EXPERIMENTS ON THE STRENGTH AND STIFFNESS OF SMALL SPRUCE BEAMS.

By F. E. KIDDER, B.C.E.

Read at the American Academy of Arts and Sciences, by Prof. Chas. R. Cross,
S.B., February 9, 1881.

The following experiments were made in the physical laboratory of the Massachusetts Institute of Technology, being a part of the fourth year work in the course in architecture.

The object of the experiments was to determine the moduli of elasticity and of rupture in small beams of white spruce (*Abies alba*), and such other information as might be derived from the data obtained.

The machine used for the purpose consists of two solid wooden frames, carefully leveled and placed 40 inches apart. Upon the top of each frame is placed a movable plate of iron, which is carefully adjusted, so that the two plates shall be directly opposite each other, and exactly 40 inches apart between the faces. These plates form the supports for the beams.

The loads were applied by means of a scale-pan suspended from a $\frac{3}{4}$ -inch bolt, which rested upon the centre of the beam. By means of an iron strap suspended from a horizontal beam placed above the test piece, and resting on two screws, the bolt from which the load was suspended, could be raised from or lowered upon the test piece as easily and gradually as could be desired.

The deflections of the beams were measured by means of a micrometer screw reading to $\frac{25}{10000}$ of a millimetre, or $\frac{1}{10000}$ of an inch. As the bolt from which the load was suspended rested on the centre of the beam, it was necessary to measure the deflections at a distance of one inch from the centre, but the deflections used in calculating the values of the modulus of elasticity were corrected, so as to give the deflection at the centre, supposing the curve assumed by the beam to be the arc of a circle, from which, in fact, it deviates but little under such small loads. In reading the micrometer, the principle of electrical contact was taken advantage of.

The chances for error in using the machine are as follows:

In measuring the deflections $\frac{1}{10000}$ of an inch. In the breaking load, possibly 1 pound; but in the small loads there could be no appreciable error. In measuring the dimensions of the test pieces $\frac{2}{1000}$ of an inch.

The experiments were conducted with the utmost care, and every possible precaution taken to prevent errors.

In arranging for the experiments, and while making them, the writer was greatly assisted by Mr. Holman, of the Massachusetts Institute of Technology, to whom he extends hearty thanks.

TABLE I.

No. of test piece.	Clear span. <i>l</i> .	Breadth. <i>B</i> .	Depth. <i>D</i> .	<i>E</i>	<i>E</i> ₁	<i>R</i>	Centre break- ing w't for beam. $\frac{1}{16} \times \frac{1}{16} \times \frac{1}{16} \times A$.
	Ins.	Ins.	Ins.	Lbs.	Lbs.	Lbs.	Lbs.
1	40	1.475	1.45	1,731,000	1,657,000	11,380	632
2	40	1.445	1.52	1,556,000	1,528,000	10,330	574
3	40	1.469	1.448	1,765,000	1,732,000	10,710	595
4	40	1.42	1.498	1,736,000	1,636,000	10,830	601
5	40	1.45	1.485	1,688,000	1,578,000	11,980	665
6	40	1.48	1.44	1,795,000	1,686,000	11,040	613
7	40	1.464	1.46	1,682,000	1,561,000	10,570	587
8	40	1.42	1.48	1,647,000	1,556,000	11,280	626
9	40	1.46	1.46	1,704,000	1,638,000	11,180	621
10	40	1.441	1.46	1,616,000	1,550,000	12,440	691

Average value of *E*, 1,692,000 lbs.; of *E*₁, 1,612,000 lbs.

“ “ *R*, 12,170 lbs.; of *A*, 620 lbs.

Weight of cubic foot, 25.7 lbs.

The pieces of wood experimented on were sawn from a spruce plank that had been cut in Eastern Maine in the spring of 1880, and the following summer shipped to Boston, where it had lain in the open air until it was cut up in October. The pieces were carefully planed to an approximate size of $1\frac{1}{2}$ inches square, and 4 feet long.

They were nearly all straight grained, and had but few defects, and

in testing the beams they were placed so that the defects should have the least possible effect upon the strength of the beams.

The exact dimensions of the test pieces are given in Table I.

In making the experiments, each beam was first subjected to a load of 30 pounds and the deflection noted. The weight was then left on the beam for a period of time varying from 1 to 4 hours, and in one case 44 hours, and the deflection again noted. The load was then removed from the beam, and the set noted, after which the beam was allowed a certain time to recover from the set.

After the piece had returned, or at least nearly returned, to its original position, it was subjected to a load of 40 pounds in the same manner.

Table II gives the deflections of each piece under the loads of 30 and 40 pounds, both immediately after the weight was applied, and after it had rested upon the beam the length of time designated.

The value of the modulus of elasticity, calculated from these deflections, is also given.

Moduli of elasticity, obtained from the deflection of the beams immediately after the weight was applied, have been denoted by E , and those obtained from the deflection of the pieces, after the weight had been applied one or more hours, by E_1 .

Table I gives the values of E and E_1 , for each piece, obtained by taking the average of the values given in Table II.

The values of E were computed by the formula $E = \frac{Wl^3}{4\Delta BD^3}$, in

which W denotes the weight in pounds producing the deflection; the clear span in inches; Δ the deflection of the beam at the centre B the breadth of the beam, and D the depth, both in inches.

After all the beams had been tested in this way, piece No. 3 was again put in the machine and subjected to a load of 100 pounds, which was allowed to remain upon the beam for about two hours, the deflection being measured directly after the weight was applied, and just before it was removed. The beam was then allowed a certain time to recover its set. In two cases the beams, after having been subjected to a load of 100 pounds, finally returned to their original position, and it appeared probable that all would have done so had sufficient time been allowed for the purpose.

After the piece had nearly recovered from the effects of the load of

100 pounds, a load of 150 pounds was put on the beam, and gradually increased until the breaking point was reached.

The remaining pieces were tested with a load of 100 pounds in the same way, and then subjected to a load of 400 pounds for one or two minutes, for the purpose of getting the deflection under that load, and immediately after subjected to the full load of 500 pounds, which was gradually increased until the piece broke. As the load approached the breaking weight, it was increased by the addition of only one or two pounds at a time, so that the breaking weight could be obtained with sufficient accuracy. In fact, the breaking weight is so much modified by the time occupied in breaking the beam that it is difficult to ascertain exactly what it really is. For any load, over three-fourths of what is called the breaking weight, would probably break the beam if applied for a sufficient length of time.

Table I gives the values of the modulus of rupture of each piece, computed by the formula $R = \frac{3}{2} \cdot \frac{Wl}{BD^2}$, in which R denotes the modulus of rupture; W the breaking weight of the beam, and the other letters have the same significance as in the formula for E . The load which would break a beam of the same wood, one inch square, and one foot between supports, if applied at the centre, is also given in the same table. This load is $\frac{1}{8}$ of the modulus of rupture.

When the weight of 400 pounds was applied to piece No. 7, it immediately cracked at a gnarl in one of the lower edges, about three-fourths of an inch from the centre of the beam. As it was thought that the beam would soon break entirely, the load of 400 pounds was allowed to remain on the beam; but at the end of 100 hours the deflection had only increased 0.2224 inch, and, as it was evident that it would at that rate take a long time for the beam to break, the load was then gradually increased until the piece broke at 550 pounds, giving a modulus of rupture considerably above the average. It was noticed in this beam that the deflections under loads above 500 pounds were considerably greater than in the other beams under the same loads.

Piece No. 5 gave a very high breaking weight, and broke very suddenly, more like the harder kinds of wood. The fracture was very perfect; the upper half of the fibres being very evidently compressed, and the lower half suddenly pulled apart, with almost no splintering. This piece had a small knot on the upper side, 5 inches from the centre

of the beam, but it appeared to have no effect upon the strength of the beam.

TABLE II.

No. of test piece.	Weight. W.	Time applied.	Deflection. Δ .	E	Weight. W.	Time applied.	Deflection. Δ	E
	Lbs.	H. M.	Ins.	Lbs	Lbs.	H. M.	Ins.	Lbs.
1	30	0 00	0·0610	1,744,000	40	0 00	0·0826	1,719,000
1	30	2 25	·0630	1,689,000	40	3 30	·0873	1,627,000
2	30	0 00	·0616	1,536,000	40	0 00	·0801	1,576,000
2	30	1 45	·0639	1,481,000	40	3 00	·0842	1,499,000
3	30	0 00	·0606	1,774,000	40	0 00	·0815	1,757,000
3	30	1 00	·0610	1,759,000	40	1 00	·0840	1,705,000
4	30	0 00	·0582	1,725,000	40	0 00	·0765	1,748,000
4	30	2 25	·0616	1,627,000	40	3 20	·0813	1,645,000
5	30	0 00	·0580	1,740,000	40	0 00	·0774	1,637,000
5	30	2 30	·0618	1,630,000	40	44 15	·0874	1,527,000
6	30	0 00	·0605	1,792,000	40	0 00	·0862	1,676,000
6	30	3 00	·0639	1,697,000	40	5 12	·0803	1,799,000
7	30	0 00	·0624	1,683,000	40	0 00	·0833	1,682,000
7	30	3 00	·0663	1,584,000	40	16 00	·0911	1,538,000
8	30	0 00	·0632	1,645,000	40	0 00	·0839	1,650,000
8	30	4 15	·0666	1,561,000	40	16 30	·0894	1,552,000
9	30	0 00	·0619	1,701,000	40	0 00	·0823	1,707,000
9	30	1 30	·0643	1,638,000				
10	30	0 00	·0661	1,614,000	40	0 00	·0881	1,618,000
10	30	2 00	·0691	1,544,000	40	1 15	·0915	1,556,000

Piece No. 4 broke in a rather peculiar manner. While under a load of 575 pounds the lower fibres, for about a depth of one-tenth of an inch, snapped apart, and the beam gradually settled down until the next layer of fibres had apparently the same deflection as had the lower

ones at the time of breaking, when they also snapped, making a layer of about the same thickness. In this way the whole lower half of the beam seemed to divide itself into layers of about one-tenth of an inch thick, and to break separately, under about the same deflection, so that the beam was a long time in breaking.

Observing that under every load that had been applied the deflection kept increasing with the length of time the weight remained on the beam, piece No. 7 was subjected to a load of 275 lbs. for 98 hours, during which time the deflection increased 0.079 in. The weight was then taken off, and the beam allowed to recover for 24 hours, when it had a set of .0446 in. The same weight was again applied, and it was found that the deflection, obtained by taking the difference between the readings just before and after the weight was applied, was less than it was the first time that the weight was applied, and the rate of increase of the deflection was about the same as before. The beam was thus subjected to a load of 275 lbs. for 300 hours in all, and it was then broken in the same manner as the others. It was expected that the effect of such a severe strain on the beam for so long a time would diminish the strength; but, on the contrary, it appeared to increase it, as the beam gave a higher modulus of rupture than any of the others, although it did not appear to be of as good quality as many of them. The ultimate deflection of this beam greatly exceeded any of the others.

Table III shows the deflection of each beam under loads of 30, 40, 100, 400, 500 and 550 lbs. immediately after the load was applied and at a distance of one inch from the centre. The *italic figures* under each deflection show what it would be if Hooke's law held true, taking the deflection under 30 lbs. as the starting point.

From these experiments I think we may draw the following conclusions:

That the modulus of elasticity depends not only upon the elasticity of the material, but also upon the length of time that the load is applied.

That when subjected to loads not exceeding one-sixth of the breaking weight spruce beams do not take a permanent set.

That even under very small loads, if applied for any length of time, there will be a temporary set.

That knots and gnarls in beams loaded at the centre, when not

within one-eighth of the span of the centre of the beam, do not materially affect the elasticity under small loads.

TABLE III.

No. of test piece	DEFLECTIONS IN INCHES, UNDER						Ultimate deflection, in inches.	Breaking Weight.
	30 lbs.	40 lbs.	100 lbs.	400 lbs.	500 lbs.	550 lbs.		
	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.		Lbs.
1	0·0610	0·0826	0·2071	0·8303	1·0906	1·2791	1·5646	588
	·0610	·0813	·2030	·8120	1·0150	1·1165		
2	·0616	·0801	·2043	·8177	1·0677	1·2811	1·3941	575
	·0616	·0820	·2050	·8200	1·0250	1·1275		
3	·0606	·0815	·2023	·8976	1·2764	1·48*	1·48*	550
	·0606	·0808	·2020	·8080	1·0100	1·1111		
4	·0582	·0765	·1929	·7929	1·1146	1·3197	1·4658	575
	·0582	·0776	·1940	·7760	·9700	1·0670		
5	·0580	·0774	·2004	·7876	1·0170	1·1827	1·5788	637
	·0580	·0773	·1933	·7732	·9665	1·0631		
6	·0605	·0803	·2138		1·2520	1·4662		565
	·0605	·0806	·2016		1·0080	1·1088		
7	·0624	·0833	·2083	·8961	1·3595			550
	·0624	·0832	·2080	·8320	1·0400			
8	·0632	·0839	·2102	·8315	1·1111	1·3331	1·5709	585
	·0632	·0842	·2106	·8424	1·0530	1·1583		
9	·0619	·0823	·2083		1·0800	1·2830	1·4254	580
	·0619	·0825	·2063		1·0315	1·1346		
10	·0661	·0884	·2220	·9276	1·1772	1·3775	1·81*	637
	·0661	·0881	·2203	·8812	1·1015	1·2116		

That the deflection is very nearly proportional to the load, far beyond the customary limits of strain, and that the modulus is consequently very nearly constant for all moderate deflections.

* Approximately.

That a high modulus of elasticity does not always accompany high transverse strength. For, as shown by Table I, piece No. 10, which had the greatest transverse strength, gave next to the lowest value of E .

That in spruce beams the upper fibres commence to rupture by compression under about four-fifths of the breaking weight, and the neutral axis, at the time of rupture, is very near the centre of the beam, as shown by the fracture.

That beams which are subjected to severe strains, for a long time, bend more before breaking than those which are broken in a comparatively short time.

That the modulus of elasticity of small spruce beams, of a quality such as is used in the best buildings, may be taken at from 1,600,000 to 1,700,000 lbs., and the modulus of rupture at 11,000 lbs.

The only other experiments on American spruce with which the writer is familiar are those made by Mr. Hatfield, on small beams, 1·6 feet between supports, and some experiments by Mr. Thomas Laslett, of England, on pieces of Canada spruce, 2 inches square and 72 inches between supports.

Mr. Hatfield gives as the average value of the transverse strength of a unit beam 612 lbs.,* which would give 11,016 lbs. for the modulus of rupture.

From data given by Laslett† we obtain as the value of R , 9045 lbs.

The value generally given for the modulus of elasticity of spruce is 1,600,000 lbs.

Length of the Second's Pendulum.—C. S. Peirce has re-examined the estimates of the length of the seconds' pendulum at Paris, by Borda, Biot and Kater. He finds that if allowance is made for the inertia of the air which is dragged by the friction of the pendulum, the estimated length should be increased about $\frac{1}{110}$ of 1 per cent. He therefore regards the true length as 993·934 millimetres (39·132 in.) Faye compliments Peirce's investigation very highly, and he congratulates the French Academy upon the great accuracy of the previous experiments, which were conducted largely under its auspices, when the facilities for minute measurements were not so great as they are now.—*Comptes Rendus*. C.

* Hatfield's "Transverse Strains," Table XLII.

† "Timber and Timber Trees, Native and Foreign," by Thomas Laslett, Inspector to the Admiralty. London, 1875.