the present case there is

* See page 203 of the paper already referred to.

a hint as to the manner in which the current along the wire influences the phenomenon. It would appear, in short, that the stronger the current along the wire the lower is the tension which produces the maximum twist in a given longitudinal field. Hence, for currents of considerable strength along the wire, it is quite possible that this maximum twist, occurring at a tension lower than the lowest used, could not be observed. For it must be remembered that in these experiments with a hanging wire, it is impossible to begin at zero tension, since the dipping arrangement needed for making contact and other necessary additions must have a definite weight.

Thus we see that when nickel wire twists under the influence of circular and longitudinal magnetising forces, the amount of twist in certain specimens is influenced by tension in a manner very similar to what occurs in the case of iron—namely, the twist diminishes as the tension increases. In other specimens, however, a maximum twist is obtained for a certain intermediate tension; and the tension which in a given longitudinal magnetic field corresponds to the maximum twist appears to be smaller for higher values of the circularly magnetising current along the wire.

I am not aware that any experiments have been made upon the effect of tension on the contraction of nickel in longitudinal magnetic fields. Mr BIDWELL does not seem to have studied the phenomena in nickel with anything like the thoroughness with which he has worked out the phenomena in iron. All I can find is the statement that “a nickel wire stretched by a weight undergoes retraction when magnetised,” but whether this retraction is greater or less than the retraction in the unstretched case is not mentioned.*

The tensions to which these various iron and nickel wires were subjected were obviously carried beyond the approximate limits of so-called perfect elasticity. A very natural inquiry to make was as to the effect of permanent strain upon the amount of twist for a given combination of magnetising forces. In the experiments on the iron wire of radius .032 (No. 2 in Table I.), observations were made as the tension was diminished again to its first and lowest value. The numbers will be found in the table, being in each case the last two rows of figures in the column. When the tension is reduced to its original value, there seems to be a slight decrease in the twist; but this does not seem to hold for the intermediate conditions. All that can be safely said is that the effect of the permanent strain, after the stress is removed, is hardly appreciable in the case of iron.

In the case of nickel, however, the effect of permanent strain is very marked, as the following small table will show. In this table two pairs of columns of tensions and twists are given. In the first of these are entered the tensions and twists before the wire had been subjected to a high tension of above 6000 kgs. weight per sq. cm.; and in the second column are entered the tensions and twists after the wire had been subjected to this high tension and relieved.

* See *Proc. Roy. Soc.*, vol. xl. p. 133.

Before Strain.		After Strain.	
T	$10^4\theta$	T	$10^4\theta$
160	5.43	206	0.59
657	3.68	847	.40
1155	1.79	1487	.43
1652	1.88 ?	2121	.43
2150	1.03	4150	.19

These twists are produced by the combination of a longitudinal field of 27.4 electromagnetic (C.G.S.) units, with a current along the wire of 1.52 ampères. The radius of the wire, originally .0253 cm., was after the strain permanently reduced to .0223 cm.

It is at once apparent that the stretched wire twists much less than the unstretched wire. The marked diminution in the radius is, of course, a sufficient indication of the great molecular change which stretching has produced in the wire. The wire has no doubt been considerably hardened by the process, and is no longer to be regarded as the same material. It should be mentioned that the numbers just given are only samples of the observations taken. A comparison of the twists in other fields than the one chosen leads, however, to the same general conclusion.

It remains, finally, to consider the influence of change of temperature on the Wiedemann effect. In order to carry out such an experiment, it was necessary to coil the magnetising helix upon a double-walled tube, between the concentric walls of which steam or other vapour could be passed. The upper end of the bore through which the wire hung was plugged up with cotton wool, so that no current of air could pass up or down. Under these conditions, the temperature of the wire may be assumed to be not very different from the temperature of the vapour after that has been for some time passed through the space within the walls. In the experiments as conducted, steam at 100° C. and water at 11° C. were passed in succession through the double-walled tube; and observations on the twist made in the usual way. The following table (Table III.) gives a few specimen numbers for both iron and nickel.

As before, C is the current along the wire, and H' the corresponding magnetic force at the circumference of the wire. H is the longitudinal field. H and H' are measured in electromagnetic (C.G.S.) units.

A glance at these numbers shows that the general tendency is for the twist to diminish as the temperature is raised, although at the highest tensions for iron there seems to be a tendency the other way. There is nothing as yet known regarding the influence of temperature on the simple Joule effect; but we might safely argue from the results just given that in general change of length of iron and nickel in a given longitudinal field is greater at the lower temperature.

In discussing these experiments, I have throughout assumed the truth of MAXWELL'S explanation of the Wiedemann effect in terms of the Joule effect. WIEDEMANN himself,

TABLE III.

Iron.			Nickel.		
$r = .0415$ cm. $C = 1.6$ ampères. $H = 10.5$. $H' = 7.7$.			$r = .05$ cm. $C = 1.52$ ampères. $H = 28.5$. $H' = 6.1$.		
T	$10^4\theta$		T	$10^4\theta$	
	11°	100°		11°	100°
48	3.04	2.89	41	2.90	2.18
243	2.65	2.60	168	2.94	2.39
439	2.36	2.17	296	2.84	2.42
634	2.07	1.93	423	2.80	2.50
828	1.83	1.83	550	2.63	2.65
1030	1.76	1.64	736	2.68	2.55
1260	1.45	1.52			
1450	1.06	1.33			

however, does not admit the sufficiency of this explanation. He entrenches himself behind the argument, that however neatly MAXWELL'S explanation may seem to explain the twist due to superposed magnetisms, it takes no cognizance of the reciprocal phenomena (see *Beiblatter*, 1886, vol. x. p. 728). Professor J. J. THOMSON has shown (see his book *Applications of Dynamics to Physics and Chemistry*) that the existence of the twist produced by passing a current along a magnetised wire requires that when a current is passed along a twisted wire, or when a wire conveying a current is twisted, the wire becomes magnetised. This result is deduced simply from the application of recognised dynamical principles, and in no way takes account of any possible explanation of either phenomenon in terms of simpler ones. There is, so to speak, no stepping behind the scenes. It may well be doubted, however, if, after all, the experiments in which a twisted wire conveying a current is found to become magnetised, can be regarded as showing phenomena reciprocal to those in the experiments on the twisting of a magnetised wire under the influence of a current passing along it. For in the latter the twist produced in the wire by the superposed magnetisms is always very small, far within the limits of torsional elasticity; whereas in the former it is necessary to apply comparatively large twists before any pronounced magnetic effect is obtained. In Professor WIEDEMANN'S own experiments the twists applied are very large indeed for the length of wire used, amounting to 7° per centimetre—that is, more than 200 times any of the twists obtained in the experiments just discussed, and 40 times the largest twist which I have ever obtained in like experiments. I have further found, by direct experiment, that it requires an applied twist of 1° per centimetre to produce any pronounced magnetic polarity in a wire conveying a current of half an ampère; so that it seems to be quite

and hence the equation to the strain ellipsoid is

$$(1-2\sigma)X^2+(1-2\sigma)Y^2+(1-2\varpi)Z^2-2\theta xYZ+2\theta yXZ=1.$$

By choosing the y or Y axis along the line from the centre of the section to the point $x y$, we get this equation in the somewhat simpler but no less general form—

$$(1-2\sigma)X^2+(1-2\sigma)Y^2+(1-2\varpi)Z^2+2\theta yXZ=1 \quad . \quad . \quad . \quad (2)$$

It is evident the Y axis is a principal axis of the strain ellipsoid; and in finding the others we may confine our attention to the plane XZ .

Let $\lambda \mu$ be the direction cosines of a principal axis in this plane, and r the corresponding radius, then

$$\begin{aligned} -2\sigma\lambda^2-2\varpi\mu^2+2\theta y\lambda\mu &= \frac{1}{r^2}-1 \\ \lambda^2+\mu^2 &= 1. \end{aligned}$$

Differentiating and remembering that $1/r^2$ is either a maximum or minimum, we get, on reduction,

$$\frac{\lambda}{\mu} = \frac{\theta y}{2\sigma-p} = \frac{2\varpi-p}{\theta y} \quad . \quad . \quad . \quad . \quad (3),$$

where

$$p = 1 - \frac{1}{r^2}$$

and satisfies the equation

$$p^2-2p(\varpi+\sigma)+4\varpi\sigma-(\theta y)^2=0, \quad . \quad . \quad . \quad . \quad (4).$$

Now let $r_1 r_2$ be the maximum and minimum values of r , then we may write

$$\begin{aligned} r_1 &= 1 + \varpi_1 & r_2 &= 1 + \sigma_1 \\ r_1^2 &= 1 + 2\varpi_1 & r_2^2 &= 1 + 2\sigma_1 \\ 1 - \frac{1}{r_1^2} &= 2\varpi_1 & 1 - \frac{1}{r_2^2} &= 2\sigma_1, \end{aligned}$$

where ϖ_1 and σ_1 are the principal elongations, and $2\varpi_1$ $2\sigma_1$ are the roots of equation (4). Hence

$$\begin{aligned} \varpi_1 + \sigma_1 &= \varpi + \sigma \\ 4\varpi_1\sigma_1 &= 4\varpi\sigma - (\theta y)^2 \end{aligned}$$

from which we find easily,

$$\begin{aligned} 2\varpi &= \varpi_1 + \sigma_1 \pm \sqrt{(\varpi_1 - \sigma_1)^2 - (\theta y)^2} \\ 2\sigma &= \varpi_1 + \sigma_1 \mp \sqrt{(\varpi_1 - \sigma_1)^2 - (\theta y)^2} \end{aligned}$$

The equation of the strain ellipsoid, referred to its own principal axes, may then be put in the form

$$(1-2\varpi_1)X^2 + \{1-(\varpi_1+\sigma_1) \pm \sqrt{(\varpi_1-\sigma_1)^2-(\theta y)^2}\}^2 Y + (1-2\sigma_1)Z^2 = 1.$$

To find the angle which the major axis of this ellipsoid makes with the axis of z , we

have to solve equation (3), putting for p its proper value for the maximum radius; in this case $2\varpi_1$. Then if θ is the required angle, we get

$$\tan \theta = -\frac{\lambda}{\mu} = -\frac{\theta y}{2\sigma - 2\varpi} = +\frac{\theta y}{\varpi_1 - \sigma_1 \pm \sqrt{(\varpi_1 - \sigma_1)^2 - (\theta y)^2}} \quad (5).$$

And now let us make the assumption, plausible enough and certainly the simplest that can be made in the circumstances, that this direction of maximum elongation coincides with the direction of the resultant magnetising force, as it may be assumed to exist in the experiments which give the Wiedemann effect. It will be remembered that we are applying the calculation to a cylindrical tube, although so far as the problem is an "elasticity" one there is no necessity for such a limitation. By so confining our attention to a thin-walled tube, we are able to regard the equation $\zeta = \varpi z$ as a sufficiently near approximation to what might reasonably be expected to hold good if the experiment giving the Wiedemann effect were made with a tube instead of a wire, the tube being under the influence of an axial current and of a uniform longitudinal field. The magnetic distribution in an iron or nickel wire due to a current passing along it is something of which we know absolutely nothing, so that to make a calculation based upon an assumed expression for ζ involving x and y in some simple manageable way, would probably ill repay the extra labour.

Returning then to the equation (5), let us take α and β as the circularly and longitudinally magnetising forces. Then, assuming that the maximum elongation ϖ_1 takes place in the direction of the resultant magnetising force, we may put $\tan \theta = \alpha/\beta$, and hence by a simple reduction,

$$\frac{\theta y}{2(\varpi_1 - \sigma_1)} = \frac{\alpha\beta}{\alpha^2 + \beta^2}.$$

Here y is the radius of the tube. Writing it r , we obtain finally,*

$$\theta = \frac{2(\varpi_1 - \sigma_1)}{r} \frac{\alpha\beta}{\alpha^2 + \beta^2} \quad (6),$$

that is, the twist θ , which measures the Wiedemann effect, is given in terms of the magnetising forces α , β , the radius of the tube r , and the JOULE elongation ϖ_1 , together with σ_1 , the accompanying elongation at right angles to ϖ_1 . Of σ_1 we have no direct measurement. JOULE, however, found that iron longitudinally magnetised did not change appreciably in volume. This would make $\sigma_1 = -\varpi_1/2$ for the moderate magnetising forces with which JOULE worked.

In comparing this formula with results of experiments as obtained till now, we must remember that in the experiment we are dealing with a wire circularly magnetised throughout its interior in a complicated and altogether unknown manner; whereas in the expression just given we are dealing with a thin-walled tube. Nevertheless, it is easy to see that the formula does to a certain extent apply even to the wire. Thus the twist

* This expression differs from the one given in my earlier paper (p. 198). That, however, was incompletely worked out with a too early assumption of the law connecting the elongation with the magnetising force.

is greater for smaller values of r . It is positive when $(\varpi - \sigma)$ is positive, as in iron in moderate fields; it is negative when $(\varpi - \sigma)$ is negative, as in nickel throughout, and as in iron in high fields. The existence of a maximum twist for some intermediate value of either α or β (β or α remaining constant) will depend upon the particular way in which $(\varpi - \sigma)$ depends on the quantities α and β . Let us suppose, for instance, that α , the circularly magnetising force, is constant, and that β is allowed to vary through a large range. This supposition is a near enough approximation to the case of a wire conveying a steady current, and then longitudinally magnetised. Now, it is clear that even if $(\varpi - \sigma)$ is constant, there is a maximum value for the twist given by the condition $\beta = \alpha$. A maximum value of $(\varpi - \sigma)$ for intermediate values of the field is not then a necessary condition for the existence of a maximum twist. Hence, it is not surprising that the field at which the maximum twist occurs should not be the same as the field at which the maximum elongation occurs. The maximum twist may exist without any maximum elongation; as for example in nickel, in which I have recently obtained a maximum twist about a field of 200 or 300. According to Mr BIDWELL'S recent experiments (see *Nature*, July 1888), nickel goes on distinctly contracting in magnetic fields up to 750, after which up to 1300 or higher the length remains apparently constant.

If we look closely at Mr BIDWELL'S curves of elongation for iron in ascending magnetic fields, we see that at first the curve is concave upwards, then becoming convex it reaches a maximum, after which it proceeds nearly straight in a long slope down to, and finally below, the zero line. For a considerable range of field near the point of inflexion we may regard the elongation as a linear function, of the most general form, of the magnetising force; and such an assumption gives a maximum twist, or rather the possibility of it, except in the very special case of simple proportionality of the elongation to the magnetic force.

Let us now test the expression for θ by a direct numerical calculation, taking for this purpose the numbers given in Table I. for wire No. 1. Here $r = \cdot 02$, $\alpha = 11\cdot 4$, $\beta = 10\cdot 5$, and we may take $(\varpi_1 - \sigma_1)$ to be approximately $\cdot 000004$. With these values, we get

$$\theta = \cdot 00002,$$

about $2\frac{1}{2}$ times smaller than the observed value for the wire.

A similar calculation can very easily be made for nickel. Now the contraction for nickel in magnetic fields is considerably greater than the expansion for iron; and yet the twist numbers given in Table III. are sensibly of the same magnitude as those in Table I. The reason of this, however, is not far to seek. For it will be noticed that the factor $\alpha\beta/(\alpha^2 + \beta^2)$ is, because of the greater inequality of α and β , much smaller than in the case of iron.

At first sight, this may not seem to be a very promising result; but when all the circumstances of the case are borne in mind, it will, I think, be admitted that the result is really as satisfactory as we could reasonably expect. The calculated twist for the tube is, at any rate, of the same order of quantity as the observed twist for the wire. The

calculation is perhaps interesting as being, I believe, the first of its kind, namely, a numerical comparison of what are at first sight different phenomena depending on the relations of magnetic and mechanical stress and strain. The result of the comparison, in my opinion, demonstrates the *sufficiency* of MAXWELL'S explanation of the Wiedemann effect in terms of the simpler Joule effect. Thus MAXWELL'S explanation has, to a first and simple approximation, stood the test of numerical calculation; whereas it is impossible even to imagine how to begin in applying such a test to WIEDEMANN'S theory.

Another serious objection to WIEDEMANN'S theory is, that it gives a so-called explanation of a particular kind of magnetic strain, but furnishes no insight into the mechanism of simpler magnetic strains. And then, again, it seems to me—in making this statement I may simply be showing how little I understand the mechanism of the frictionally restrained rotating molecules—but it seems to me that, according to WIEDEMANN'S theory, the direction of twist in an iron or nickel wire should depend on the order in which the circular and longitudinal magnetising forces are applied to the wire. Thus, let the wire, hanging vertically, be magnetised with north pole downwards, and then let a current be passed down it. Then the originally vertically polarised surface molecule facing the spectator will tend to rotate like the hands of a watch. But, if the wire is first magnetised circularly by a current flowing down it, and then subjected to the influence of the longitudinal field, the originally horizontally polarised surface molecule will tend to rotate contrary to the hands of a watch. Now, it is difficult to see how such contrary tendencies can possibly cause a similarly directed twist.

I have recently found by experiment that the *amount* of twist, due to a given combination of magnetising forces, does depend upon the order in which the forces are applied; but, except in a very particular case, the direction of twist never does. As the experiments, however, are not quite completed, I reserve their discussion for a second paper.