lines, runs down to the right of the 3–2 (1·85) line, bending towards the line. Loomis's lies between the 3–2 and 6–4 lines. A mean curve would be just to the right of the 3–2 line, and might readily run out at the 3–2 (1·85) intersection. This would indicate single molecules in dilute solutions dissociating into two ions, an early occurrence of doubling of molecules, and steady increase in the extent of association as dilution diminished, the double molecules formed dissociating into 4, 3, or 2 ions, but not into more. Although the coefficients with which the curves are plotted are doubtful, the curves are so nearly parallel to the axis of coefficients, that even a considerable error in their determination would not affect the above result.

General Conclusions.

Although the observations on which the above discussion is based are defective, and the particular conclusions drawn are consequently tentative, I think it may be held with some confidence: (1) that the curves of equivalent depression against ionization-coefficient have positions, forms, and slopes such as they might be expected to have on reasonable assumptions as to mode of ionization and constitution in solution, according to the Van 't Hoff-Arrhenius theory of the depression of the freezing-point in solutions of electrolytes; (2) that they are consistent with the depression-constant having a common value of about 1·85 for all the electrolytes examined, and that in the case of the electrolyte for which we have the best data, its curve is not consistent with a greater limit of error in this value than about 0·01, unless improbable assumptions are made with respect to the constitution of the electrolyte in solution; and (3) that the diagram enables us to reach in some cases conclusions of considerable probability with respect to the constitution of the electrolyte in solution and its mode of ionization.

L. *The Motion of a Sphere in a Viscous Fluid.*

By H. S. Allen, *M.A.*, *B.Sc.*

[Continued from p. 338.]

[Plates I. & II.]

III.

8. *Photographic Method of Determining Velocities.*

It was only possible to apply the method already described to cases in which the terminal velocity was small. In order to extend the range of observation it was decided to have recourse to a photographic method. Several ways of employing photography in the determination of the velocity might be suggested, but the plan finally adopted was as follows.

A photographic plate was placed at the back of the rectangular glass vessel in which the balls were allowed to fall. The plate was illuminated by a series of flashes of strongly actinic light emerging from a small aperture at some distance from the front of the glass vessel. In this way the *shadow* of the ball was thrown upon the plate by each successive flash, giving a permanent record of its position at the corresponding instant. In order to secure a succession of flashes at equal intervals of time, twelve equidistant radial slits were cut round the circumference of a disk, 20 centim. in diameter, kept revolving at a uniform rate ; in the course of a revolution each slit was brought opposite a fixed vertical slit on which a beam of light was concentrated by a short-focus condensing lens. It is clear that this arrangement would give a series of images of the ball in a vertical line on the plate, but any one image would be fogged by the light from the remaining flashes. To prevent such a result a metal screen having a vertical rectangular opening was placed between the plate and the back of the glass vessel, and the plate was drawn horizontally past this opening so as to expose a fresh surface to each flash. The glass vessel, screen, and plate were inclosed in a wooden case, in the front of which was a circular aperture closed by a Thornton-Pickard photographic shutter to limit the duration of the exposure.

It will be seen that the degree of success attainable in the measurement of the velocity depends in the first place on the constancy of the rate of rotation of the revolving disk and our ability to measure that rate, and in the second place on the sharpness of the images on the developed plate.

The first requirement was satisfied with a very high degree of accuracy by attaching the disk, with its twelve radial slits, to the fly-wheel of the modification of Froment's electromagnetic engine devised by Lord Rayleigh*. The motor was driven by a current from storage-cells rendered intermittent by a tuning-fork interrupter making about 30 complete vibrations per second. The speed of rotation was obtained by means of a counting-wheel geared to an endless screw on the axle of the motor.

In 36 min. 7 secs. the counting-wheel made 197 revolutions. This gives exactly 11 secs. as the time of one revolution. The counting-wheel possessed 45 teeth, and since

* Phil. Trans. clxxiv. pp. 316-321 (1883).

the movement of a single tooth corresponds to one revolution of the fly-wheel, the period of rotation of the latter is $\frac{1}{4\frac{1}{5}}$ sec. Hence the interval of time between two successive flashes is $\frac{1}{12} \times \frac{1}{4\frac{1}{5}}$ sec. or ·02037 sec., very nearly $\frac{1}{50}$ sec.

The second requirement, which is far more difficult to meet, is that the image of the ball should show a sharply-defined outline. To ensure this result the light should issue from as small an opening as possible so as to avoid the formation of a penumbra or region of partial illumination, and at the same time the duration of the flash should be so short that no appreciable motion of the ball takes place. In practice this means that both the fixed and rotating slits should be as narrow as possible.

On the other hand, if the slits are made too narrow the amount of light transmitted will be insufficient to give a developable image.

As the work proceeded considerable improvements were made in order to overcome these difficulties, and the later results show much greater sharpness than the earlier ones, although in the later photographs the balls were moving with greater velocities.

Several sources of light were experimented with, but that which proved efficient, with the additional advantage of simplicity, was a strip of burning magnesium ribbon held in position by a fixed clip.

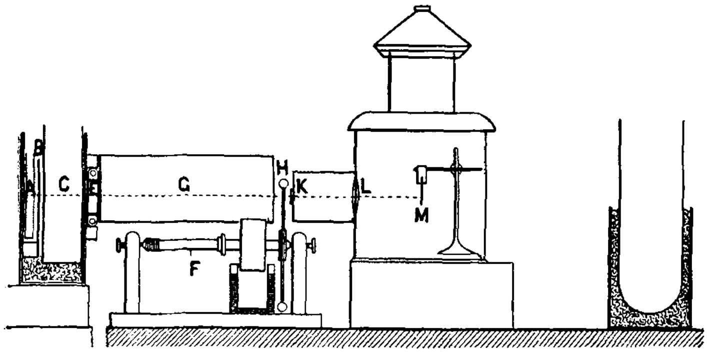
The plates used were Cadett Lightning, developed with hydroquinone and intensified when necessary.

In order to find the actual and not merely the relative velocities of the falling spheres, it is necessary to determine the relation between the distances measured on the photographic plate and the distances traversed by the spheres in the liquid. This was done by photographing a glass scale suspended in the liquid in the same vertical plane as that passed through by the spheres in their descent. Then the actual velocity of a sphere must bear to the velocity obtained by direct measurement of the plate the same ratio as the distance between two fixed points on the scale bears to the distance between their images.

The plates were measured by means of a travelling microscope, the vernier of which read correctly to the tenth of a millimetre.

The main features of the photographic apparatus are shown in section in the diagram, drawn approximately to a scale of $\frac{1}{12}$. The light produced by burning a strip of magnesium ribbon M, inside the lantern, is brought to a focus by the lens L on an adjustable slit K. It passes through one of the slits in the revolving disk H, driven by the Froment's engine F, and then in succession through the cardboard shade G, the Thornton-Pickard shutter E, the glass vessel C, and a rectangular aperture in the screen B on to the plate at A.

Fig. 1.



Method of Release.

In order to obtain undisturbed motion through the liquid, it is clear that the sphere must be released beneath the surface. Worthington and Cole* have shown that when a sphere is allowed to fall from air into water, a disturbance of a large body of water takes place and the sphere carries down with it a bubble of air. It therefore became necessary to devise some method of instantaneously releasing the ball under water without giving rise to any appreciable disturbance of the liquid, and without communicating any velocity of translation or rotation to the ball. After trying some mechanical devices, it appeared that by far the simplest method was to suspend the spheres by a small straight electromagnet. As this involved the use of magnetic material for the spheres, it was decided to experiment with bicycle bearing-balls. These could be obtained in six different sizes, with diameters ranging from about 0·3 centim. to 0·8 centim., and careful measurements showed extremely small divergence from true sphericity.

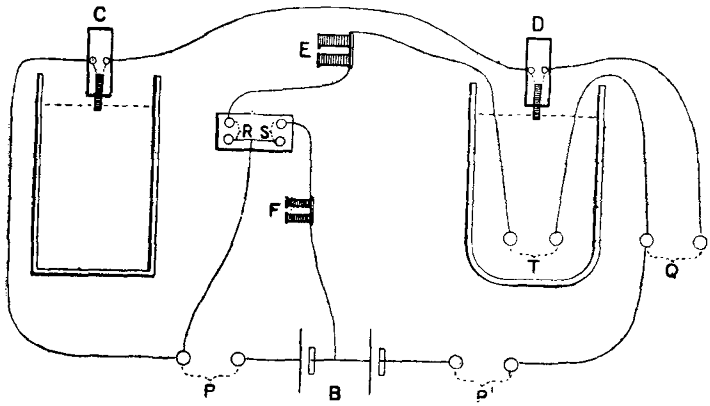
The following sketch shows diagrammatically the method employed for releasing the shutter and the falling weight which gives the plate its horizontal motion, at appropriate instants of time after the ball to be photographed has commenced falling. The release is effected by a second ball dropping in another vessel on to a light platform of aluminium

* Phil. Trans. clxxxix. (A), pp. 137–148 (1897).

foil capable of rotation about a horizontal axis. The depression of this platform makes contact between two mercury cups at T.

The method of procedure is as follows :—The trap at T is adjusted, the rocking-key is placed so as to give contact at R,

Fig. 2.—Diagrammatic View showing the Method of Release.



B is the battery comprising a couple of two-volt storage-cells.

C is the small electromagnet by which the ball to be photographed is suspended.

D is an electromagnet similar to C by which the ball working the release is suspended.

E is a larger electromagnet which releases the weight for moving the plate horizontally.

F is an electromagnet for releasing the Thornton-Pickard shutter.

P, P′, Q are mercury keys; R, S is a rocking-key; and T is a trap which makes contact when the ball falls from D.

and contact is made at P, P′, and Q. The balls are suspended from the electromagnets at C and D, through which the current now runs. The room having been darkened, the photographic plate is inserted in its sheath, and the strip of magnesium ribbon is ignited. When it is well alight contact is destroyed at Q, and the exposure proceeds automatically. For breaking contact at Q sets free the balls simultaneously; the ball falling from D makes contact at T, and a current is sent through the electromagnet E so that the suspended weight is released. This weight in its fall strikes the rocking key so as momentarily to give contact at S instead of R. The current of one cell now passes through P, S, and F, and the Thornton-Pickard shutter is opened.

When the correct height of D above T has been determined for some one position of C, we have only to raise or lower C and D by the same amount to ensure a successful exposure for any desired fall. In practice failures sometimes occurred, mainly in consequence of the contact at T having to be made under water, the mercury surfaces there being easily contaminated.

The general appearance of the apparatus may be gathered from the figure (Plate I. fig. 1). The vessel in which the fall took place may be seen on the extreme left. In order to prevent the transmission of vibrations to the liquid from the electromagnetic engine (which cannot be seen in the figure), this vessel rested on a stand separated from the table supporting the rest of the apparatus. The aluminium shutter is partly visible between the front of the vessel and the cardboard shade. The inside of this shade, as well as other parts of the apparatus exposed to the light, was painted a dull black. Immediately in front of the lantern are the two storage-cells for working the release. On the right of the picture is the vessel in which the timing sphere falls. The falling weight and its release are on the side of the table not shown in the figure.

The experiments were carried out in a cellar in the Cavendish Laboratory.

Results.

An inspection of the reproductions in Plates I. & II. will show the character of the photographs obtained. Fig. 1 (Pl. II.) in which a glass scale is photographed, shows the actual size of the aperture in the screen. Special care was taken that the upper and lower edges of this aperture should be sharply defined and horizontal, so that they might serve as fixed lines from which to measure the vertical displacements of the spheres. Figs. 2–3 (Pl. I.) & 2–4 (Pl. II.) illustrate the manner in which successive images of the aperture are formed on the moving plate. It will be noticed that at least three flashes occurred during the time the shutter remained open. Since the shutter opened from above downwards, the lower part of the first image and the upper part of the third are generally cut off. In some of the photographs the horizontal velocity of the plate has not been great enough to separate the images completely.

When the falling sphere happens to be in the path of the beam of light, we obtain two or more images of the shadow. An examination of one of these images will show that the opposite edges are most sharply defined in a direction sloping downwards from left to right. This is a consequence of the finite duration of a flash. The direction of the sharp edge is that of the velocity compounded of the separate velocities of the ball and the plate.

9. *Accelerated Motion of the Falling Sphere.*

We may conveniently consider the motion of a sphere falling in a viscous fluid to be divided into two stages. In the first stage the sphere is moving with continually diminishing acceleration; in the second it is moving with constant "terminal" velocity. Theoretically the second stage is reached only after the lapse of an infinite time, during which the limiting value is approached asymptotically; practically this stage is reached after a very short time.

Measurement of the distance between two images on a plate during the accelerated motion gives the average velocity of the sphere in moving from one position to the other. This average velocity is the velocity at the middle of the time between the two positions, and therefore, except at the very beginning of the motion, is nearly the same as the velocity at the point of space halfway between the two positions.

If a series of photographs of the same sphere at different depths is obtained, we have the means of plotting a curve showing the velocity attained after falling through any height. Such a curve is drawn for the smallest ball, radius ·1590 centim., falling in a vessel 11·5 centim. long and 3 centim. wide, in fig. 3. The result would be more regular if greater care had been taken in measuring the height of fall; when these photographs were taken this was only done roughly to serve as a check in determining whether the terminal velocity had been attained.

In the same diagram is shown, as far as limits of space allow, a portion of the corresponding curve for the fall of a body *in vacuo*. Comparison of these two curves shows how effective is the resistance of the fluid in destroying the acceleration of the sphere. The velocity of the sphere has become practically constant after a fall of 20 centim.

Fluid Resistance in Accelerated Motion.

In the case of the smallest ball used, the determinations of velocity at different depths were sufficiently numerous to enable a curve to be drawn showing the relation between the velocity and the fall. The slope of this curve at any point gives the value of $\frac{dV}{ds}$ at the corresponding depth, and so the value of the acceleration $V\frac{dV}{ds}$ may be found.

The ratio of this acceleration to the acceleration of a body falling *in vacuo* is the same as the ratio of the effective force on the sphere to its weight.

$$\therefore \frac{f}{g}=\frac{\mathrm{W}-\mathrm{R}}{\mathrm{W}},$$

$$\therefore \mathrm{R}=\left(1-\frac{f}{g}\right)\mathrm{W},$$

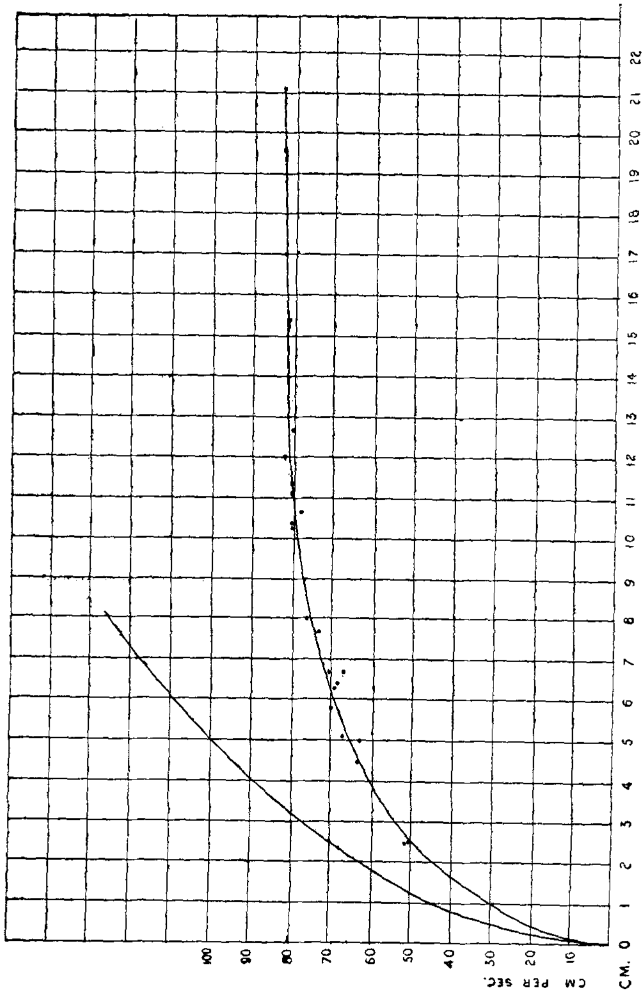


Fig. 3.—Fall of a Steel Ball in Water.

where R represents the resultant upward force due to the pressure of the fluid. If we subtract from R the weight of fluid displaced by the sphere, we obtain the force due to the motion through the liquid. This will include at least two terms, one arising from the motion in a viscous fluid with velocity V, and the other involving the acceleration of the sphere. As we have no means of separating the effects, we cannot in this way deduce the resistance experienced by a sphere moving with constant velocity through a viscous fluid.

The following values of R were calculated from the curve:—

TABLE VI.

S, cm.	V, cm./sec.	f, cm./sec.^{2}	R.	$R - \frac{\rho}{\sigma} W.$
3	54·0	415	$\frac{566}{981}$ W.	$\frac{441}{981}$ W.
5	65·2	294	$\frac{687}{981}$ W.	$\frac{562}{981}$ W.
7	72·2	227	$\frac{754}{981}$ W.	$\frac{629}{981}$ W.
9	77·5	151	$\frac{830}{981}$ W.	$\frac{705}{981}$ W.
11	80·2	75	$\frac{906}{981}$ W.	$\frac{781}{981}$ W.
Terminal.	83·0	0	W.	$\frac{856}{981}$ W.

The rate at which the acceleration falls off increases suddenly when the velocity is about 75 cm./sec. This would seem to show that the law of resistance to the *steady* motion of the sphere employed undergoes a sudden change for this particular velocity.

10. *Terminal Velocities of Steel Balls.*

In the case of the two smallest balls, the depth of the glass vessel was great enough to allow of the terminal velocity being attained. But for the larger balls it was found necessary to increase the possible height of fall. This was done by allowing the ball to fall through a vertical glass tube filled with water and having its open end beneath the surface of the water in the vessel. In order to suspend the ball from the electromagnet hermetically fitted into the top of the tube, and then fill the latter with water, a circular opening was made in the side of the tube close to the top. Through this opening the ball was introduced. Then the aperture was closed by a rubber bung, through which passed a short tubulure for the purpose of filling the tube by suction

A similar tube for increasing the height of fall was provided for the timing-sphere.

In Plate I. fig. 1, one of these two tubes may be seen supported by a retort-stand, on the extreme left of the picture.

The dimensions of the glass vessel first used were as follows:—length 11·5 centim., width (from front to back) 3 centim., depth 28 centim., all internal measurements. The internal diameter of the tube used to produce a higher fall was 2·4 centim.

An increase in the height of fall from 34 centim. to 46 centim. produced an increase in the velocity of the largest ball of less than 1 per cent. It was therefore assumed that with a fall of 45 centim. (the vertical distance from the electromagnet to the top of the rectangular aperture) all the balls would have practically attained their terminal velocity. The results are given in Table VII., but it should be noted that the velocities of balls V. and VI. were obtained from falls of 23·9 and 21·2 centim., respectively.

The dimensions of the balls are given later in Table VIII.

Table VII.

Steel Balls in Water.—Small Vessel.

	Velocity V.	log V.	log a V.	Temperature.
	cm./sec.			
I.	128·0	2·1073	1·7051	11°·2 C.
II.	121·6	2·0849	1·5864	14·6 C.
III.	116·5	2·0665	1·5114	14·6 C.
IV.	102·3	2·0100	1·3864	14·0 C.
V.	95·4	1·9797	1·2793	18·9 C.
VI.	83·1	1·9194	1·1208	17·8 C.

The object aimed at in the experiments was to approach, as far as practicable, the ideal case of a sphere falling in a fluid of infinite extent. The width of the glass vessel in the experiments already quoted was only four times the diameter of the largest sphere. It might, therefore, fairly be presumed that the walls of the vessel would have considerable influence on the motion of the fluid, and consequently on the velocity of the sphere. In order to test this point, a larger vessel was constructed 11·5 centim. long, 6 centim. wide (from back to front), and 21 centim. deep. A wider fall-tube was also made 4·5 centim. in diameter. The vessel was connected by a siphon with a large vessel of water, so as to keep the water-level nearly unaltered when this tube was filled by suction.

Table VIII.

Steel Balls in Water.—Large Vessel.

	Weight W.	log W.	Radius a.	log a.	Velocity V.	log V.	log aV.	Temp.
	gm.		cm.		cm./sec.			
I....	2·010	·3034	·3961	$\bar{1}$·5978	133·2	2·1245	1·7223	10°·8 C.
II....	1·036	·0153	·3173	$\bar{1}$·5015	126·3	2·1016	1·6031	11°·7 C.
III....	0·7006	$\bar{1}$·8455	·2786	$\bar{1}$·4449	120·5	2·0808	1·5257	12°·6 C.
IV....	0·4354	$\bar{1}$·6389	·2379	$\bar{1}$·3764	110·5	2·0432	1·4196	12°·4 C.
V....	0·2542	$\bar{1}$·4051	·1993	$\bar{1}$·2996	100·5	2·0020	1·3016	11°·4 C.
VI....	0·1316	$\bar{1}$·1192	·1590	$\bar{1}$·2014	90·9	1·9586	1·1600	11°·5 C.

The observed velocities are given in Table VIII., which also contains the radius and weight of each ball.

These velocities were all obtained from a fall of more than 34 centim.

A comparison of these results with those already given in Table VII. for the same balls falling in a smaller vessel, shows that the effect of increasing the width of the vessel from 3 centim. to 6 centim. is to increase the velocity by only about 4 per cent. Hence we may fairly conclude that in the larger vessel the circumstances attending the fall do not differ in any material respect from those in an infinite fluid for a corresponding fall, and that even if the velocities could be still further increased by increasing the size of the vessel, the manner in which the velocity depends on the size of the sphere would not be affected.

11. *Law of Resistance.*

In order to determine from these results the relation between the resistance and the velocity of the sphere, recourse was had to the method of logarithmic coordinates. It has been shown that if the resistance can be represented by a single term, it must be proportional to $(a\mathrm{V})^n$.

The values of log aV were calculated and employed as ordinates, while the abscissæ were given by the values of log W, since for spheres of the same density the resistance is proportional to the weight. It was then found that save for the largest sphere the points lay almost exactly on a straight line (fig 4)*.

The slope of the straight line on which the observed points fall determines the value of n. The straight line drawn in the diagram is that line passing through the point VI., for which $n=2$. Hence it appears that *the resistance is proportional to the square of the velocity.*

Referring to § 3 we obtain

$$R=k\rho a^2V^2,$$

indicating that the resistance to the steady motion of a sphere with velocity V is independent of the viscosity of the fluid.

It should be borne in mind that this does not imply that equal spheres moving with the same velocity in two liquids, one of great the other of small viscosity, necessarily experience the same resistance. For in the more viscous liquid the sphere will require a greater velocity before the régime indicated by the above law can be entered upon.

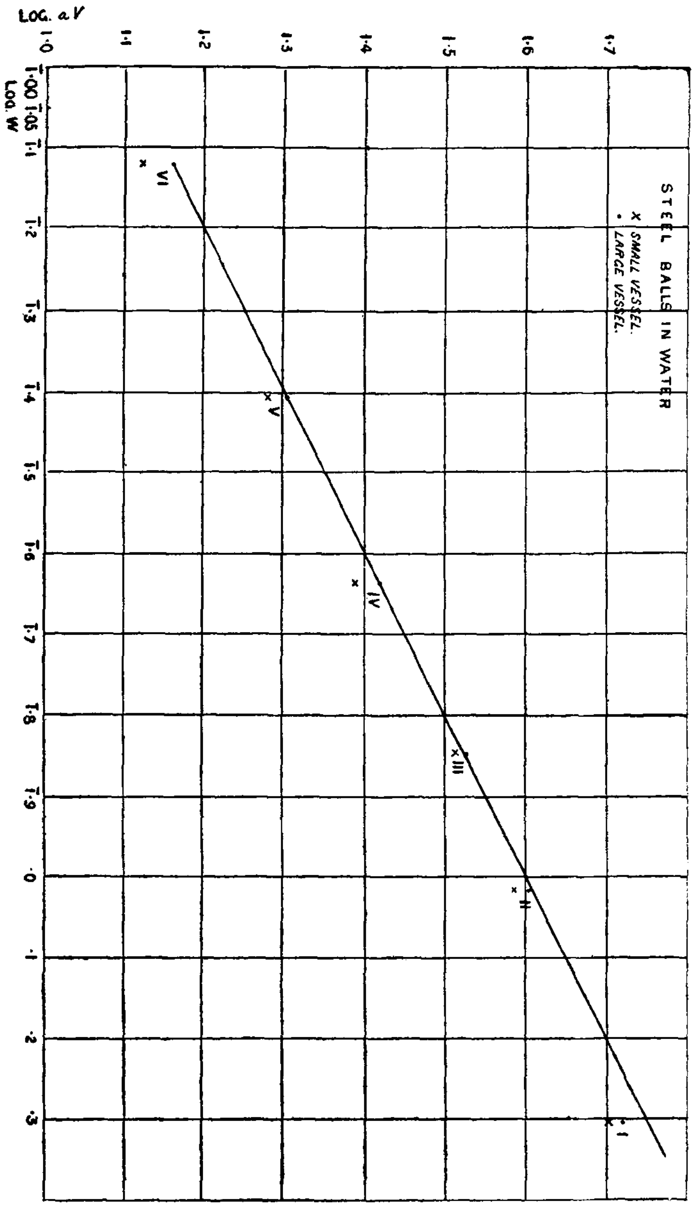
The exceptional case of the largest sphere presents some difficulty. The observed values of $\log aV$ fall short of those required by the assumption of a resistance proportional to the square of the velocity. It is scarcely possible to suppose that any higher power than the square could be involved, for this would necessitate a resistance *decreasing* with increasing viscosity.

Perhaps the simplest explanation is to be found in the supposition that the influence of the walls of the fall-tube and vessel has become appreciable in the case of this sphere, which possesses the greatest diameter and the greatest terminal velocity.

In this connexion it is interesting to recall a remark of Sir I. Newton in a discussion of experiments on the resistance experienced by a pendulum oscillating in water. "I found (which will perhaps seem strange) that the resistance in the water was augmented in more than a duplicate ratio of the velocity. In searching after the cause I thought upon this, that the vessel was too narrow for the magnitude of the pendulous globe, and by its narrowness obstructed the motion of the water as it yielded to the oscillating globe" (Mathematical Principles of Natural Philosophy, Book II. Sect. vi.).

Assuming that $n=2$ we can determine the value of the

* In the diagram (fig. 4) observations in the small vessel are indicated by crosses, those in the large vessel by points. The latter are the more reliable (see § 8).


LOG. aV
1·0
1·1
1·2
1·3
1·4
1·5
1·6
1·7
LOG. W
1̄·00 1̄·05 1̄·1
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1̄·4
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1̄·8
1̄·9
0·
·1
·2
·3
STEEL BALLS IN WATER
× SMALL VESSEL.
• LARGE VESSEL.
VI
V
IV
III
II
I

Fig. 4.

constant k in the relation

$$R = k\rho a^2 V^2 ;$$

for if W is the actual weight of a sphere,

$$R = \frac{\sigma - \rho}{\sigma} W,$$

and so

$$\log k = \log W + \log \left(\frac{\sigma - 1}{\sigma} \right) - 2 \log aV.$$

The mean density of the steel balls was found to be 7·731.

The values of k for the spheres, excepting the largest, are the following :—

II.	$5{\cdot}61 \times 10^{-4}$.
III.	$5{\cdot}42 \times 10^{-4}$.
IV.	$5{\cdot}49 \times 10^{-4}$.
V.	$5{\cdot}52 \times 10^{-4}$.
VI.	$5{\cdot}48 \times 10^{-4}$.
Mean	$5{\cdot}50 \times 10^{-4}$.

The formula giving the terminal velocity will be

$$k\rho a^2 V^2 = \tfrac{4}{3}\pi g(\sigma - \rho) a^3$$

$$V^2 = \frac{1}{k} \cdot \tfrac{4}{3}\pi g \frac{\sigma - \rho}{\rho} \cdot a.$$

R. S. Woodward * has made a preliminary series of experiments on metallic spheres falling in water. Spheres of steel, silver, aluminium, and platinum were dropped in a tube of water 16 feet long and 1 foot in diameter. The spheres varied in diameter from one inch to two inches. All the spheres acquired a constant velocity inside of the first metre. Newton's law † that resistance to motion is proportional to the square of velocity seemed to be verified. The times of falling were determined with a chronoscope.

No further details have yet been published.

* Trans. New York Acad. Sci. xv. p. 2 (1895).

† "But, yet, that the resistance of bodies is in the ratio of the velocity, is more a mathematical hypothesis than a physical one. In mediums void of all tenacity, the resistances made to bodies are in the duplicate ratio of the velocities. For by the action of a swifter body, a greater motion in proportion to a greater velocity is communicated to the same quantity of the medium in a less time; and in an equal time, by reason of a greater quantity of the disturbed medium, a motion is communicated in the duplicate ratio greater; and the resistance is as the motion communicated."—Newton, 'Mathematical Principles of Natural Philosophy,' Book II. Scholium.

Newton's own experiments on the resistance experienced by falling spheres are described in Sect. VII. of the same Book.

12. *Fall of an Oiled Sphere.*

The photograph reproduced in Plate II. fig. 4 is of special interest since it shows the fall through water of a sphere oiled with Rangoon oil. It will be noticed that the greater portion of the oil has collected on the upper surface of the sphere, forming as it were a "tail" that follows the sphere in its downward motion. The velocity of the oiled sphere found from this photograph was 114·5 centim. per sec., as compared with 120·5 centim. per sec. for the same sphere unoiled. The effect of oiling the sphere has therefore been to reduce the velocity by 5 per cent.

No sensible change has been produced in the diameter of the image, so that practically all the oil must be collected in the tail. The volume of the tail is roughly estimated at ·0034 cub. centim.: the total volume of oil descending with the sphere must certainly be less than ·005 cub. centim. The approximate weight of the ball is ·70 grm., and its volume ·090 cub. centim., the density being about 7·8. The mean density of the oiled sphere, taking the upper limit for the volume of the oil, would be 7·2. The change in density would therefore be sufficient to account for the observed reduction in the velocity, so that it is not necessary to assume any change in the general character of the fluid motion.

13. *Summary and Conclusion.*

The experiments described in this paper have had for their object the measurement of the terminal velocity attained by a spherical body falling freely in a viscous fluid. From these measurements it was desired to deduce the law of resistance, and also to obtain information as to the existence of "slipping" at the boundary of a fluid mass moving in a fluid of different density.

With regard to the latter point, the experiments made on the velocity of ascent of small air-bubbles in water and aniline, show that the velocity acquired is the same as would be attained by a solid sphere of corresponding density and dimensions. No appreciable slipping has been detected in the case of a solid in contact with a liquid. We may therefore extend this conclusion to the case of two different fluids in contact.

It has also been shown that the law of resistance to the motion of a sphere moving with constant velocity in a viscous fluid depends on the magnitude of that velocity. Three distinct stages have been recognized.

(1) When the velocity is sufficiently small the motion

agrees with that deduced theoretically by Stokes for non-sinuous motion on the assumption that no slipping occurs at the boundary. In such motion the resistance is proportional to the velocity.

(2) When the velocity is greater than a definite critical value, the terminal velocity of small bubbles and solid spheres is proportional to the radius less a small constant; it may be expressed by the formula

$$V = k\left(\frac{\rho - \sigma \, . \, g}{\rho}\right)^{\frac{2}{3}} \frac{a - h\bar{a}}{\nu^{\frac{1}{3}}},$$

where $k = \frac{1}{2}$ and $h = \frac{2}{5}$ approximately.

This would indicate a resistance varying as the velocity raised to the power of three halves.

(3) For velocities considerably greater than those just considered the law of resistance is that which Sir I. Newton deduced from his experiments, namely, that the resistance is proportional to the square of the velocity. The resistance is, in fact, given by the expression

$$R = k\rho a^2 V^2.$$

In discussing the resistance during the accelerated part of the motion, we found indications of a somewhat sudden change in the law of resistance to steady motion. This change may correspond to the passage from stage 2 to stage 3 above.

In conclusion I beg to express my thanks to Professor J. J. Thomson for placing the resources of the Cavendish Laboratory at my disposal, and for his suggestions and advice in the course of the work.

Trinity College, Cambridge.

LI. *Notices respecting New Books.*

Grundriss der Allgemeinen Chemie. Von W. Ostwald. Leipzig: Wilhelm Engelmann, 1899. Pp. xvi + 549.

THE appearance of a new and revised edition of this standard text-book will be welcomed by all students interested in the subject. It is a masterly exposition by one who has himself contributed in no small measure to the development of this comparatively recent branch of science.

The book is divided into two parts. Part I. is subdivided into six sections, each section consisting of several chapters. Section 1 deals with the fundamental laws of physical chemistry, the elements and their laws of combination, and the periodic law. Section 2 is devoted to the laws of gases and the kinetic theory. In section 3 are considered the general properties of liquids, the phenomena of evaporation and condensation, the thermodynamics of liquids, their behaviour with respect to light, and the phenomena of surface-tension and viscosity. Section 4 deals with solids, crystallization,

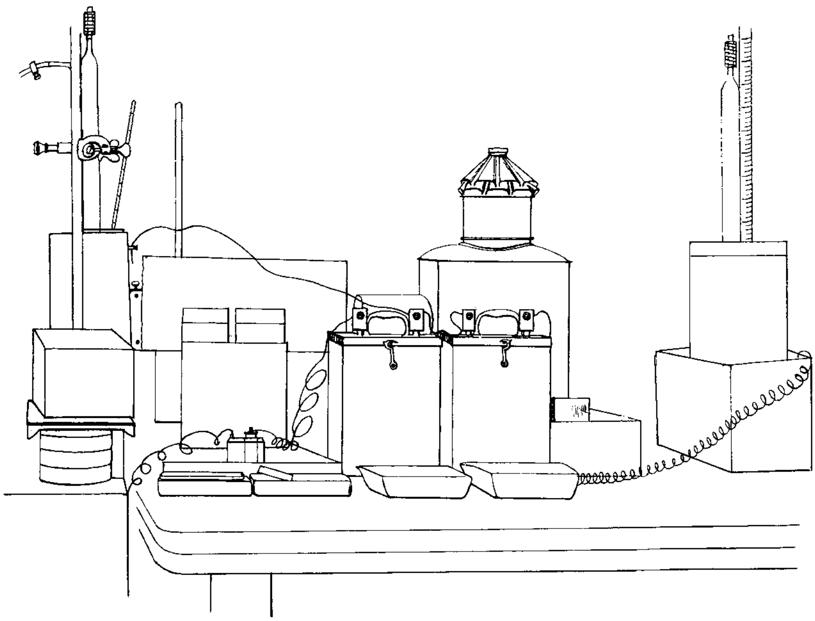


Fig. 1.

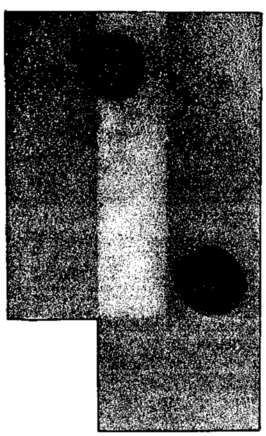


Fig. 2.

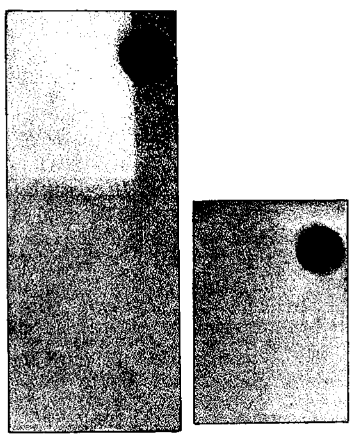


Fig. 3.

Mintern Bros. lith.

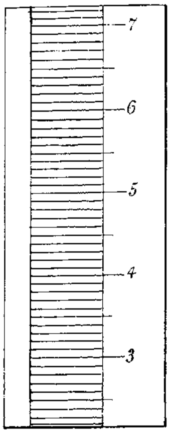


Fig. 1.

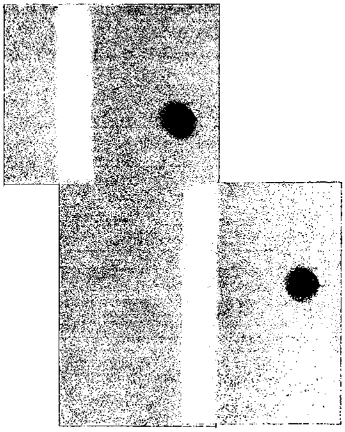
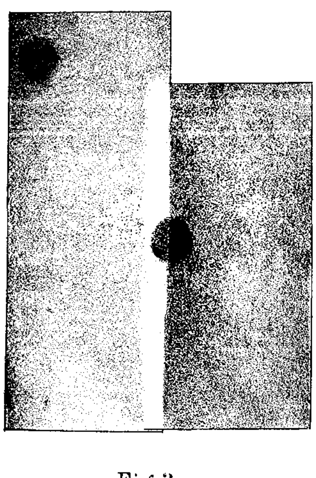


Fig. 2.



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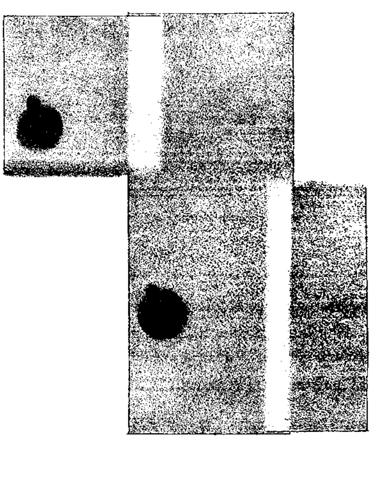

Fig. 4.

Mintern Bros. lith.