THE ELLIPTICITY OF PLANETS.

By L. D'AURIA.

Admitting that planets were originally in a fluid state, however heterogeneous their respective masses may be considered, their form of equilibrium when fluid, must have been such that the direction of the resultant of all the forces acting upon any point of the free surface must have been normal to the surface itself. Now, the form of equilibrium of any fluid mass when at rest is the sphere, and when any fluid sphere is made to revolve around any one of its diameters, it becomes more or less flattened on its axis of rotation on account of centrifugal force. This causes the sphere to become an oblate ellipsoid, whose minor diameter coincides with the axis of rotation itself; and, therefore, the equation of any meridian line on the surface of any fluid mass revolving around an axis of inertia can be expressed by

$$a^2y^2 + b^2x^2 = a^2b^2.$$

Differentiating this equation we have

$$\frac{dy}{dx} = -\frac{xb^2}{ya^2};$$

and observing that the first member represents the cotangent of the

latitude φ of a point M , whose co-ordinates are x and y (Fig. 1), and

that $\frac{x}{y}$ represents the cotangent of the latitude θ of the same point referred to the centre of the ellipsoid; we have

$$\frac{\sqrt{1 - \sin.^2 \varphi}}{\sin. \varphi} = - \frac{b^2 \cos. \theta}{a^2 \sin. \theta};$$

and

$$\sin. \varphi = \frac{\sin. \theta}{\sqrt{1 - \cos.^2 \theta e^2 (2 - e^2)}},$$

where $e^2 = \frac{a^2 - b^2}{a^2}$.

On the other hand, let MR represent the direction and the intensity of the attraction of the whole ellipsoid upon the point M , which attraction we designate by g ; and put $MO = \rho$; angle $MRE = \theta_1$; angular velocity $= \omega$. Then, the resultant of the attraction g , and the centrifugal force at M , must coincide with the normal MN , for centrifugal force is parallel to EE ; and since this latter force is expressed by $\omega^2 \rho \cos. \theta_1$, we have for the resultant itself,

$$f = g \sqrt{1 - \cos.^2 \theta_1 \frac{\omega^2 \rho}{g} \left(2 - \frac{\omega^2 \rho}{g}\right)};$$

and, therefore,

$$\sin. \varphi : \sin. \theta_1 = g : g \sqrt{1 - \cos.^2 \theta_1 \frac{\omega^2 \rho}{g} \left(2 - \frac{\omega^2 \rho}{g}\right)};$$

$$\sin. \varphi = \frac{\sin. \theta_1}{\sqrt{1 - \cos.^2 \theta_1 \frac{\omega^2 \rho}{g} \left(2 - \frac{\omega^2 \rho}{g}\right)}}.$$

Comparing this value of $\sin. \varphi$ with the preceding one, we find

$$e^2(2 - e^2) = \text{tg. } \theta \cotg. \theta_1 \frac{\omega^2 \rho}{g} \left(2 - \frac{\omega^2 \rho}{g}\right) + \frac{\sin.^2 \theta_1 - \sin.^2 \theta}{\sin.^2 \theta_1 \cos.^2 \theta}.$$

When θ and θ_1 become equal to each other, that is, when the point M is considered very near to the equator of the ellipsoid, then

$$\frac{\sin.^2 \theta_1 - \sin.^2 \theta}{\sin.^2 \theta_1 \cos.^2 \theta} = 0; \text{tg. } \theta \cotg. \theta_1 = 1; \rho = a;$$

and

$$e^2(2 - e^2) = \frac{\omega^2 a}{g} \left(2 - \frac{\omega^2 a}{g}\right);$$

$$e^2 = \frac{\omega^2 a}{g}.$$

Since the ellipticity is expressed by $\epsilon = \frac{a-b}{a}$; we have

$$2\epsilon - \epsilon^2 = \frac{\omega^2 a}{g}; [a]$$

and neglecting ϵ^2 , we have, approximately,

$$\epsilon = \frac{\omega^2 a}{2g}.$$

Now in the case of our planet,

$$\omega = \frac{2\pi}{86184}; a = 6377278 \text{ meters}; g = 9 \text{ meters } 81462;$$

(including centrifugal force). Hence,

$$\epsilon = \frac{1}{578}.$$

This value is entirely too small compared with $\frac{1}{300}$ as deduced from geodetical measurement of arcs of meridians and parallels; but the discrepancy cannot be ascribed to incorrectness of equation $[a]$, since the latter has been deduced from a well established mathematical principle. It may be due partly to the fact that since the time the earth became compact enough to retain its form, its volume has been considerably contracted uniformly all around, which evidently increases its ellipticity; and partly to a retardation of the earth's axial rotation since that time, which also increases the value of ϵ , by increasing ω , in equation $[a]$. Both these causes, however, are based upon the assumption that our planet must have lost its fluidity long ago, although in its interior it may be yet intensely hot; and since there is no other way to account for the discrepancy in question, we must recognize in this an excellent confirmation of the opinion lately advanced by Sir William Thomson on the constitution of the earth as regards the observed phenomena of precession, nutation, and tides.

That the earth's axial rotation is subject to a certain retardation, has been almost evidently shown by astronomers in order to account for the apparent acceleration in the moon's orbital motion; and such retardation has been estimated at about ten seconds in a century.

The ellipticity of the planet Jupiter, computed approximately after the formula

$$\epsilon = \frac{\omega^2 a}{2g},$$

is found to be $\frac{1}{24}$, while, according to observations, it should be about

17. The discrepancy here, which is due to the same causes above explained, is much less than in the case of the earth, and this seems to show that if at present the planet Jupiter is not perfectly fluid throughout, it is yielding enough to obey in a certain measure the forces which tend to decrease its ellipticity, a conclusion which is in perfect accordance with the observed physical conditions of this planet, and otherwise gives strength to our assumptions.

TO CHICAGO IN EIGHTEEN HOURS.

By ROBERT GRIMSHAW, M. E.

[Read at the Stated Meeting of the FRANKLIN INSTITUTE, April, 1884.]

I have the honor this evening to submit to the Franklin Institute the outlines of a bold, but feasible project for making the regular run between New York or Philadelphia and Chicago, in eighteen hours, with comfort, safety, and cheapness. At present, the transit occupies from 27 to 37 hours, according to the route and the character of the train.

To accomplish the desired result, necessitates changes in engine, train, method, and permanent way.

Naturally, the limits of this paper do not permit me to rehearse all the details of the proposed selections and innovations; but the most important of them will be outlined, as a basis for discussion.

This paper is the outcome of a discussion started by me in the *American Journal of Railway Appliances* in December last; I have no patents to advertise and no axes to grind. Many of the suggestions in this paper are mine, others have been contributed by eminent mechanics, practically familiar with the building, repairing and running of high speed locomotives. I do not, however, propose to designate which are mine and which are not, having been much amused by hearing ideas, contributed or indorsed by leading builders and master mechanics, and some of which have been for some time in successful operation, characterized as impracticable and visionary. All I will say in this connection is, that whatever any of my hearers may consider as good and practical, no matter what it is, I do not claim. For anything which any one may consider particularly good for nothing, or old, or wild, or unfeasible, or idiotic, I accept full responsibility. I am perfectly aware that this