429. The Teaching of Numerical Trigonometry

Author(s): H. S. Carslaw

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Let the homogeneous coordinates of P, Q, R, S be

$(1, 1, 1), (-1, 1, 1), (1, -1, 1), (1, 1, -1)$ respectively.

Any conic of the system is $lx^2 + my^2 + nz^2 = 0$ with the condition $l + m + n = 0$. Hence O_1 and O_2 are conjugate if their coordinates are related in the form

$$x_1x_2 = y_1y_2 = z_1z_2.$$

Now the points where the lines joining O_1 to P, Q, R, S cut $x=0$ are given by the four equations :

$$y(x_1 \mp z_1) - z(x_1 \mp y_1) = 0,$$

$$y(x_1 \mp z_1) + z(x_1 \pm y_1) = 0.$$

Hence

$$O_1[PQRS] = \frac{x_1^2 - z_1^2}{x_1^2 - y_1^2}.$$

Again O_2O_1 cuts $x=0$ where $y(x_1z_2 - x_2z_1) = z(x_1y_2 - x_2y_1)$, and O_2A cuts it where $y z_2 = z y_2$.

Hence

$$\begin{aligned} O_2[O_1ABC] &= \frac{x_1z_2 - x_2z_1}{x_1y_2 - x_2y_1} \cdot \frac{y_2}{z_2} = \frac{x_1^2 - z_1^2}{x_1^2 - y_1^2} \\ &= O_1[PQRS]. \end{aligned}$$

It may be noticed that with this notation Dr. Milne's cubic and polar conic have the real equations :

$$\begin{vmatrix} x^2 & y^2 & z^2 \\ xx_1 & yy_1 & zz_1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \quad \text{and} \quad \begin{vmatrix} x^2 & y^2 & z^2 \\ x_1^2 & y_1^2 & z_1^2 \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

F. J. W. WHIPPLE.

429. [v. a. θ .] *The Teaching of Numerical Trigonometry.*

To the *Gazette* of December, 1913, which has just reached me, Mr. Mercer contributes a paper on *The Teaching of Numerical Trigonometry*.

Two points suggested by that paper, I think, require further discussion. First, I hold that the pupils who are learning *Easy Numerical Trigonometry* are quite capable of appreciating a reasoned geometrical proof of the constancy of the ratios; and the experimental method advocated by Mr. Mercer, and others, seems to me, in the circumstances, wholly wrong. Secondly, the tendency to extend the work past the *Easy Numerical Trigonometry of the Right-Angled Triangle*, and to give it the dignity of a separate subject and a text-book of about 150 pages, instead of associating it with Elementary Geometry, Algebra and Arithmetic, is, I think, equally objectionable.

To take the first point: Mr. Mercer proceeds as follows: "... Let every boy draw an angle of 35° , AOB , take any point P in one arm and draw PM perpendicular to the other. Then let him measure carefully OM , MP and work out $\frac{MP}{OM}$ correct to two decimal places. They will all have different lengths for OM and MP , but should obtain the same value for $\frac{MP}{OM}$. This number ($\cdot 70$) is called the tangent of 35° , and a definition is suggested for the tangent of any acute angle. From any point P in one arm of the angle AOB , draw PM perpendicular to the other arm; then $\frac{MP}{OM}$ is called the tangent of the angle. ..."

If it were impossible, early in the school course, to give a reasoned proof that the ratio $\frac{MP}{OM}$ is independent of the position of the point P upon the arm of the angle, then we might, though unwillingly, put up with Mr.

Mercer's argument. But surely, when it is quite a simple matter to treat this question geometrically towards the beginning of the second year's work in Geometry, if not earlier, it is a pity to fall back upon one of these so-called proofs, with which we are becoming too familiar in the mathematical work of the school.

In dealing with parallels, the pupils have learned that if $P_1, P_2, P_3, \dots$ are any points at equal distances along one of two parallel lines (Fig. 1), and P_1p_1, P_2p_2, P_3p_3 , etc., the perpendiculars from $P_1, P_2, P_3, \dots$ to the other line, then $P_1P_2 = p_1p_2 = p_2p_3$, etc.

FIG. 1.

It is natural to examine what is the corresponding result in the case of any two intersecting lines OA and OB , enclosing an acute angle AOB . In this case it is equally easy to show that if $P_1, P_2, P_3, \dots$ are any points on one arm OB , such that $OP_1 = P_1P_2 = P_2P_3$, etc., and P_1p_1, P_2p_2, P_3p_3 , etc., the perpendiculars from $P_1, P_2, P_3, \dots$ to the other arm (Fig. 2), then $Op_1 = p_1p_2 = p_2p_3$, etc.

FIG. 2.

This theorem could, of course, be stated for any parallel lines cutting off equal intercepts from one of the intersecting lines.

Now let P and Q be any two points on the arm OB , and PM, QN the perpendiculars from P and Q to the other arm OA . We may assume that the segments OP and OQ are commensurable. Thus we may take $OP = m$ units, say equal to $m \cdot OP_1$; and $OQ = n$ units, say equal to $n \cdot OP_1$ (Fig. 3).

FIG. 3.

Then, from the theorem quoted above, we have

$$OM = m \cdot Op_1 \text{ and } ON = n \cdot Op_1, P_1p_1 \text{ being perpendicular to } OA.$$

Therefore

$$\frac{OM}{OP} = \frac{ON}{OQ}.$$

Having established this result, it is a simple matter to treat the trigonometrical ratios—the sine, cosine, and tangent would alone be required in his numerical work. It is unnecessary to take the tangent first. Indeed,

from this point of view, it is more satisfactory to begin with the sine and cosine.

The ratios will be defined in the usual way : Let OAB be any acute angle, and P any point upon one of the arms of the angle, say OB . From P draw the perpendicular PM to the other arm OA . Then the ratios

$$\frac{MP}{OP}, \quad \frac{OM}{OP}, \quad \text{and} \quad \frac{MP}{OM}$$

are called the sine, cosine, and tangent of the angle.

The first thing to do is to prove that these ratios are independent of the position of the point P upon OB .

In the case of the sine, it is only necessary to draw the line through O perpendicular to OA , and from P the perpendicular PN to this line. Then we have $MP=ON$. But, by the geometrical theorem, $\frac{ON}{OP}$ is independent of the position of P upon OB . It follows that $\frac{MP}{OP}$ is also independent of the position of P upon OB .

The corresponding result for the ratio $\frac{OM}{OP}$ follows immediately from the geometrical theorem ; and since

$$\frac{MP}{OM} = \frac{MP}{OP} \div \frac{OM}{OP},$$

the ratio $\frac{MP}{OM}$ is also independent of the position of the point P upon OB .

It is not difficult to go further, and show that, as the angle increases from 0° to 90° , the sine and tangent continually increase, and the cosine continually diminishes. So that every acute angle has a definite sine, cosine, and tangent, which belongs to it and no other acute angle. If the angle is given, the ratios are known. If one of the ratios is given—a number between suitable limits—the angle is known.

In this argument there is nothing beyond the powers of the pupils early in their geometrical course. The experimental method may help in driving the point home ; but the pupils themselves would be the first to admit that a proof was still lacking. Naturally the teacher will treat the different ratios separately, and illustrate each by numerous examples, and a certain amount of practical work. But, in my opinion, the geometrical theorem should be the foundation of the whole theory.*

Space does not permit me to do more than touch upon the second point referred to in my opening sentences. We are all agreed that it is advisable to introduce the *Easy Numerical Trigonometry of the Right-Angled Triangle* into the Secondary School Course quite early. But I submit that this can best be done in conjunction with the elementary work in the other mathematical subjects, Arithmetic, Algebra, and Geometry ; and that it is unnecessary—indeed inadvisable—to extend this treatment further at this stage. The work can be made extremely interesting and instructive ; and, if I may say so, Mr. Mercer's remarks upon the typical examples and illustrations are admirable. But where I disagree with him again—and I refer here to his text-book entitled *Numerical Trigonometry*, for the second part of his paper will not be in my hands yet for some time—is that I do not think it is a good thing at that early stage to cover in this way so large a part of the subject. Plane Trigonometry, as now generally taught, offers

* Cf. *A School Course in Geometry*, by W. J. Dobbs (Longmans' Modern Mathematical Series), Chapter IV. Mr. Nunn's *Teaching of Elementary Algebra (including the Elements of Trigonometry)*, in the same series, has not yet reached me, but I shall be surprised if he does not adopt some such method as I have advocated above. The companion volume, *Exercises in Algebra (including Trigonometry)*, is a most useful book.

little difficulty to the pupils of 15 years and upwards who learn it. No teacher will now fail to give prominence to the numerical work, even at the higher stage. But this work must be, and ought to be, different from that which many of us understand by the *Easy Numerical Trigonometry of the Right-Angled Triangle*. H. S. CARSLAW.

Sydney, Feb. 17, 1914.

430. [K. 1.] The construction (and its justification) to solve the problem, "Given a finite straight line, it is required to cut off therefrom a segment equal to one-seventh of its length,"

may be made to depend entirely upon the first book of Euclid.

Describe any equilateral triangle abc , and along bc set off $bd = (1/3)bc$. Produce ad to D so that AD is equal in length to the given line; and through D draw BDC parallel to bdc , meeting AB , AC produced in B , C respectively. Along CA take $CE = BD$, and let BE intersect AD in Z . Then $ZD = (1/7)AD$.

Along AB take $AF = BD$ or CE , and let CF intersect BE , AD in X , Y respectively.

[Since $2bd = dc$, $2\triangle abd = \triangle adc$ and $2\triangle bbd = \triangle cdc$, wherefore
 $2\triangle abd = \triangle adc$ and $2BD = DC$.]

Contemplate the three-fold symmetry of the figure.

Since $AB = 3AF$, $\triangle ABY = 3\triangle AFY = 3\triangle BDZ$ (of equal sides); hence $AY = 3ZD$. Since $DC = 2BD$, $\triangle AZC = 2\triangle AZB = 2\triangle CYA$ (of equal sides); hence $AZ = 2AY$.

Thus $AY = YZ = 3ZD$ and $ZD = (1/7)AD$.

Generally, if $BD : DC = m : n$, $ZD = \{m^2 / (m^2 + mn + n^2)\}AD$, so that, for instance, if an equilateral triangle was described on AD and distances each $= ZD$ set off along the sides, we could obtain a segment $= 1/43rd$ of a certain line. R. F. DAVIS.

431. [B. 8. a.] Reduction of a ternary cubic to the form

$$X^3 + Y^3 + Z^3 + 6mXYZ$$

by a real transformation.

Prof. Elliott has given a proof of the reduction of a ternary cubic to canonical form in § 229 of his *Algebra of Quantics*, which (in the revised form which recently appeared in the second edition of this book) leaves